Lifetime of mixed phase clouds
(theoretical consideration)

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Phase transformation of mixed-phase clouds

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SUMMARY

The glaciation time of a mixed-phase cloud due to the Wegener–Bergeron–Findeisen mechanism is calculated using an adiabatic one-dimensional numerical model for the cases of zero, ascending, descending and oscillating vertical velocities. The characteristic values of the glaciation time are obtained for different concentrations of ice particles and liquid-water content. Steady state is not possible for the ice-water content/total water content ratio in a uniformly vertically moving mixed-phase parcel. The vertical oscillation of a cloud parcel may result in a periodic evaporation and activation of liquid droplets in the presence of ice particles during infinite time. After a certain time, the average ice-water content and liquid-water content reach a steady state. This phenomenon may explain the existence of long-lived mixed-phase stratiform layers. The obtained results are important for understanding the mechanisms of formation and life cycle of mixed-phase clouds.

KEYWORDS: Glaciation time

1. INTRODUCTION

The saturation vapour pressure over liquid water is higher than that over ice at low temperatures. Therefore, the growth rate of ice particles and liquid droplets under the same conditions will be different. As a result an *adiabatic* colloidal three-phase component system consisting of water vapour, ice particles and liquid droplets becomes condensationally unstable and may exist during a limited period of time. At the final stage, all the liquid droplets will evaporate and the system will consist only of ice particles and water vapour. The glaciation process due to ice growth by deposition at the expense of co-existing liquid droplets is known as the Wegener–Bergeron–Findeisen (hereafter WBF) mechanism (Wegener 1911; Bergeron 1935; Findeisen 1938).

Interactions between ice particles and liquid droplets in mixed-phase clouds have been considered in a large variety of numerical models (e.g. Scott and Hobbs 1977; Hall 1980; Cotton and Anthes 1989; Houze 1992; Reisin *et al.* 1996; Lohmann and Roeckner 1996; Tremblay *et al.* 1996; Harrington *et al.* 1999; Jiang *et al.* 2000; Rotstayn *et al.* 2000; Zawadzki *et al.* 2000 to name a few of them). Most numerical models considering mixed-phase clouds take into account ice growth due to the WBF mechanism. However, the role of the WBF mechanism is in many ways masked by the effects of ice multiplication, riming, aggregation, and terminal fall velocity.

The purpose of this paper is to study the glaciation time of mixed-phase clouds

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Supersaturation of Water Vapor in Clouds

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ABSTRACT

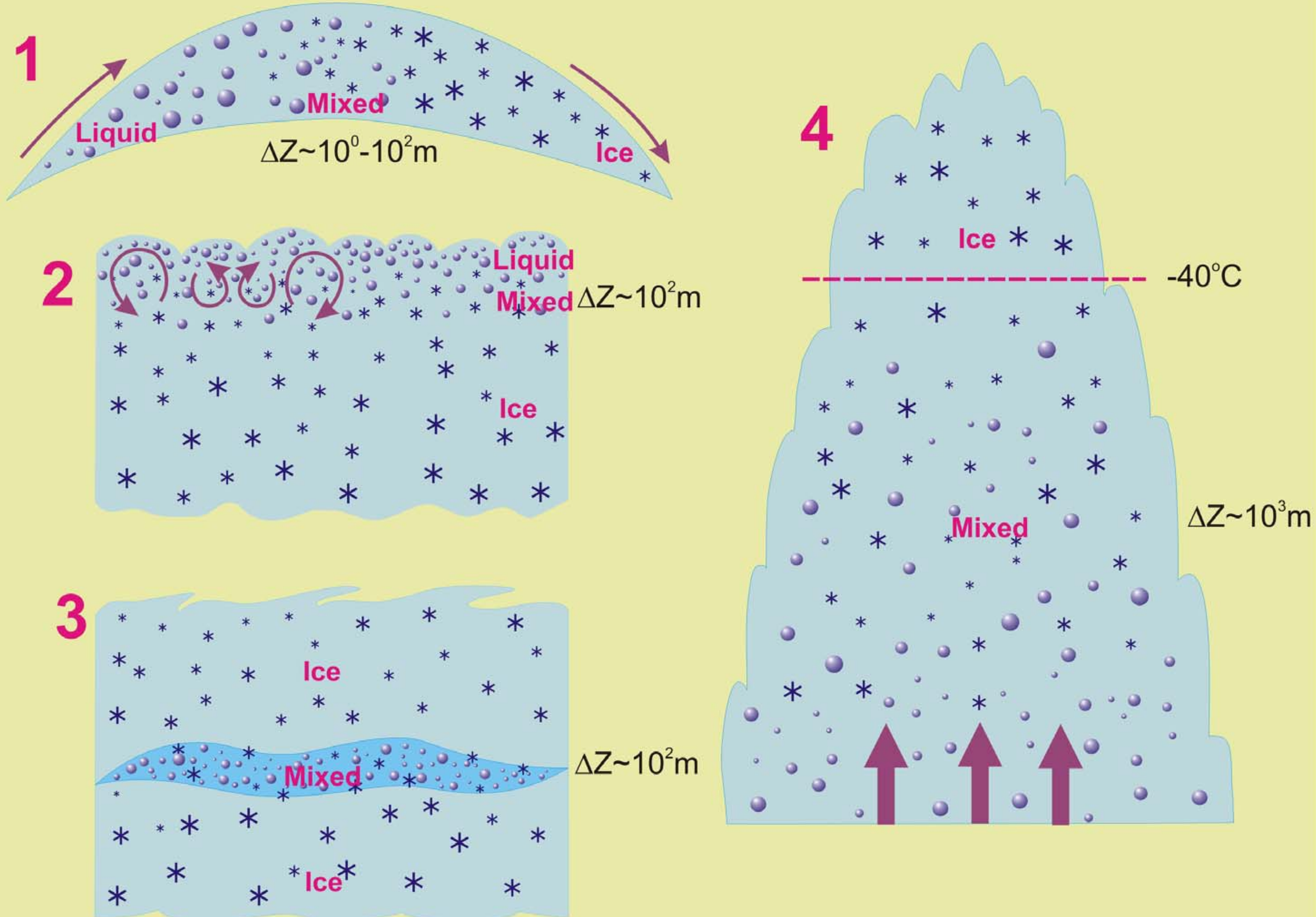
A theoretical framework is developed to estimate the supersaturation in liquid, ice, and mixed-phase clouds. An equation describing supersaturation in mixed-phase clouds in general form is considered here. The solution for this equation is obtained for the case of quasi-steady approximation, that is, when particle sizes stay constant. It is shown that the supersaturation asymptotically approaches the quasi-steady supersaturation over time. This creates a basis for the estimation of the supersaturation in clouds from the quasi-steady supersaturation calculations. The quasi-steady supersaturation is a function of the vertical velocity and size distributions of liquid and ice particles, which can be obtained from in situ measurements. It is shown that, in mixed-phase clouds, the evaporating droplets maintain the water vapor pressure close to saturation over water, which enables the analytical estimation of the time of glaciation of mixed-phase clouds. The limitations of the quasi-steady approximation in clouds with different phase composition are considered here. The role of phase relaxation time, as well as the effect of the characteristic time and spatial scales of turbulent fluctuations, are also discussed.

1. Introduction

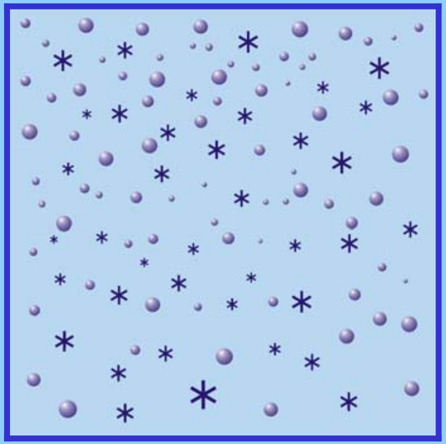
The phase transition of water from vapor into liquid or solid phase plays a pivotal role in the formation of clouds and precipitation. The direction and rate of the phase transition "liquid-to-vapor" or "ice-to-vapor" are determined by the vapor supersaturation with respect to liquid or ice, respectively. The early numerical modeling by Howell (1949) and Mordy (1959) has shown that the supersaturation in a uniformly ascending cloud parcel reaches a maximum near the cloud base during centration and its average size (Squires 1952; Mazin 1966, 1968). Questions relating to the formation of supersaturation in clouds were discussed in great details by Kabanov et al. (1971). Rogers (1975) studied the supersaturation equation for liquid clouds numerically and considered analytical solutions for some special cases. Subsequent refinements of the supersaturation equation and the time of phase relaxation were made by Korolev (1994). Most studies of the supersaturation are related to liquid clouds. Only few works consider supersaturation in glaciated clouds (e.g., Juisto 1971;

Outline

1. Types of mixed phase clouds
2. Cloud glaciation $u_z=0$
3. Cloud glaciation $u_z \neq 0$
4. Effect of the spatial mixed phase inhomogeneity
   on the glaciation time
4. Conclusions



1
Liquid
Mixed
Ice
$\Delta Z \sim 10^0 - 10^2$m
2
Liquid
Mixed
$\Delta Z \sim 10^2$m
Ice
3
Ice
Mixed
$\Delta Z \sim 10^2$m
Ice
4
Ice
-40°C
Mixed
$\Delta Z \sim 10^3$m



Assumptions

1. liquid droplets and ice particles are uniformly distributed in space

2. N_i = const and N_w = const,
   i.e. the cloud particles always stay inside the parcel at all times (no sedimentation) and there is no activation of new cloud droplets and no nucleation of ice particles

3. the water vapor pressure and temperature fields, at large distances from cloud particles, are considered to be uniform and all cloud particles grow or evaporate under the same conditions (T, P, E)

$$\frac{1}{S_w+1}\frac{dS_w}{dt} = a_0 u_z - a_2 B_i^* N_i \bar{r}_i - \left(a_1 B_w N_w \bar{r}_w + a_2 B_i N_i \bar{r}_i\right) S_w$$

the supersaturation in a mixed phase cloud

N_i, r_i concentration and size of ice particles

N_w, r_w concentration and size of droplets

$a_0, a_1, a_2, b_i, b_w, B_i, B_w$ coefficients dependent on T,P,E

$$S_{qs\ w} = \frac{a_0 u_z - b_i^* N_i \bar{r}_i}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

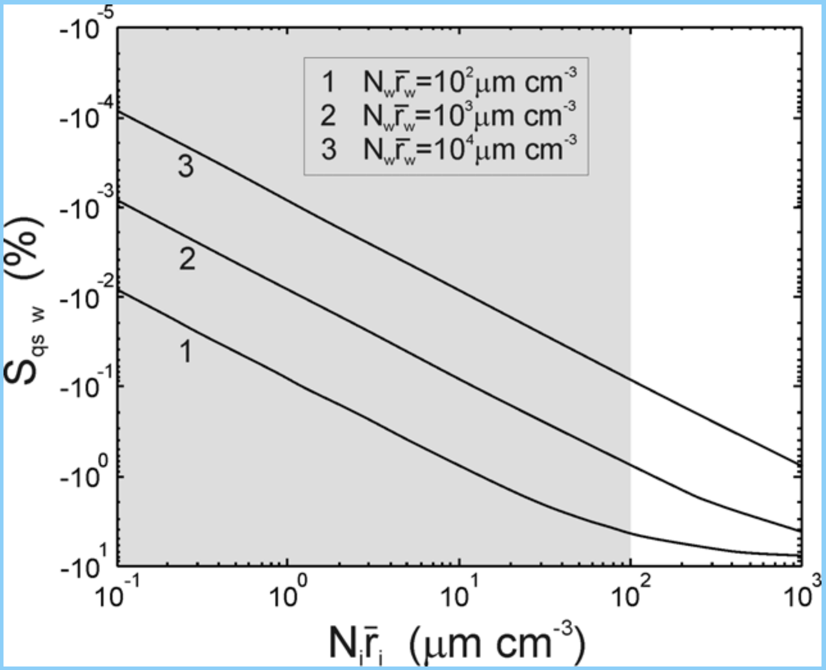
quasi-steady supersaturation

$$\tau_p = \frac{1}{a_0 u_z + b_w N_w \bar{r}_w + (b_i + b_i^*) N_i \bar{r}_i}$$

time of phase relaxation

$$S_{qs\ w} = -\frac{b_i^* N_i \bar{r}_i}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

supersaturation in mixed phase cloud for U_z=0

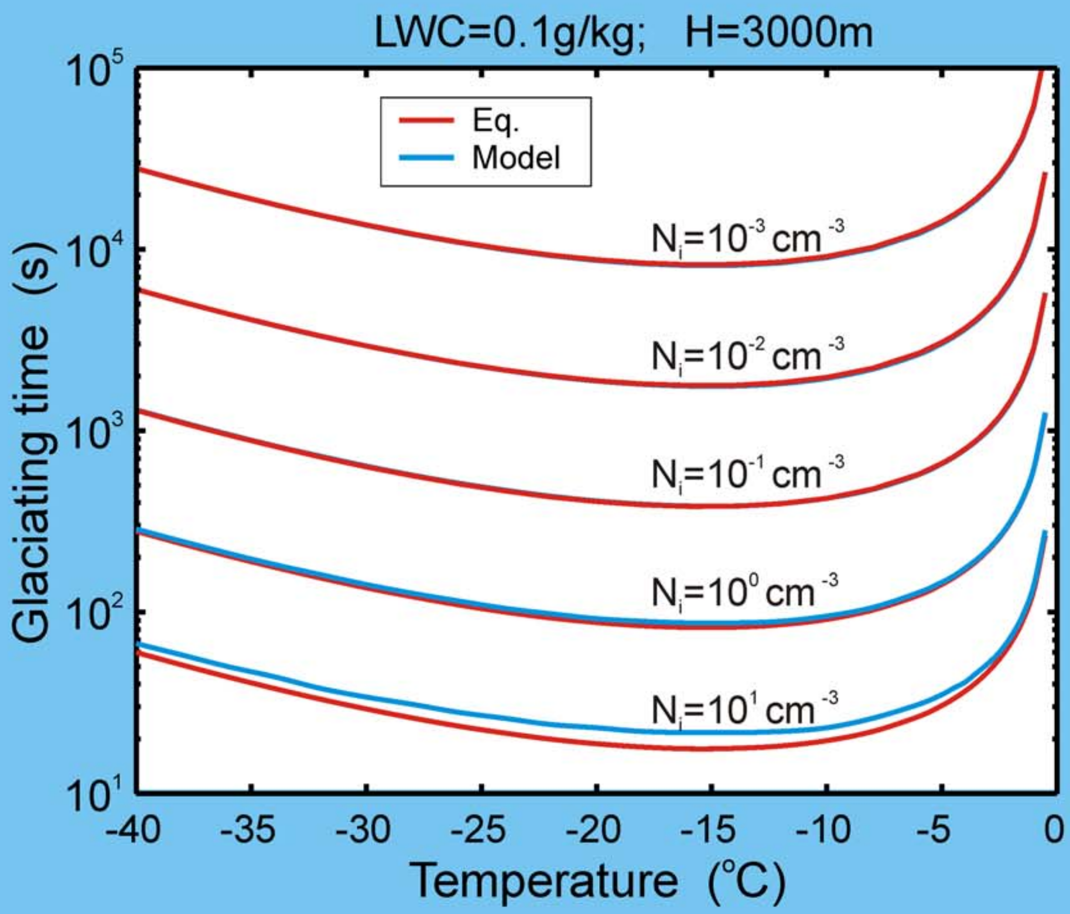


Glaciation time of a mixed phase cloud for U_z=0

$$\tau_{gl} = \frac{1}{4\pi c A_i S_i}\left(\frac{9\pi\rho_i}{2}\right)^{1/3}\left(\left(\frac{W_w(t_0)+W_i(t_0)}{N_i}\right)^{2/3} - \left(\frac{W_i(t_0)}{N_i}\right)^{2/3}\right)$$

$$\tau_{gl} = \frac{1}{4\pi c A_i S_i}\left(\frac{9\pi\rho_i}{2}\right)^{1/3}\left(\frac{W_w(t_0)}{N_i}\right)^{2/3}$$

Glaciation time $U_z=0$



$$\tau_{gl} = \frac{1}{4\pi c A_i S_i}\left(\frac{9\pi\rho_i}{2}\right)^{1/3}\left(\left(\frac{W_w(t_0)+W_i(t_0)}{N_i}\right)^{2/3} - \left(\frac{W_i(t_0)}{N_i}\right)^{2/3}\right)$$

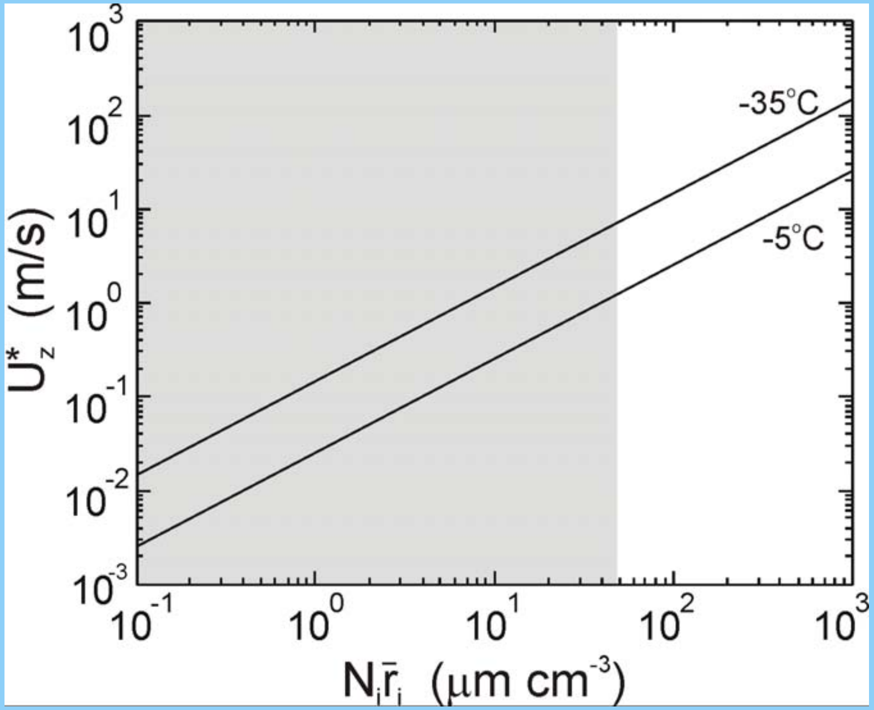
Three scenarios of the mixed phase evolution:

1. ice particles grow while liquid droplets evaporate (Wegener-Bergeron-Findeisen (WBF) mechanism)
2. liquid and ice particles both evaporate
3. liquid and ice particles both grow

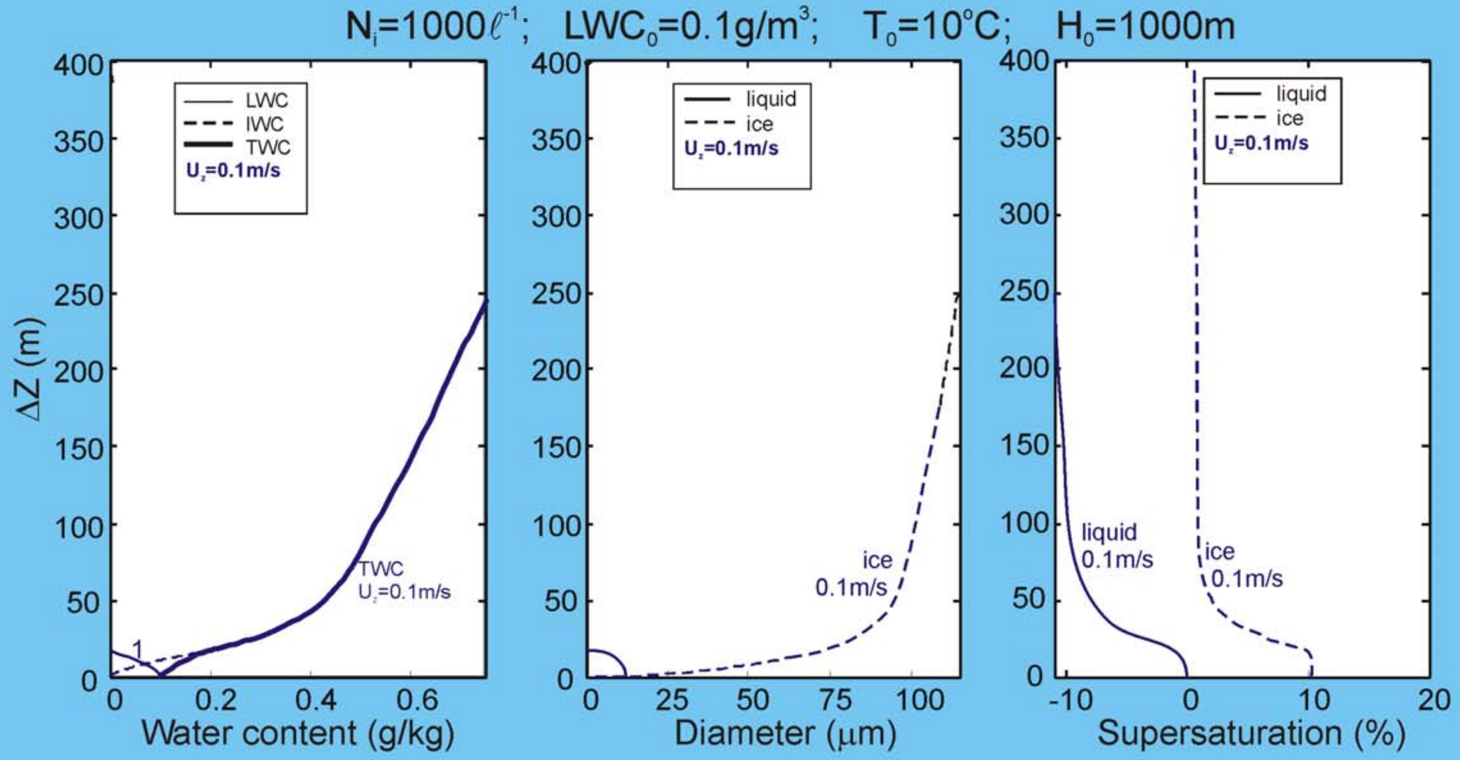
$$S_{qs\ w} = \frac{a_0 u_z - b_i^* N_i \bar{r}_i}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

$$u_z^* = \frac{b_i^* N_i \bar{r}_i}{a_0}$$

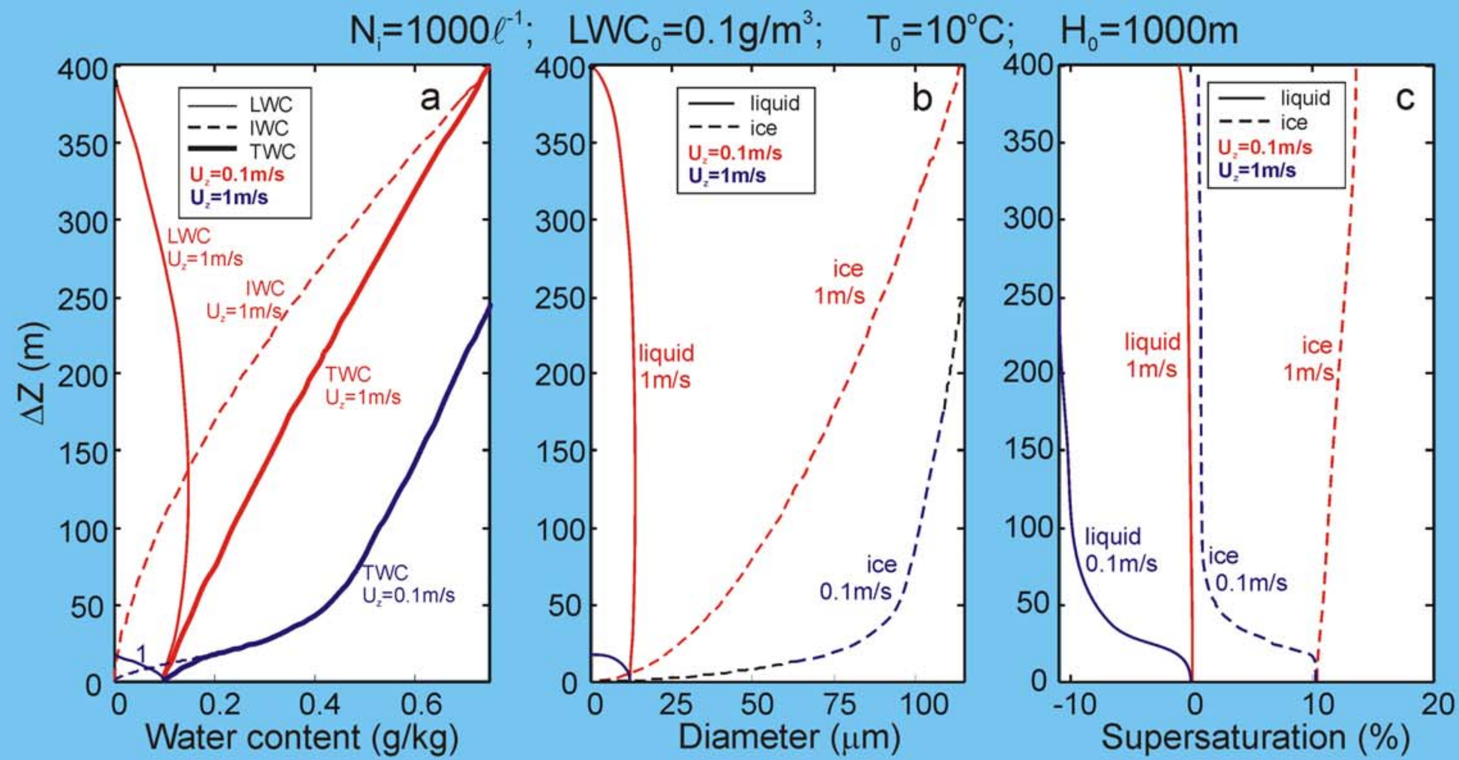
Threshold velocity for simultaneous liquid and ice particles growth



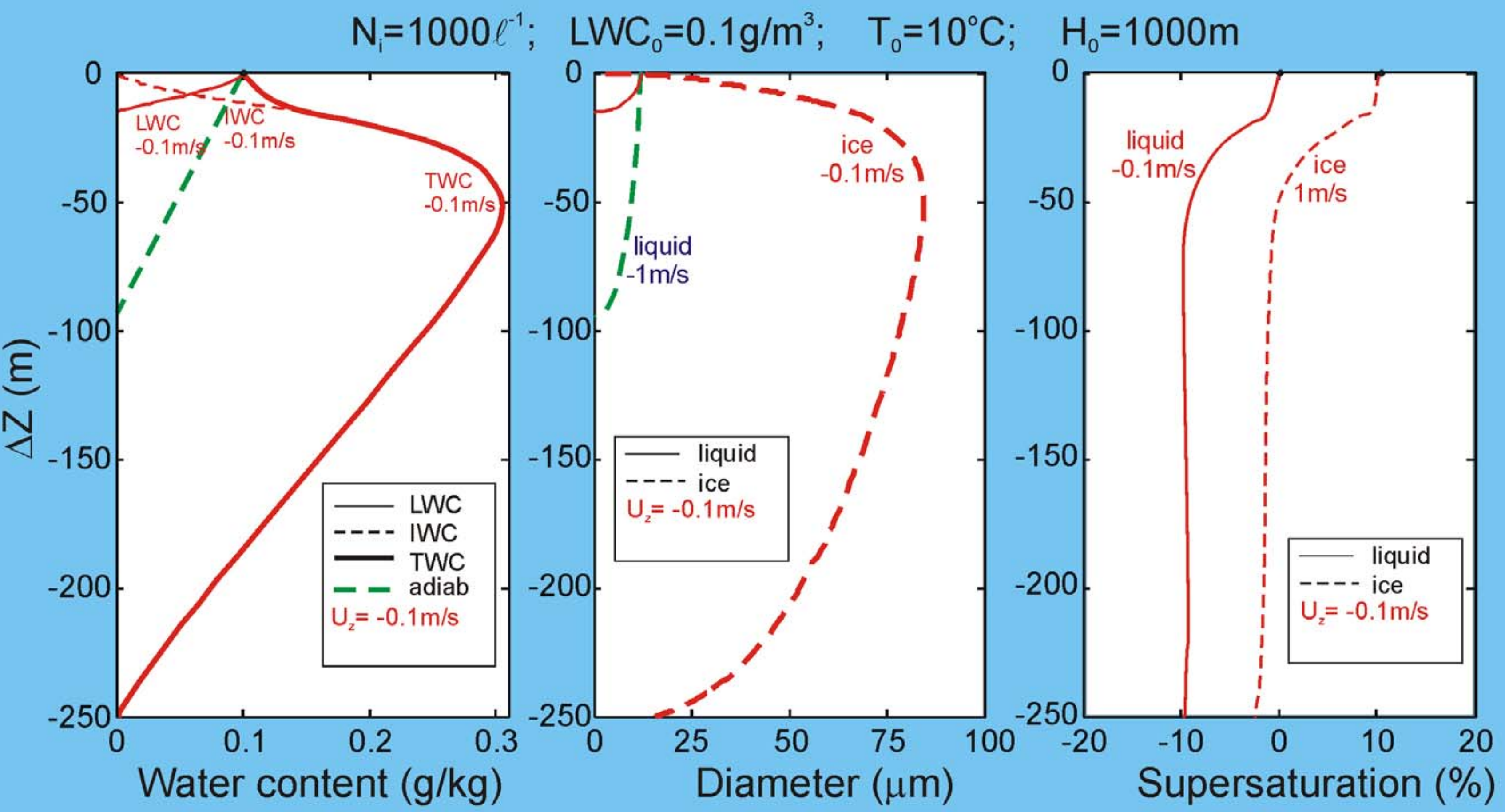
Glaciation time in updraughts



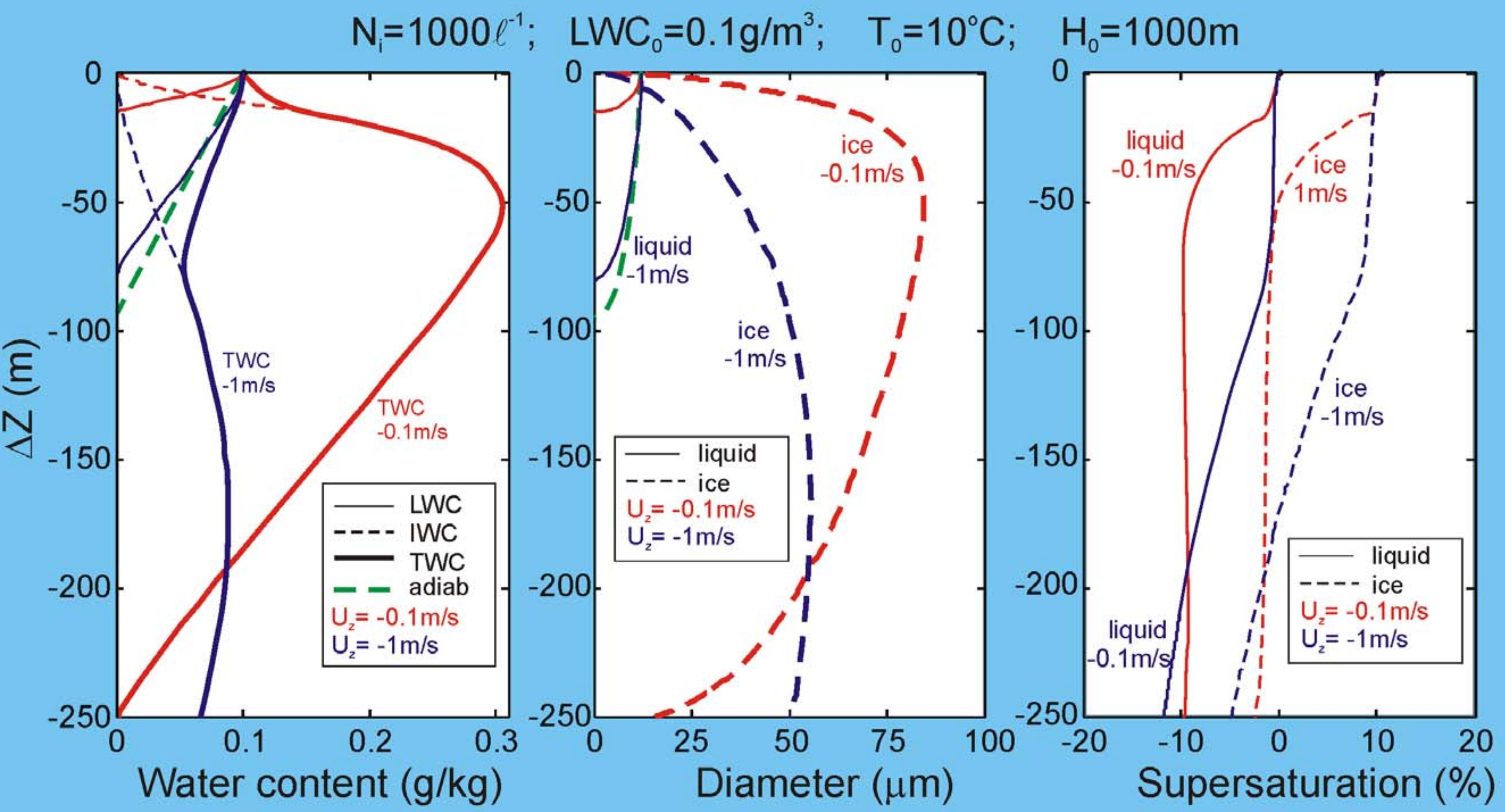
Glaciation time in updraughts



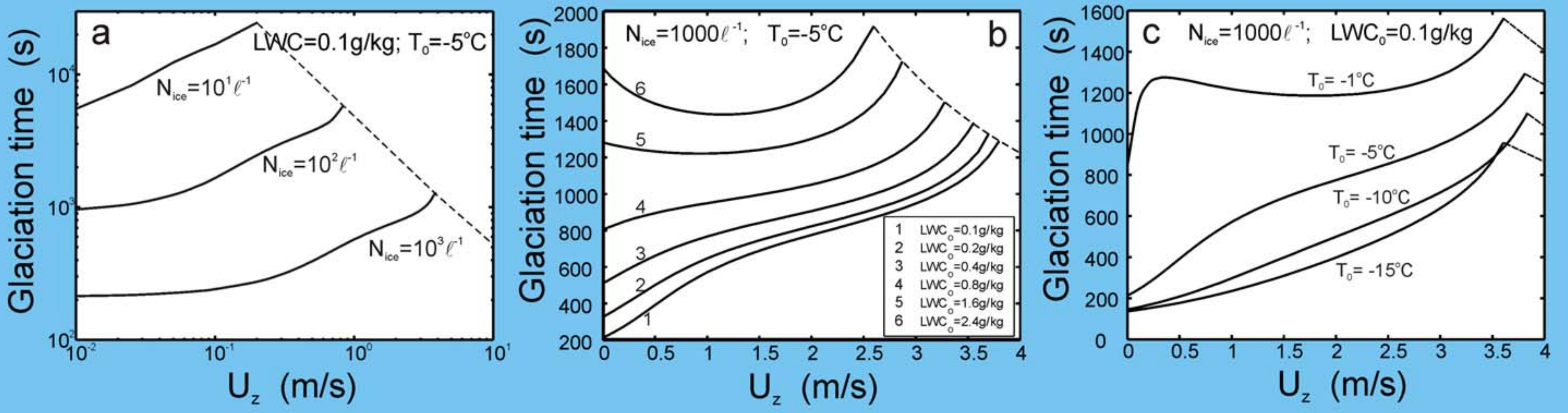
Glaciation time in downdrafts



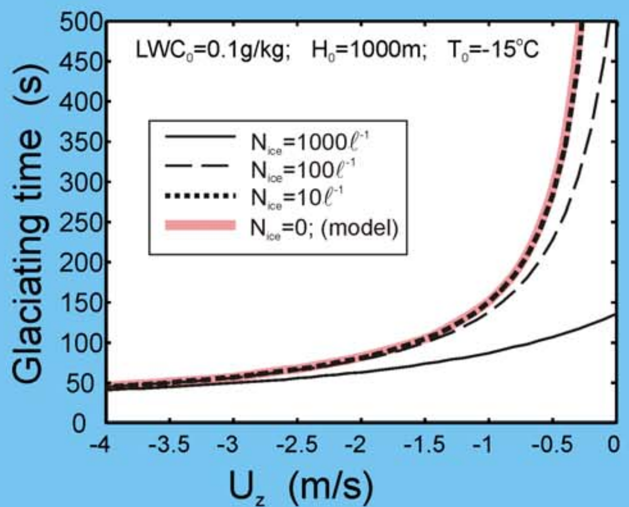
Glaciation time in downdrafts



Glaciation time $U_z>0$

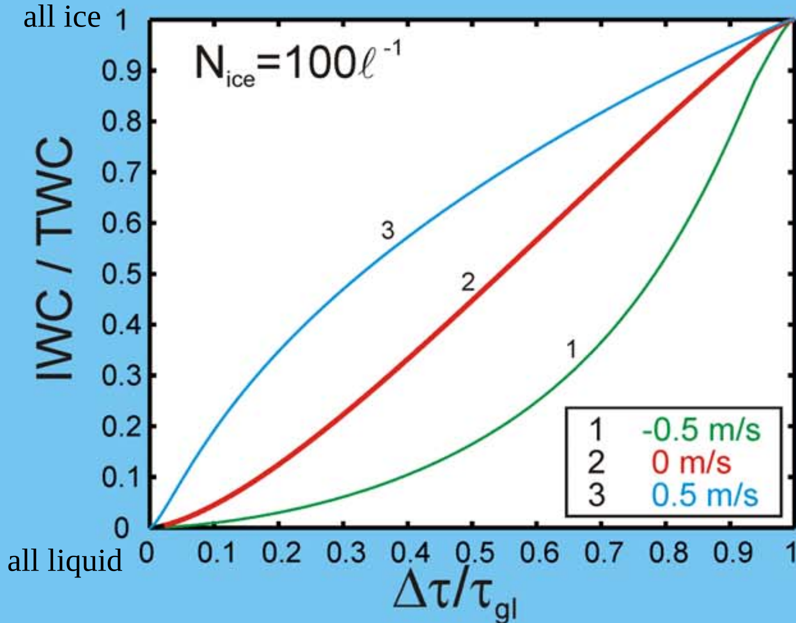


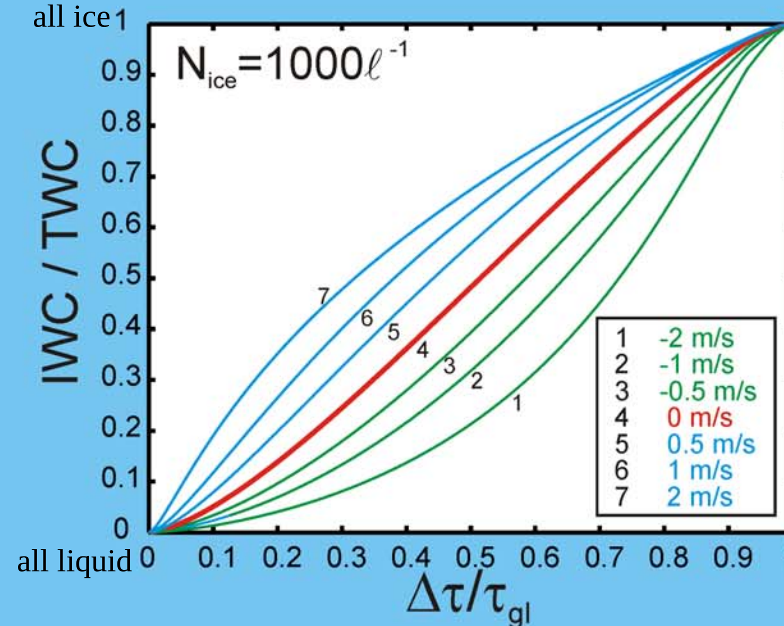
Glaciation time $U_z<0$

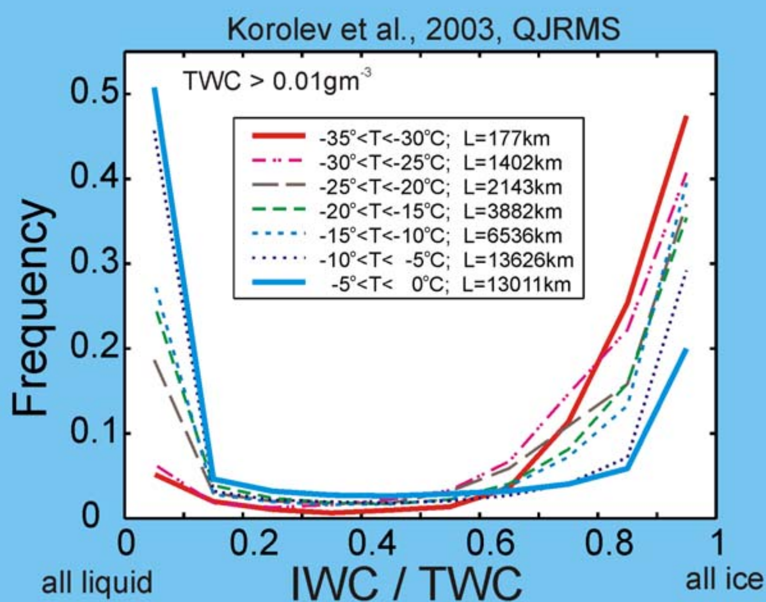


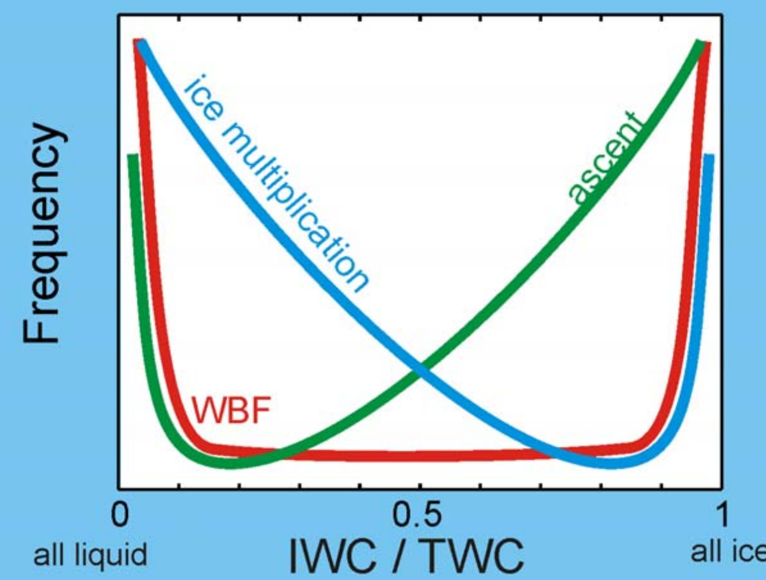
$$\tau_{ev\ ad} = \frac{W_{LWC0}}{U_z \beta_{ad}}$$ time of droplets evaporation in downdraughts

$$\beta_{ad} = \left(\frac{dW_{LWC}}{dz}\right)_{ad} \cong g \frac{\frac{LR_m}{C_p R_v T} - 1}{\frac{R_v TP}{E_s} + \frac{L^2 R_m}{C_p R_v T}}$$ adiabatic gradient of LWC

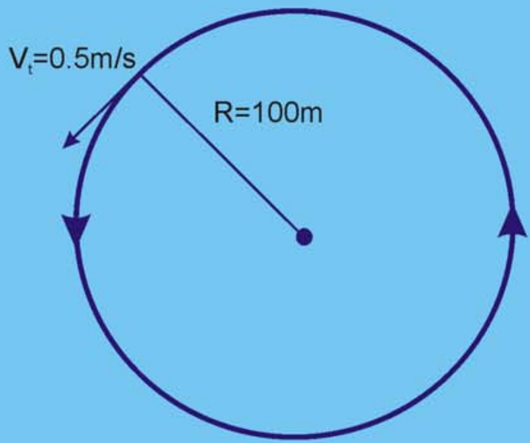

all ice
all liquid
$N_{ice}=100\ell^{-1}$
IWC / TWC
$\Delta\tau/\tau_{gl}$
1 -0.5 m/s
2 0 m/s
3 0.5 m/s


all ice
all liquid
$N_{ice}=1000\ell^{-1}$
IWC / TWC
$\Delta\tau/\tau_{gl}$
1 -2 m/s
2 -1 m/s
3 -0.5 m/s
4 0 m/s
5 0.5 m/s
6 1 m/s
7 2 m/s


Korolev et al., 2003, QJRMS
TWC > $0.01gm^{-3}$
-35°<T<-30°C; L=177km
-30°<T<-25°C; L=1402km
-25°<T<-20°C; L=2143km
-20°<T<-15°C; L=3882km
-15°<T<-10°C; L=6536km
-10°<T< -5°C; L=13626km
-5°<T< 0°C; L=13011km
Frequency
IWC / TWC
all liquid
all ice


ice multiplication
ascent
WBF
Frequency
IWC / TWC
all liquid
all ice

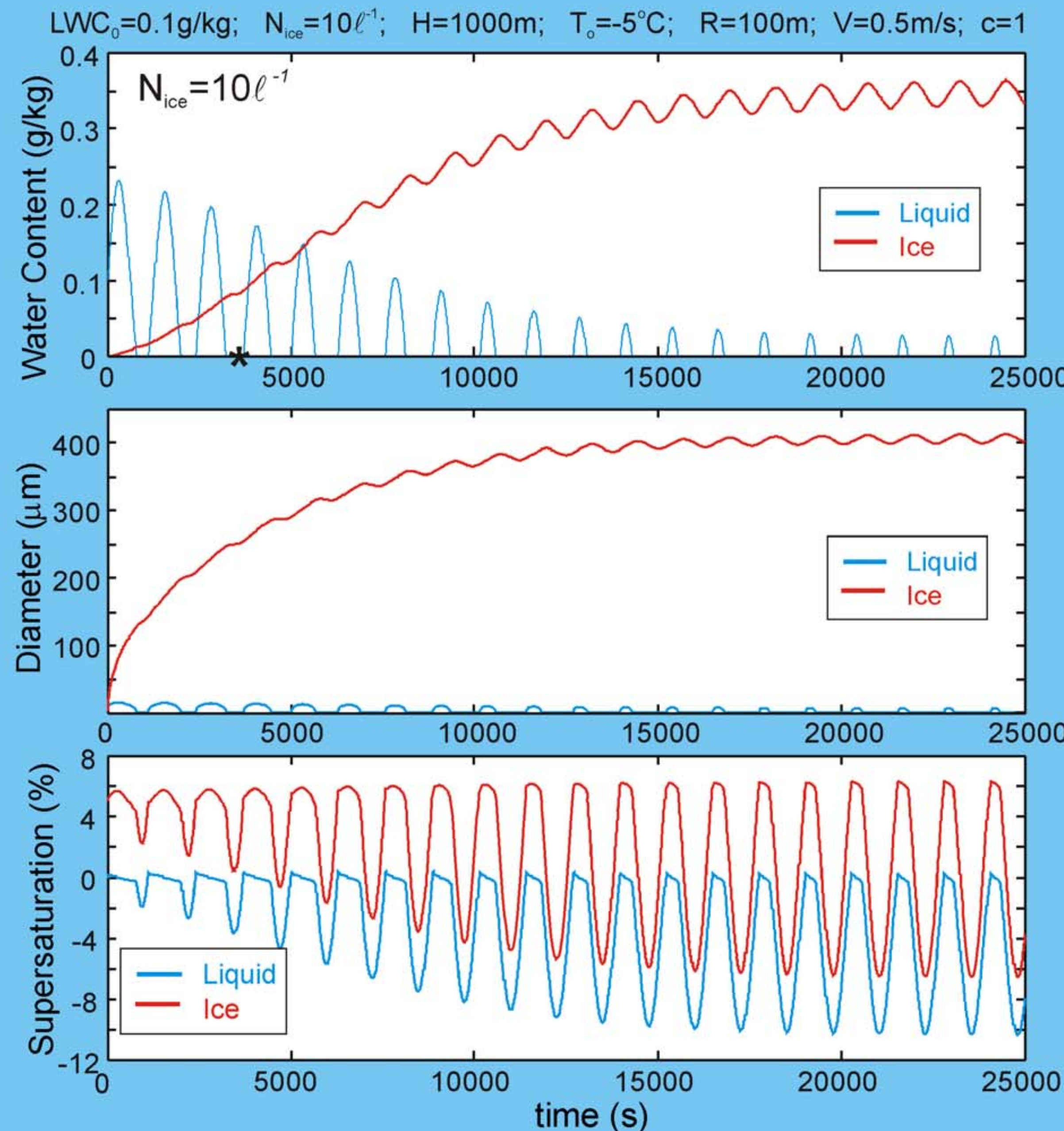
Glaciation time: $U_z \sim \sin(\omega t)$

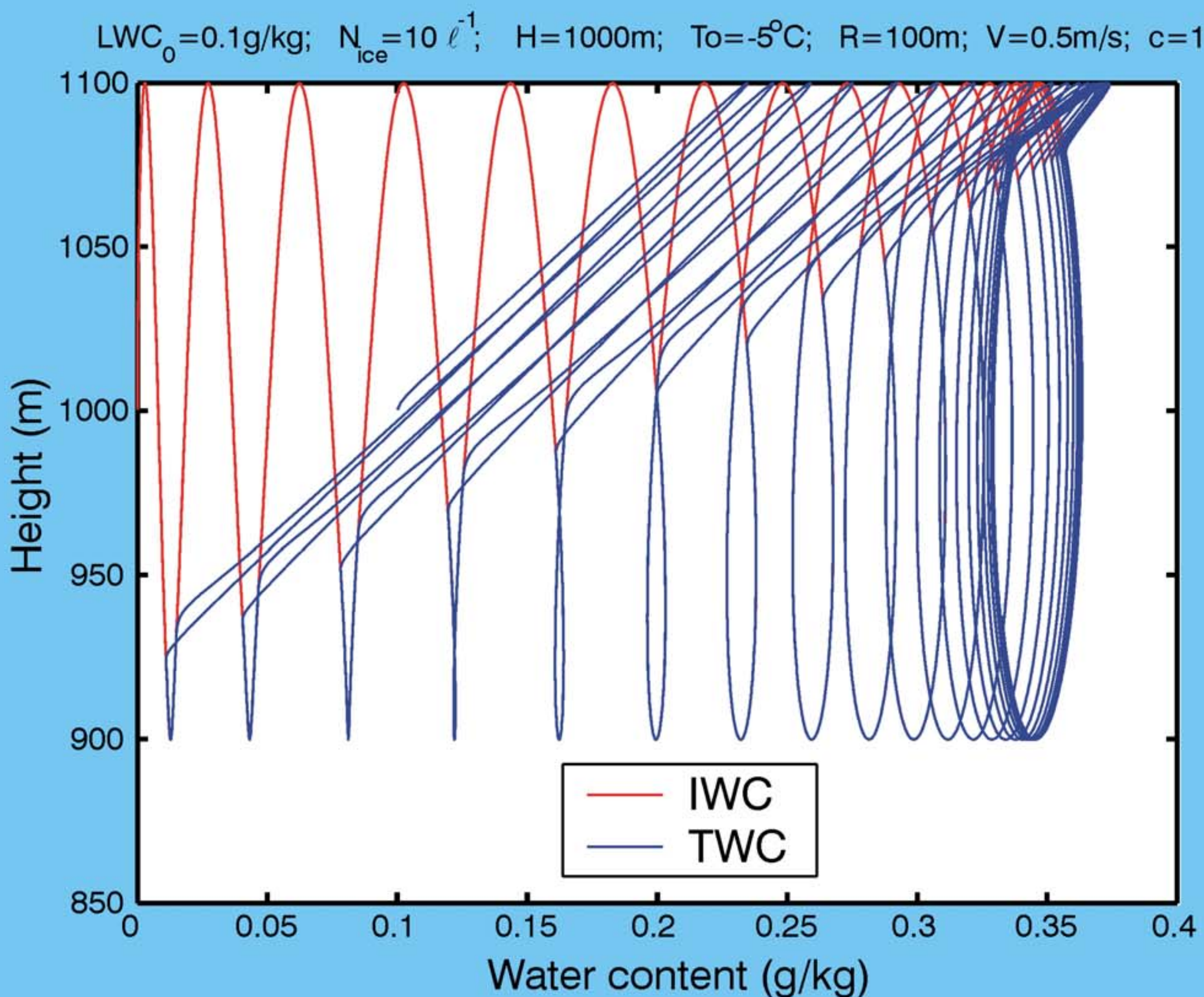


$$\tau_{ph\ ice} > \tau_t$$

$$\tau_t \sim \frac{\pi}{\omega} \sim 10^3 s$$

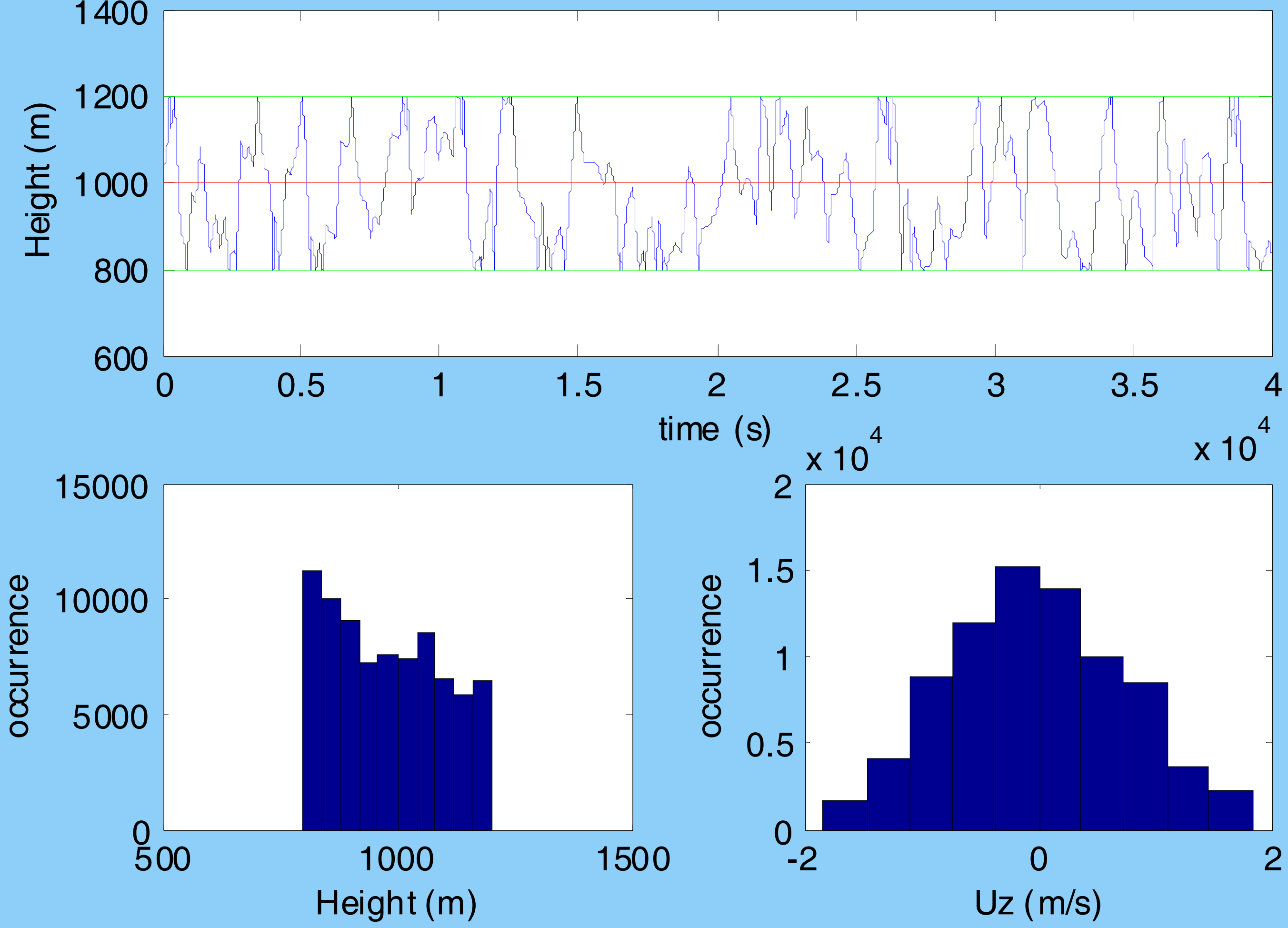
$$\tau_{ph\ ice} \approx \frac{D}{N_i \bar{r}_i} \sim 2 \times 10^3 s$$

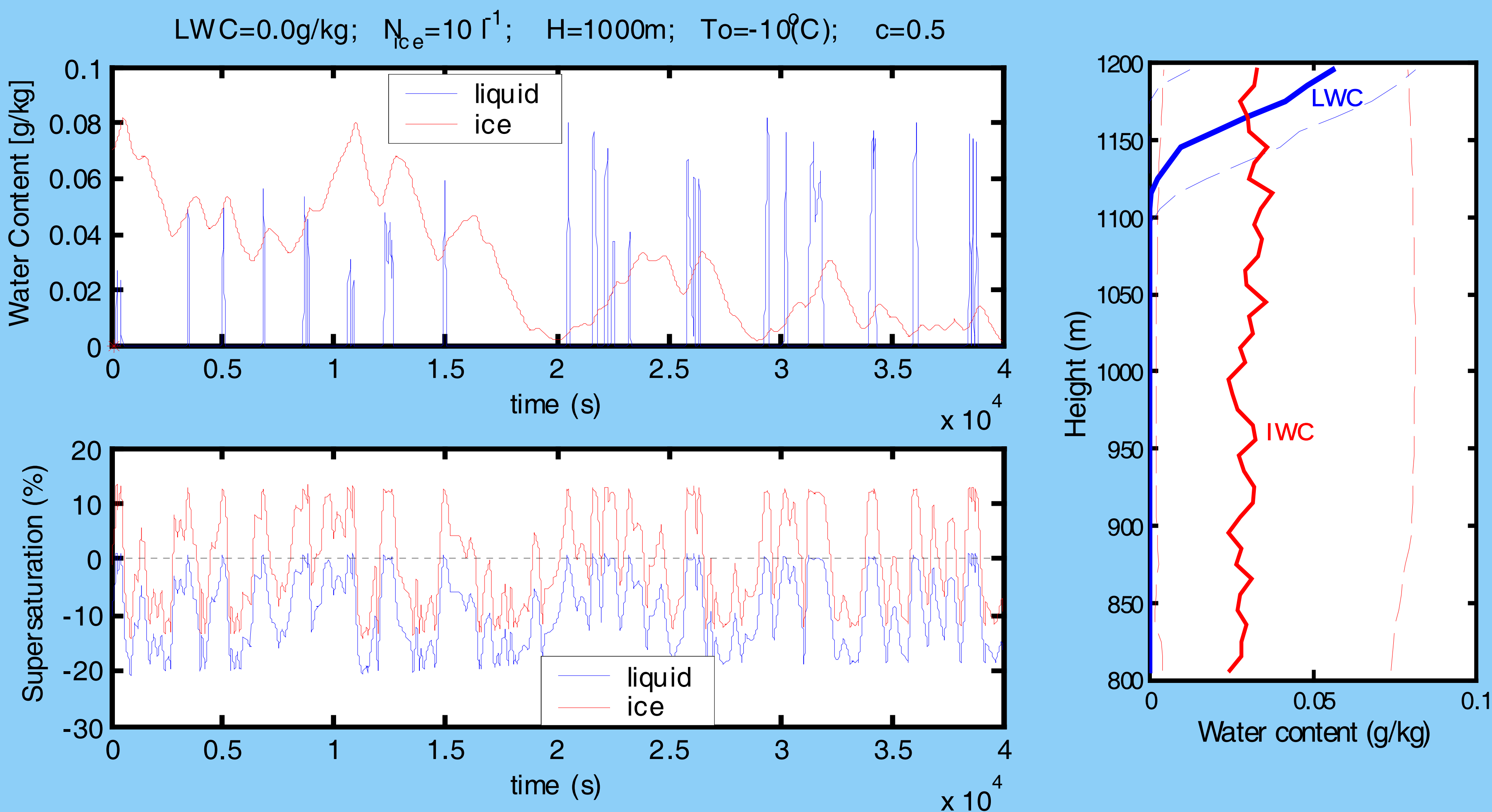




LWC0=0.1g/kg; Nice=10 ℓ-1; H=1000m; To=-5°C; R=100m; V=0.5m/s; c=1
Height (m)
Water content (g/kg)
IWC
TWC
1100
1050
1000
950
900
850
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

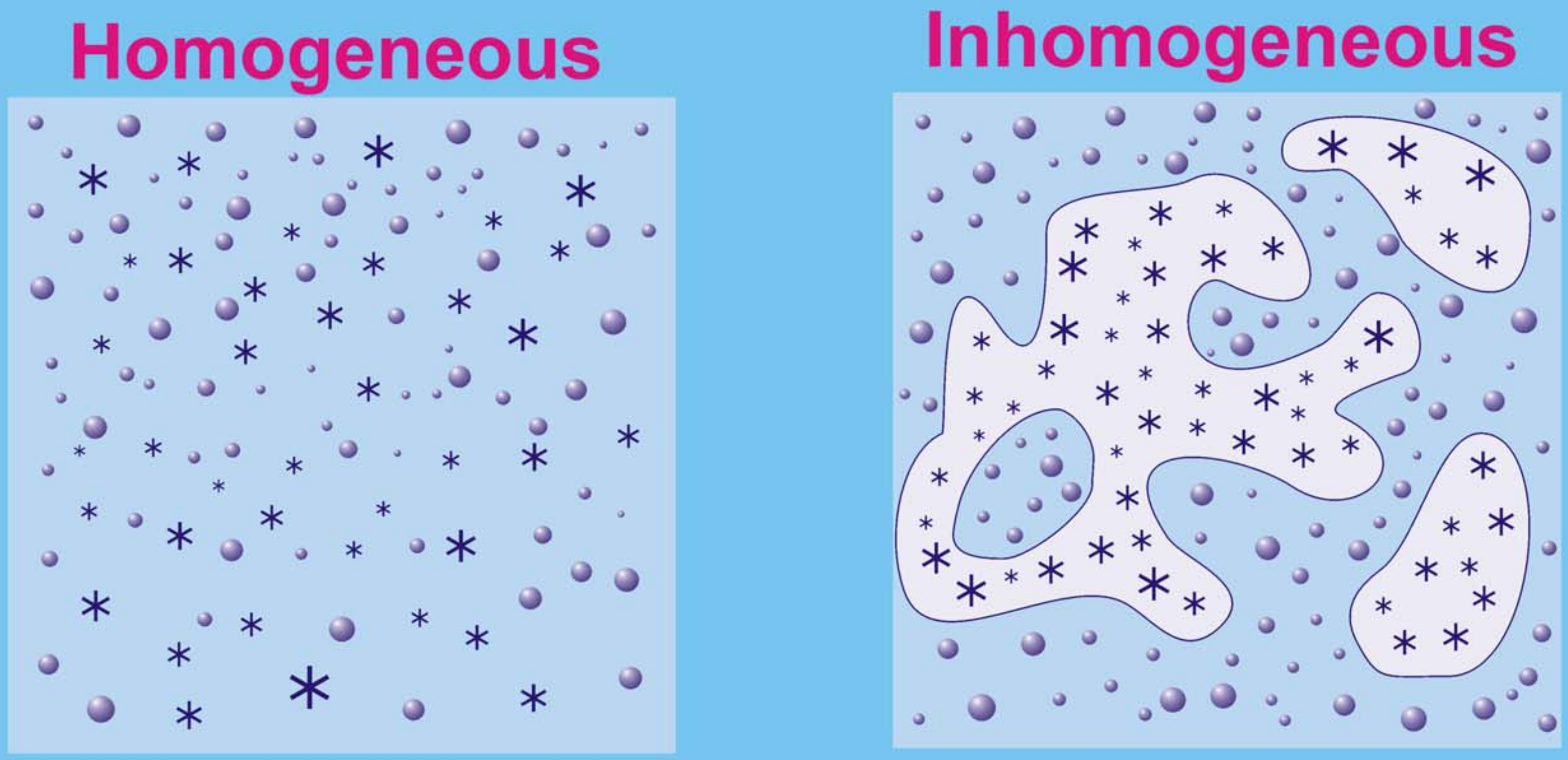
Glaciation time: random vertical fluctuations





LWC=0.0g/kg; N_ice=10 l^-1; H=1000m; To=-10(°C); c=0.5
Water Content [g/kg]
liquid
ice
time (s)
x 10^4
Supersaturation (%)
liquid
ice
time (s)
x 10^4
Height (m)
LWC
IWC
Water content (g/kg)

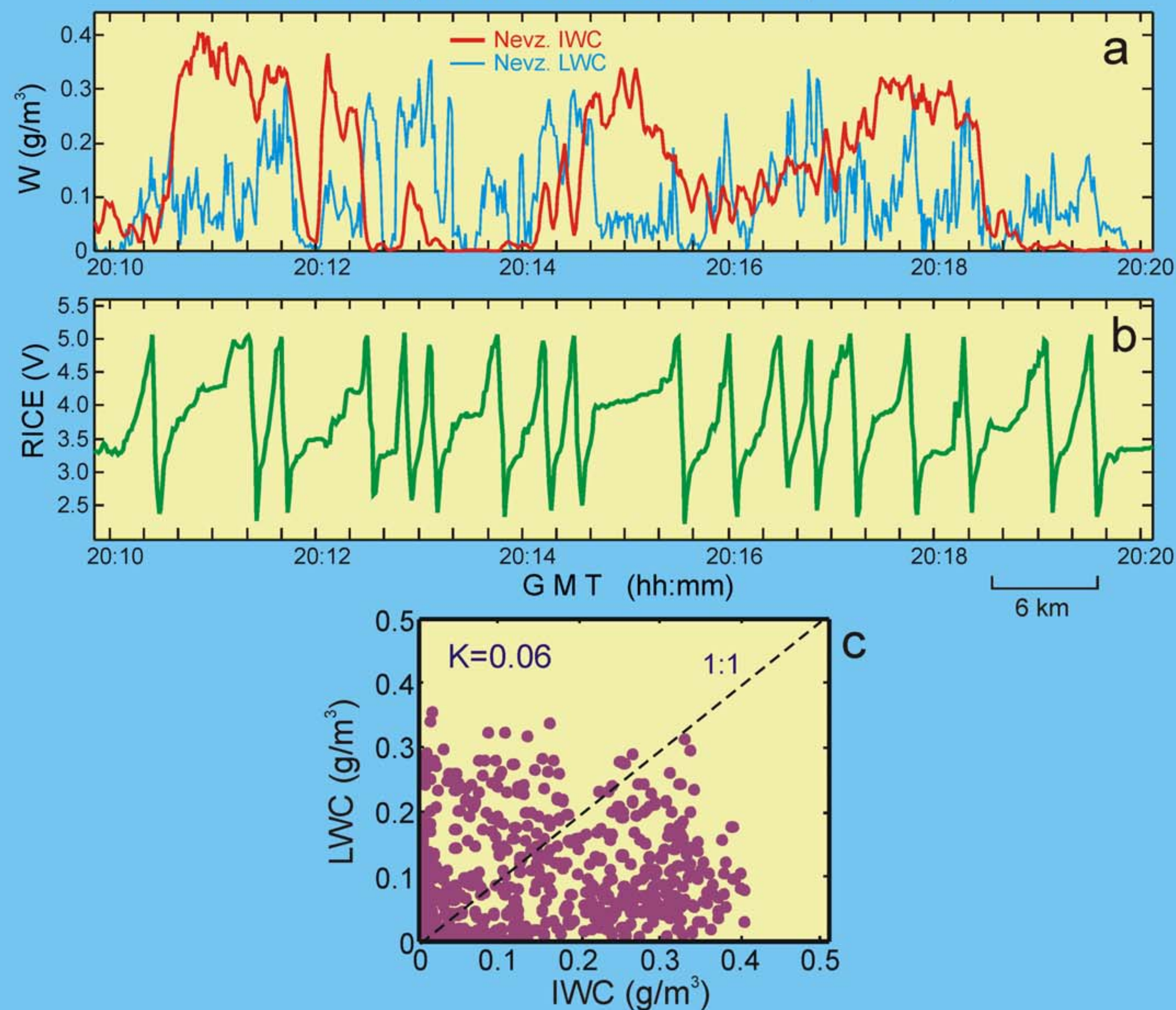
Spatial structure of mixed phase zones



What is the characteristic scale of the single phase zones in mixed phase clouds? 1 cm? 1 m? 1 km?

The existence of small scale isolated ice and liquid zones would result in extension of the lifetime of mixed phase clouds

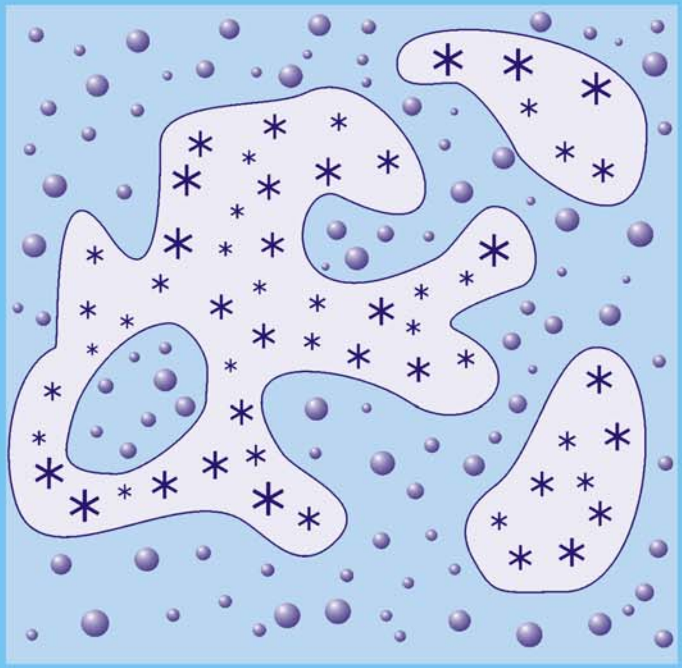
16 December 1999; AIRS   20:09:45-20:20:12; T= -6°C; H=1200m



$$\tau_{gl} << \tau_t$$

The condition for the existence of isolated single phase liquid and ice zones with the characteristic scale *L* in clouds with isotropic turbulence

$$\tau_t = \left(\frac{L^2}{\varepsilon}\right)^{1/3}$$



τ_{gl}~10^2 -10^3s;

ε~ 10^{-3} -10^{-4} m^2/s^3

The characteristic spatial scale of the single phase zones in mixed phase clouds

L_{gl}~10^1-10^3m

Conclusions

(a) $U_z=0$ $\Rightarrow$ $\tau_{gl} \sim \left(\frac{W_{LWC0}}{N_i}\right)^{2/3}$

for $N_{ice}\sim10^3\, l^{-1}$; $W_{LWC0}\sim0.1 g/m^3$; $c=1$ $\Rightarrow$ $\tau_{gl}\sim 10^2 s$
for $N_{ice}\sim10^2\, l^{-1}$; $W_{LWC0}\sim0.1 g/m^3$; $c=1$ $\Rightarrow$ $\tau_{gl}\sim 10^3 s$
for non-spherical particles τ_{gl} should be scaled proportional to c^{-1}.
to a first approximation τ_{gl} does not depend on the droplet size distribution.

(b) $U_z > 0$ or $U_z < 0$ (U_z = const)
τ_{gl} is a complex function of initial T_0, P_0, W_{LWC0}, N_{ice}, c, U_z.
τ_{gl} may both increase and decrease in updrafts with an increase of U_z.
τ_{gl} in descending parcels increases with an increase of downdraft velocity.
Numerical modeling shows that glaciation in convective clouds having $N_{ice}<1000\, l^{-1}$ and $U_z>4$m/s would never occur through the Wegener-Bergeron-Findesen mechanism before it reaches a level with $T= -40^oC$. After this, glaciation occurs through spontaneous freezing.

(c) $U_z \sim \sin(\omega t)$ or random fluctuations
The vertical oscillation of a cloud parcel may result in a periodic evaporation and activation of liquid droplets in the presence of ice particles. After a certain time, the *average* IWC and LWC reaches a steady state. This phenomenon may explain the existence of long-lived mixed phase stratiform layers.

(d) The characteristic spatial scale of the single phase zones in mixed phase clouds is $L_{gl}\sim10^1$-10^3m

$$\frac{dP}{dt} = -\frac{gPU_z}{R_m T}$$

$$\frac{dT}{dt} = -\frac{gU_z}{C_p} + \frac{L_l}{(1+q_v)C_p}\frac{dq_l}{dt} + \frac{L_i}{(1+q_v)C_p}\frac{dq_i}{dt}$$

$$\frac{dq_v}{dt} + \frac{dq_l}{dt} + \frac{dq_i}{dt} = 0$$

$$\frac{dq_l}{dt} = 4\pi A_l N_l r_l S_l$$

$$\frac{dq_i}{dt} = 4\pi c A_i N_i r_i S_i$$

$$\frac{1}{S_w+1}\frac{dS_w}{dt} = \left(\frac{gL_w}{c_pR_vT^2} - \frac{g}{R_aT}\right)u_z - \left(\frac{1}{S_w+1}\frac{pR_v}{E_wR_a} + \frac{L_iL_w}{c_pR_vT^2}\right)B_i^*N_i\bar{r}_i -$$
$$-\left(\left(\frac{1}{S_w+1}\frac{pR_v}{E_wR_a} + \frac{L_w^2}{c_pR_vT^2}\right)B_wN_w\bar{r}_w + \left(\frac{1}{S_w+1}\frac{pR_v}{E_wR_a} + \frac{L_iL_w}{c_pR_vT^2}\right)B_iN_i\bar{r}_i\right)S_w$$

$$\frac{dS_w}{dt} = AS_w^2 - BS_w + C$$

$$S_{qs\ w} = \frac{a_0u_z - b_i^*N_i\bar{r}_i}{b_wN_w\bar{r}_w + b_iN_i\bar{r}_i}$$

$$\frac{1}{S_w+1}\frac{dS_w}{dt} = a_0u_z - a_2N_i\sqrt{r_{i0}^2 + 2cA_i\int_0^t\left(\xi S_w(t') + \xi - 1\right)dt'} -$$
$$-\left(a_1B_wN_w\sqrt{r_{w0}^2 + 2A_w\int_0^t S_w(t')dt'} + a_2B_iN_i\sqrt{r_{i0}^2 + 2cA_i\int_0^t\left(\xi S_w(t') + \xi - 1\right)dt'}\right)S_w$$