P6.9

Reevaluating the Equitable Threat Score of Forecasts Using the Economic Value Concept

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1. Introduction

Equitable Threat Score (*ETS*) is a very frequently used measure of quantitative precipitation forecasts. It is defined with the use of a contingency table (Table 1),

	Observed YES	Observed No
Forecast YES	*a(C)*	*b(C)*
Forecast NO	*c(L)*	*d*

Table 1. Contingency Table.

where *a* (hits), *b* (false alarms), *c* (misses), and *d* (correct rejections) stand for probabilities of an event exceeding a certain threshold, as

$$ETS \equiv \frac{a - ch}{a + b + c - ch}, \qquad (1)$$

and chance *ch* is given by

$$ch \equiv \frac{(a+b)(a+c)}{a+b+c+d}.$$

By introducing

$$f \equiv a+b, o \equiv a+c, q \equiv \frac{b}{a},$$

ETS can be rewritten as

$$ETS = \frac{f\frac{1}{q+1} - fo}{o + f - f\frac{1}{q+1} - fo}, \qquad (2)$$

and $q \geq \frac{f}{o} - 1,$ since $o \geq a = \frac{f}{q+1}.$

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2. *ETS* Deficiency

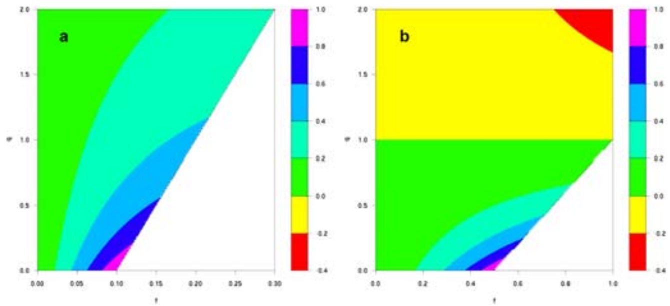

Figure 1. Dependence of ETS on forecast area f and ratio q for rare (o = 0.1, left) and frequent (o=0.5, right) events. White areas are unphysical.

By increasing forecast area *f* and assuming that hits (*a*) and false alarms (*b*) benefit proportionally so that *q=b/a=const,* model's *ETS* improves without any intuitive gain in skill. For any skillful forecast (*ETS* > *0*), fallacy of *ETS* is most evident for less frequent events where large gains can be achieved with large values of *q*.

Assuming *q=0* in Eq. 2 we arrive at

$$f_e \equiv \frac{ETSo}{1 - o + ETSo}, \qquad (3)$$

which specifies an equivalent forecast area (f_e) when a model forecast has no misses. For example, ETSs for forecasts in Fig. 2 are equal. Which is better? Which is more valuable/costs less: hit, false alarm, or miss? This needs to be determined.

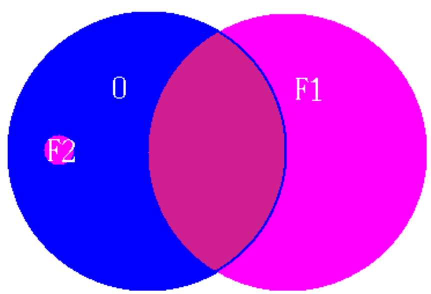

Figure 2. Forecasts F1 and F2 have the same ETSs (approx. 0.2) for the observed probability o=0.1. Radius of F2 is approximately equal to 0.1 of radius F1.

3. Reevaluating *ETS*s Using the Economic Value Concept

Using a static cost-loss model, assigning cost (*C*) and loss (*L*) to hits, misses, and false alarms (see Table 1), a formula can be derived for a potential economic value of forecasts (Richardson, 2000)

$$V = \frac{\min(o,r) - fr - o + \frac{f}{q+1}}{\min(o,r) - or}, \qquad (4)$$

where *r=C/L* is the cost-loss ratio.

Substituting in Eq. 4 for *f* from Eq. 2, dependence for the potential economic value of forecasts *V=V(o,r,ETS,q)* can be found.

The economic value of forecasts differs for different decision-makers (as their cost-loss ratios *r* vary) but can be useful in more appropriate quality assessments of different forecasts with varying *ETS*s.

For reasons outlined in Section 2, *V* is also dependent on the value of ratio *q*. In Fig. 3, plots of the potential economic value as a function of *ETS* and *q* are given for fixed *o*s and *r*.

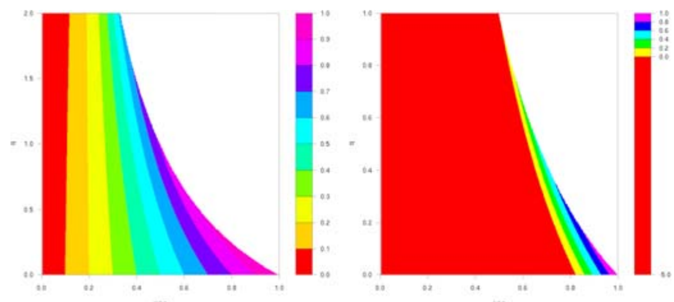

Figure 3. Potential economic value of forecasts as a function of ETS and q for rare (o=0.1, left) and frequent (o=0.5, right) events. Cost-loss ratio of 0.15 assumed. White areas are unphysical.

4. References

Richardson, D.S., 2000, Skill and relative economic value of the ECMWF ensemble prediction system, *Quart. J. Royal Meteorol. Soc.*, 126. 649 - 667.