CAPE in mixed phase cloud conditions

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Introduction

An approximate but pragmatic approach is presented to define Convective Available Potential Energy (CAPE) in mixed phase cloud conditions. The underlying process calls for mixed (i.e. liquid and ice) phase parcels and assumes the liquid fraction to be a unique function of temperature. The approach is meant to represent average conditions. Differences between this and more traditional approaches are quantified and discussed for mean tropical conditions.

Concept

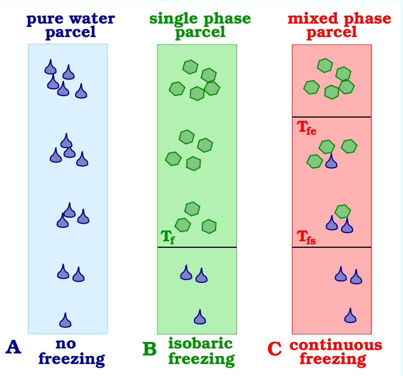

Figure 1: Schematic of analyzed processes.

We analyze the following processes defining the thermodynamical state of the parcel during ascent:

1. **Pure water parcel process:** The formation of ice is neglected. Water vapor condenses to form liquid water only (Fig. 1A).
2. **Single phase parcel process:** **Isobaric freezing** occurs at T_{f}. All condensate is liquid at temperatures T warmer than T_{f} and solid at T colder than T_{f} (Fig. 1B).
3. **Mixed phase parcel process:** Freezing of liquid water occurs within the temperature range between T_{fs} and T_{fe}, where the **liquid water fraction** $\xi = \xi(T)$ $(0 \leq \xi \leq 1)$ continuously decreases to the benefit of the ice fraction (see Fig. 1C).

Thermodynamical Background

We derive the lapse rate for mixed phase cloud conditions using the following assumptions:

- The condensate loading $m^F = m^2 + m^3$ includes liquid water, m^2, and ice, m^3 with
$$m^2 = \xi\, m^F \quad \text{and} \quad m^3 = (1-\xi)\, m^F \; .$$
- We hypothesize a 'mixed phase saturation vapor pressure' p^{F1}, defined as a weighted mean of the saturation pressure with respect to water, p^{21}, and ice, p^{31},
$$p^{F1} = \xi\, p^{21} + (1-\xi)\, p^{31} \; .$$

The mixed phase reversible lapse rate than reads

$$\boxed{\Gamma_{\mathrm{rev}} = \frac{g\left(1 + r^{F1} + r^F\right)\left(1 + \frac{r^{F1}\,\ell_{F1}}{R_0\,T}\right)}{C_p + \frac{r^{F1}\,\ell_{F1}}{p^{F1}}\,\frac{\mathcal{L}^{F1}}{R_1\,T^2} + \left[\frac{r^{F1}\,\ell_{F1}}{p^{F1}}\left(p^{21} - p^{31}\right) + \ell_{32}\, r^F\right]\frac{d\xi}{dT}}} \; .$$

A convinient way to quantify the density of a parcel is the density temperature T_ρ given by

$$T_\rho = T\,\frac{1 + \frac{R_1}{R_0}\, r}{1 + r + r^F} \approx T\left(1 + 0.608\, r - r^F\right)$$

(e.g. Emanuel 1994). The buoyancy B can be defined as the difference of the density temperature of the parcel and that of the environment, $B = T_{\rho,\mathrm{p}} - T_{\rho,\mathrm{e}}$.

In case of saturation the excess vapor condenses. In **reversible conditions** all condensate remains in the parcel and it falls out instantaneously in **pseudo-adiabatic conditions**. In real clouds only a certain fraction χ leaves the parcel as precipitation. This **precipitation efficiency** χ can be considered by replacing r^F by

$$r^F_\chi = (1-\chi)\, r^F \; .$$

Symbols: Mixed phase saturation mixing ratio for vapor $r^{F1} = R_0\, p^{F1}/R_1\, p^0$ and condensate $r^F = m^F/m^0$, gas constant R_0, partial pressure p^0, and specific mass m^0 of dry air, gas constant of vapor R_1, mixed phase latent heat $\ell_{F1} = \xi\ell_{21} + (1-\xi)\ell_{31}$, mixed phase specific heat $C_p = c_{p0} + c_{p1}\, r^{F1} + c_2\xi\, r^F + c_3(1-\xi) r^F$, specific heat at constant pressure for dry air c_{p0} and vapor c_{p1}, specific heat for liquid water c_2 and ice c_3, latent heat of fusion $\ell_{32} = \ell_{31} - \ell_{21}$, and $\mathcal{L}^{F1} = \xi\ell_{21}p^{21} + (1-\xi)\ell_{31}p^{31}$.

Temperature Profiles

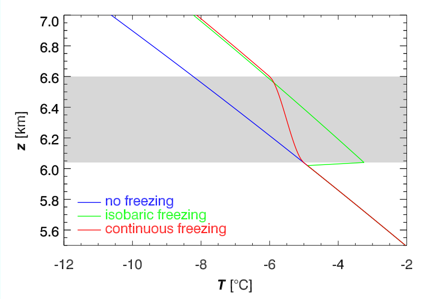

Figure 2: Temperature profiles.

We apply our approach to the standard tropical atmosphere (Anderson et al., 1986), which is used as environmental conditions and as initial thermodynamic state of the parcel.

In the **pure water parcel process** the ascending parcel cools at the wet adiabatic lapse rate. **Isobaric freezing** in a **single phase parcel** leads to an abrupt T increase due to the release of latent heat during freezing and the decrease of saturation pressure.

Continuous freezing in a **mixed phase parcel** reduces the adiabatic lapse rate.

Buoyancy Profiles

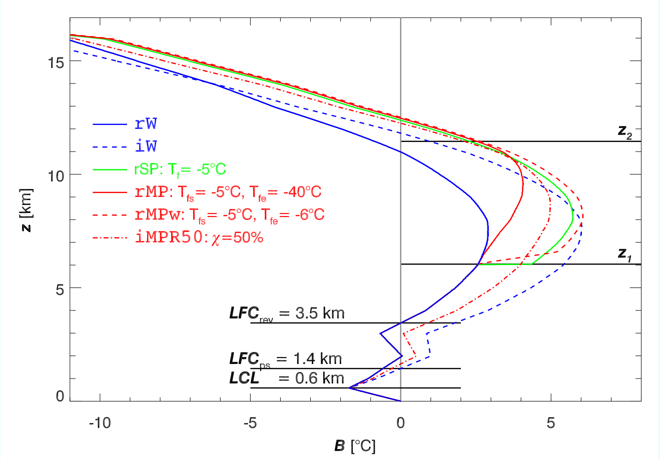

Figure 3: Buoyancy profiles.

Up to the lifting condensation level (LCL) all processes reflect the temperature resulting from the moist adiabatic lapse rate. Above the LCL, B in the pseudo-adiabatic pure water parcel `iW` is higher than in the reversible pure water parcel `rW` because of the higher T_ρ in the absence of condensate. This effect dominates up to about 14 km height. At higher levels the heat capacity of the liquid water at `rW` reduces parcel cooling during the ascent.

Isobaric freezing at $T_{\mathrm{f}} = $–5° C in the reversible single phase parcel `rSP` leads to a temperature and, thus, buoyancy increase of about 2° C at z_1=6 km height.

The reversible mixed phase parcel `rMP` including continuous freezing consists completely of water droplets at T warmer than T_{fs}=–5° C and completely of ice at T colder than T_{fe}=–40° C; within the temperature range from T_{fs} to T_{fe} both water and ice coexist with a liquid fraction according to ξ. Since the phase changes continuously, B changes gradually with altitude. `rMPw` bases on `rMP` but with the continuous phase transition restricted to the small temperature range between T_{fs}=–5° C and T_{fe}=–6° C. Apparently, there is a stronger local increase of B between 6.0 and 6.6 km altitude compared to `rMP`.

`iMPR50` bases on `rMP` but with χ=50% applied. The reduction of r^F essentially interpolates between pseudo-adiabatic and reversible conditions. In case of high χ the impact of freezing is much smaller than for low χ. A smaller amount of water has less potential for freezing and, thus, less heat of fusion is added to the parcel.

Impact on CAPE

$$\boxed{\mathrm{CAPE} = -R_0 \int_{p_{LFC}}^{p_{LNB}} B\, d\ln p}$$

Convective available potential energy (CAPE) is the buoyant energy available between the level of free convection (LFC) and the level of neutral buoyancy (LNB),

The LFC is the lowest level where buoyancy turns to positive values, while the LNB is the uppermost level where buoyancy turns to negative values.

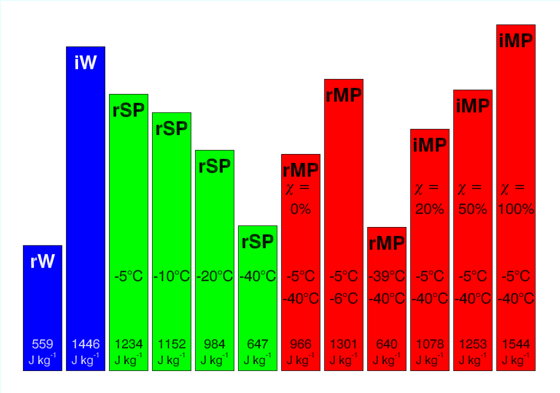

Figure 4: CAPE values for different processes, temperatures where freezing occurs, and precipitation efficiencies.

CAPE is very sensitive to . . .

- . . . the existence of condensate: CAPE(`rW`) $\ll$ CAPE(`iW`)
- . . . the phase of condensate and the temperature where isobaric freezing occurs:
  CAPE(`rSP`, -5° C) > CAPE(`rSP`, -10° C) > CAPE(`rSP`, -20° C) > . . .
  ⇒ The lower T_{f} the lower CAPE.
- . . . the temperature range where continuous freezing occurs:
  CAPE(`rMP`, [–5,–6° C]) > CAPE(`rMP`, [–5,–40° C]) > CAPE(`rMP`, [–39,–40° C])
  ⇒ The more liquid water freezes at warm T the higher CAPE.
- . . . the precipitation efficiency:
  CAPE(`rMP`, χ=100%) > CAPE(`rMP`, χ=50%) > . . . > CAPE(`rMP`, χ=0%)

Summary

Generally freezing increases parcel temperature and, hence, buoyancy. If freezing occurs isobarically (as was often assumed in the past), all water changes phase at a single level resulting in a discontinuity in buoyancy at that level. By contrast, the mixed phase parcel process implies a continuous phase transition in a finite range of temperatures $T_{\mathrm{fs}} \geq T \geq T_{\mathrm{fe}}$, leading to a gradual change of buoyancy with altitude and preventing any temperature inversion. The details of this gradual change depend on the choice of the specified temperature range $[T_{\mathrm{fs}}, T_{\mathrm{fe}}]$. High in the troposphere, where all water is frozen irrespective of the details, the differences between the buoyancy profiles are small (but finite).

CAPE is very sensitive to the treatment of the freezing process. Isobaric freezing at a relatively high temperature (e.g. –5° C) in a reversible process may increase CAPE by a factor of 2 to 3, and this increase is similar in magnitude to the difference between the pseudo-adiabatic and the reversible processes for pure water parcels. Either of these processes is considered less realistic than the reversible mixed phase process with continuous freezing over a broad temperature range $[T_{\mathrm{fs}}, T_{\mathrm{fe}}]$ = [–5° C, –40° C]; the corresponding CAPE lies about half way between the reversible and irreversible pure water processes. For clouds with finite precipitation efficiency the effect of freezing is less pronounced than for reversible conditions.

References

Anderson et al., 1986: *AFGL-TR-86-0110, MA, USA.*
Doswell & Rasmussen, 1994: *Weather and Forcasting*, **9**, 625–629.
Emanuel, 1994: *Atmospheric Convection.* Oxford University Press, New York, USA.
Zdunkowski & Bott, 2004: *Thermodynamics of the Atmosphere.* Cambridge Univ. Press, UK.