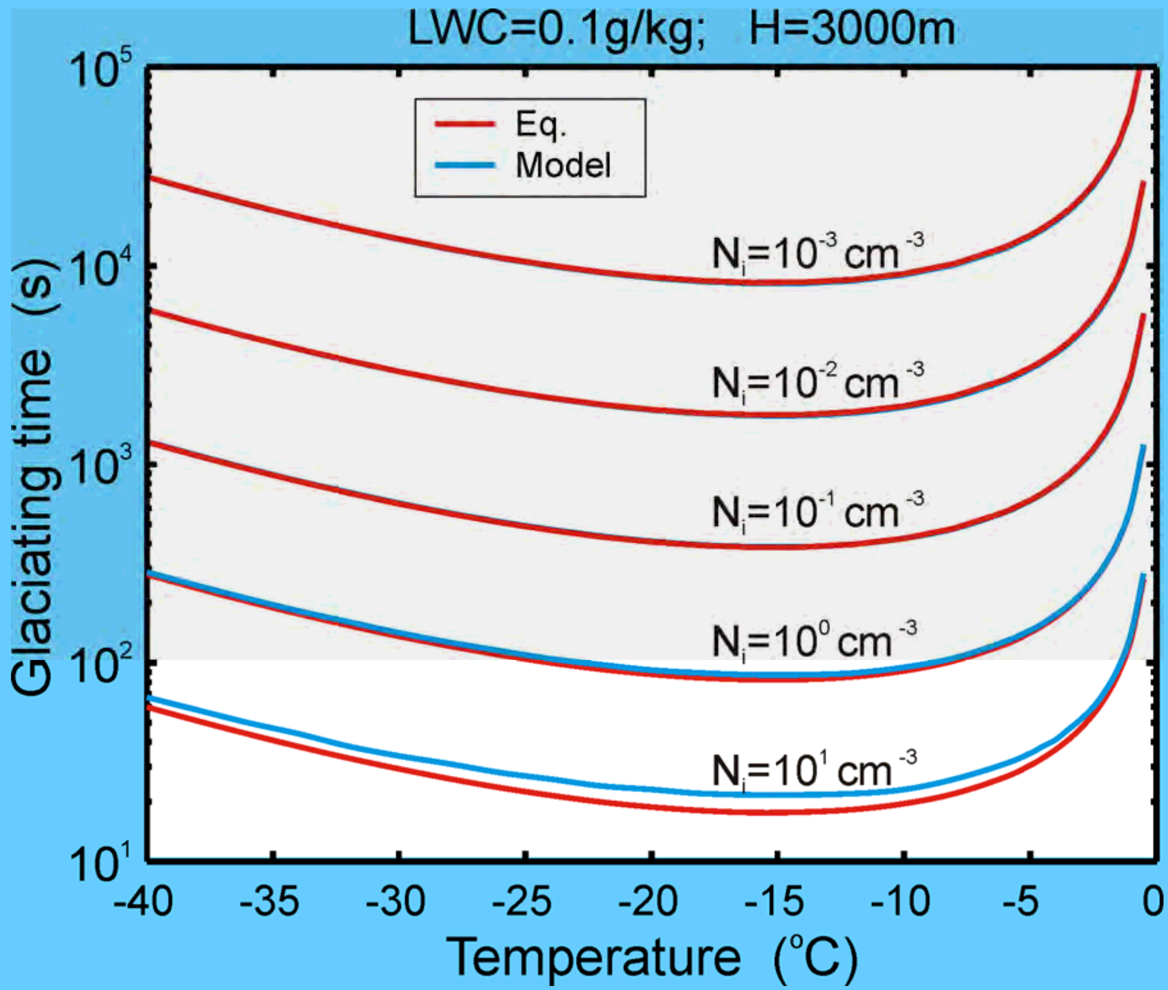
Mixed phase clouds:
recent progress in theoretical analysis

Alexei Korolev

Cloud Physics and Severe Weather Section,
Environment Canada, Toronto, Canada

Glaciation time for U_z=0

$$\tau_{gl} = \frac{1}{4\pi c A_i S_i}\left(\frac{9\pi\rho_i}{2}\right)^{1/3}\left(\left(\frac{W_w(t_0)+W_i(t_0)}{N_i}\right)^{2/3} - \left(\frac{W_i(t_0)}{N_i}\right)^{2/3}\right)$$

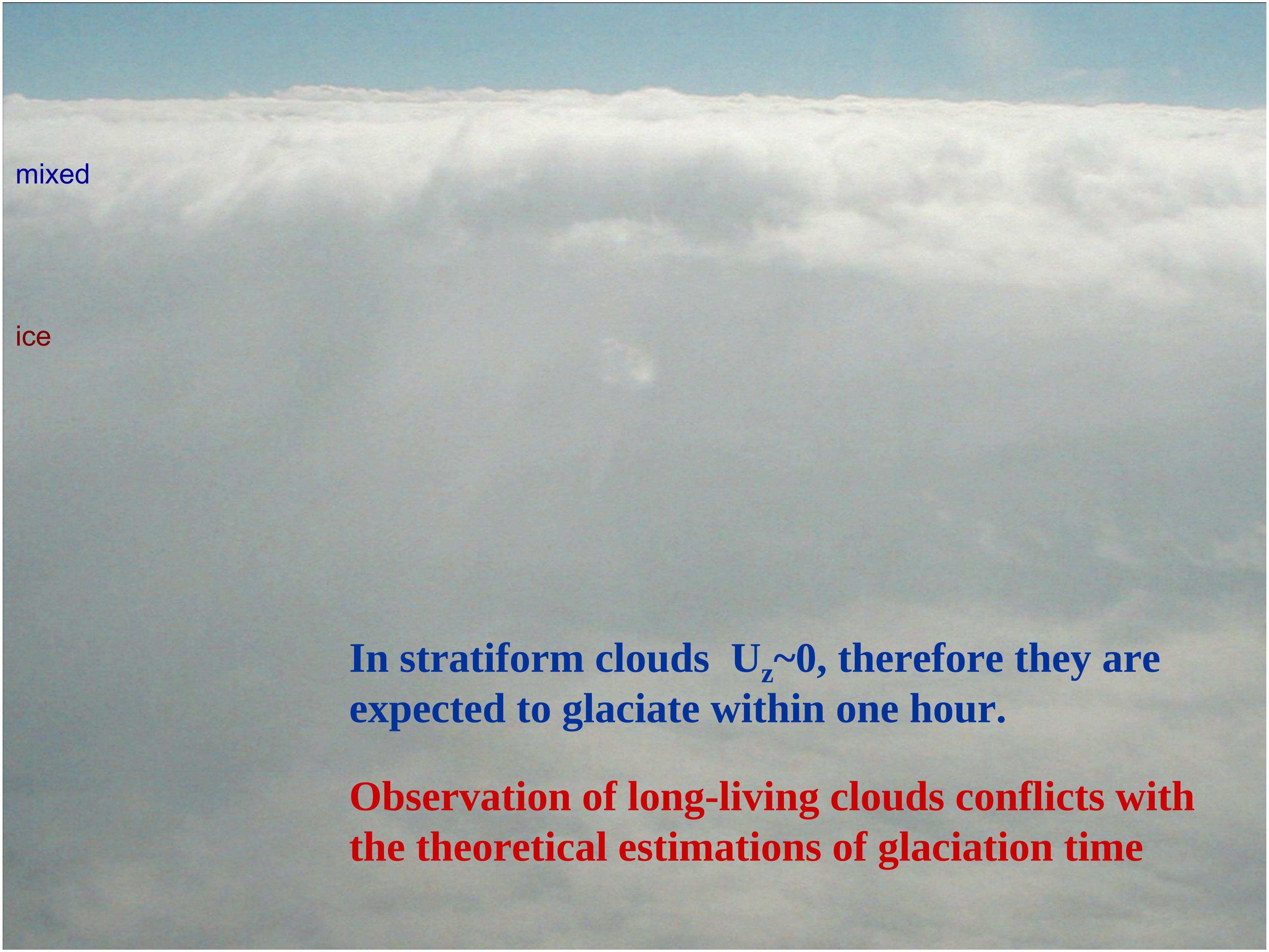


Korolev and Isaac (2003)

The documentation of long-living mixed phase stratiform layers at temperatures as low as -30C

Rauber, R.M. and A. Tokay, 1991: An explanation for the existence of supercooled water at the top of cold clouds *J. Atmos. Sci*., **48**, 1005–1023

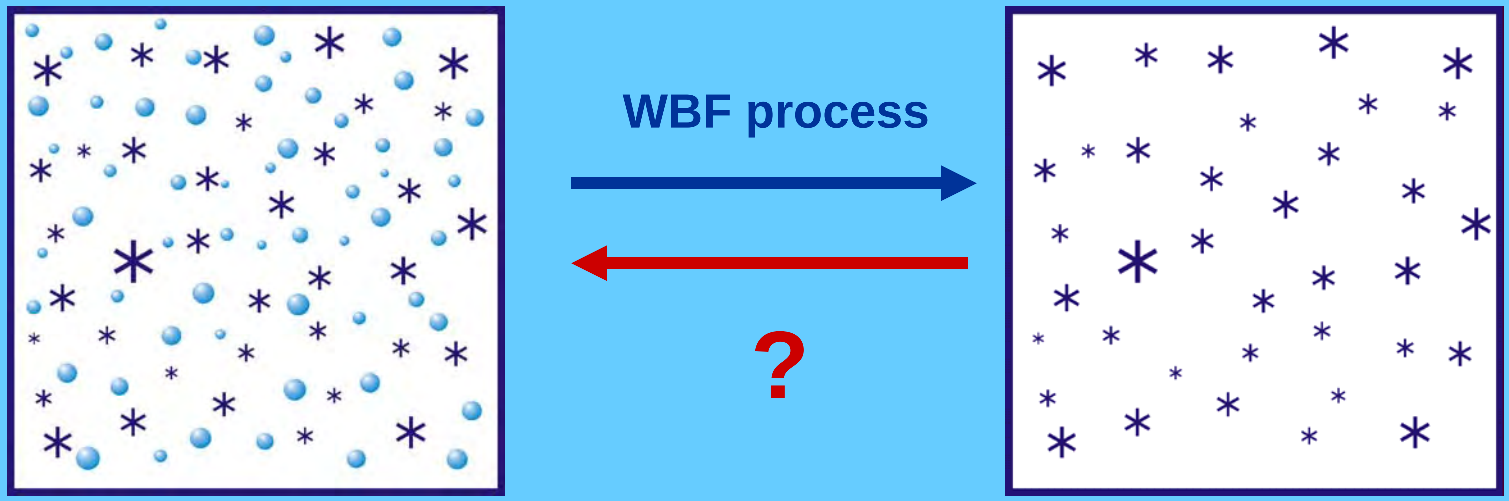
Pinto, J.O., 1998: Autumnal mixed-phase cloudy boundary layers in the Arctic. *J. Atmos. Sci*., **55**, 2016–2038



In stratiform clouds $U_z \sim 0$, therefore they are expected to glaciate within one hour.

Observation of long-living clouds conflicts with the theoretical estimations of glaciation time

Wegener, 1911
Bergeron, 1935
Findeisen, 1938



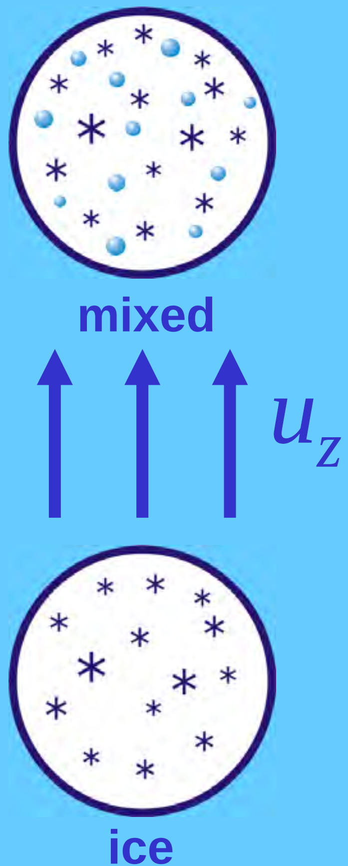
OBJECTIVE: To find conditions for the activation of liquid water in ice clouds and the maintenance of mixed phase

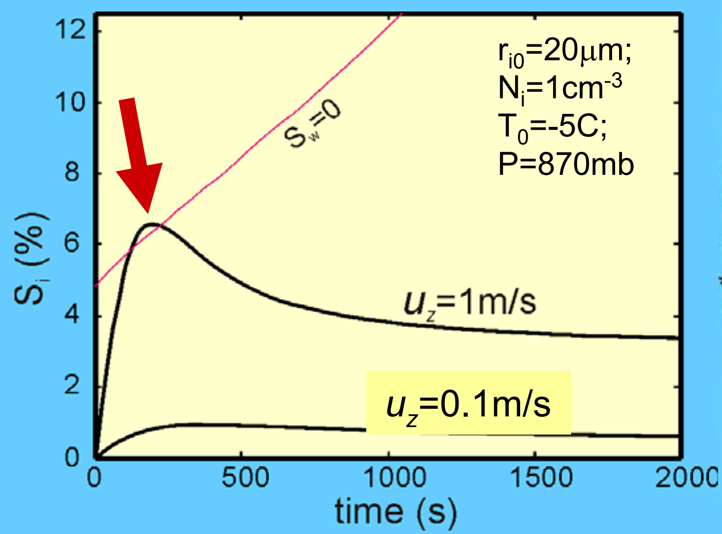
Activation of liquid in ascending adiabatic ice cloud parcel

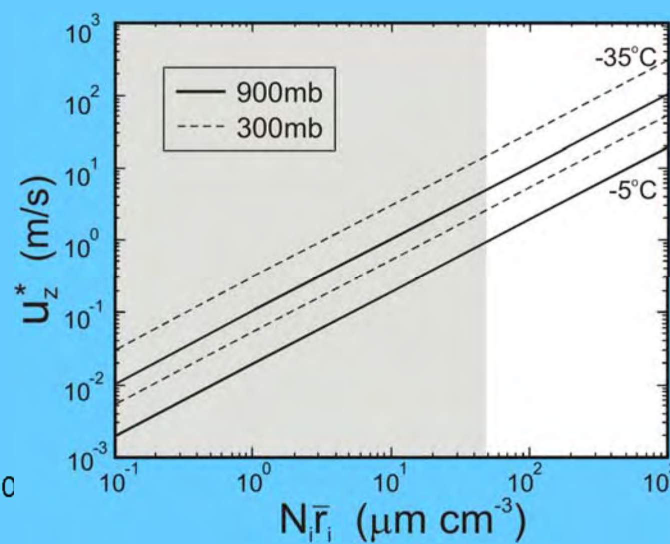
CONDITION #1

$$u_z^* = \frac{b_i^* N_i \bar{r}_i}{a_{0w}}$$ threshold vertical velocity, when $e=E_w$

$$u_z > u_z^*$$ condition for activation of liquid water in ice cloud







Korolev and Mazin, 2003, JAS

Activation of liquid in ascending adiabatic ice cloud parcel

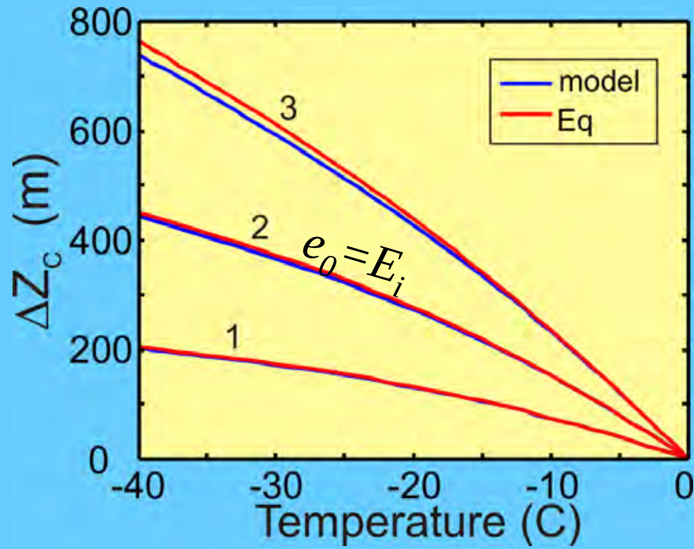
CONDITION #2

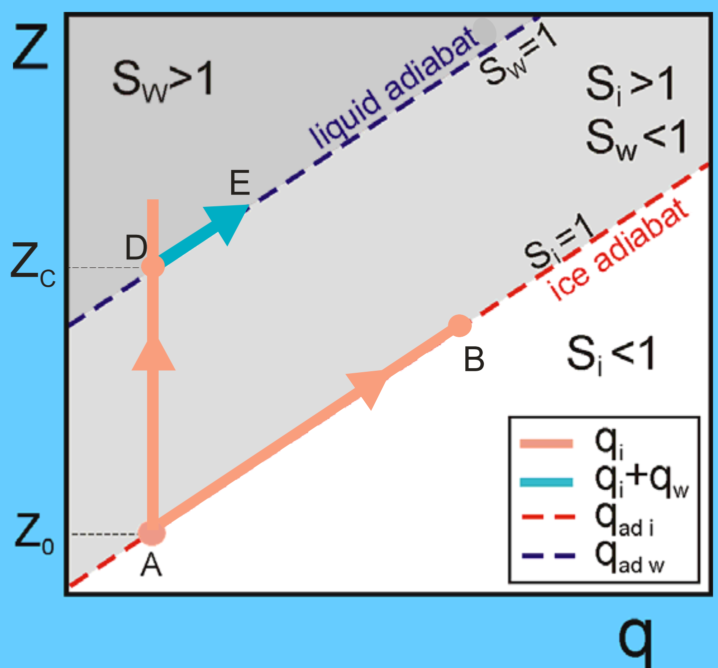
$$\tau_{ph} \approx \frac{b(T)}{N_i \bar{r}_{i0}}$$ time of phase relaxation

$$\tau_z \approx \frac{\Delta Z}{u_Z}$$ characteristic time of vertical motion

$$\tau_z >> \tau_{ph} \quad \rightarrow \text{line AB}$$

$$\tau_z << \tau_{ph} \quad \rightarrow \text{line AD}$$





$$\frac{1}{S_w+1}\frac{dS_w}{dt} = a_0 u_z - a_2 B_i^* N_i \bar{r}_i - \left(a_1 B_w N_w \bar{r}_w + a_2 B_i N_i \bar{r}_i\right) S_w$$

$$\frac{1}{S_w+1}\frac{dS_w}{dz} = a_w \quad \rightarrow \quad \boxed{\Delta Z_c = a_w^{-1} \ln\left(\frac{E_w}{e_0}\right)}$$

$$\boxed{\Delta Z > \Delta Z^*}$$ condition for activation of liquid water in ice cloud

$$\Delta Z^*_{\min} = \Delta Z_C \quad \text{for} \quad u_z \rightarrow \infty$$

$$\Delta Z < \Delta Z_C$$ no activation of liquid is possible

General formulation of the necessary and sufficient conditions for the activation of liquid in ice clouds

1st Necessary Condition: The vertical velocity of an ice cloud parcel must exceed a threshold velocity to activate liquid water.

$$u_z > u_z^*$$

2nd Necessary Condition: The activation of liquid water within an ice cloud parcel, below water saturation, requires a vertical ascent (ΔZ) above some threshold altitude (ΔZ^*) to bring the vapor pressure of the parcel to water saturation:

$$\Delta Z > \Delta Z^*$$

1st and 2nd conditions give a set of ***necessary and sufficient*** conditions

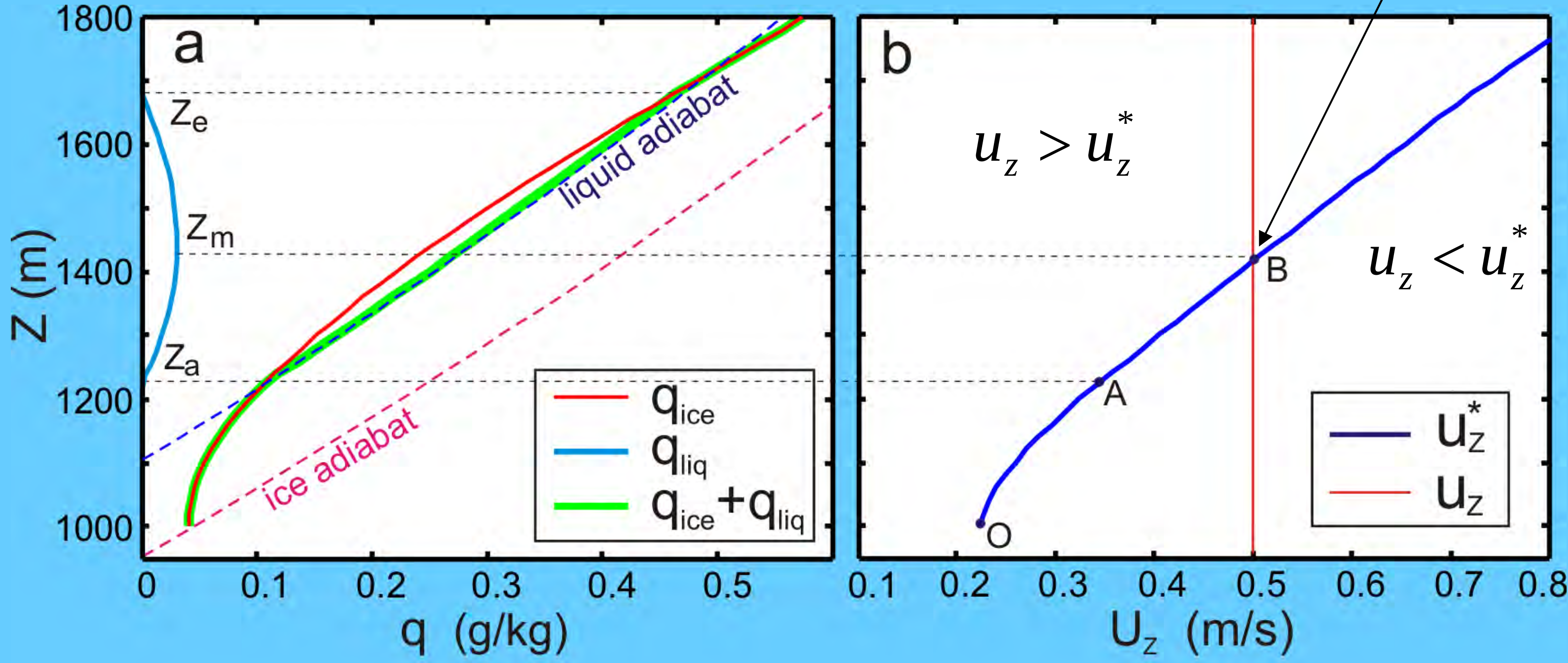
Korolev and Field, 2007, JAS

UNIFORM ASCENT
(LWC activation)

$$u_z = u_z^*$$

Numerical modeling of the activation of liquid water within ice cloud

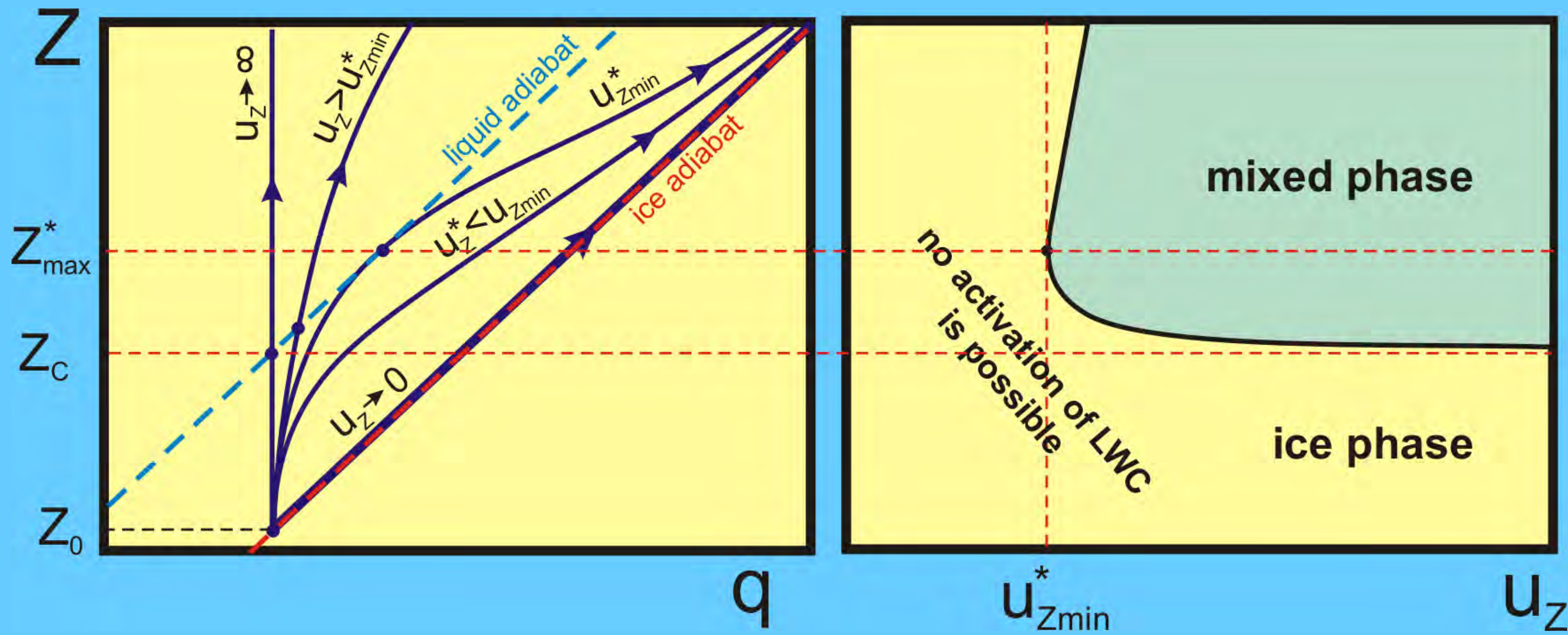
u_z=0.5m/s, N_i=100l^{-1}, r_{i0}=50μm, S_{i0}=1; T=-10C



$$u_z^* = \frac{b_i^* N_i \bar{r}_i}{a_{0w}}$$

UNIFORM ASCENT

(necessary and sufficient conditions)



Necessary and sufficient condition for the activation of liquid water in ice clouds for *uniform ascent*.

$$u_z > u_z^*$$

$$\Delta Z > \Delta Z^*$$

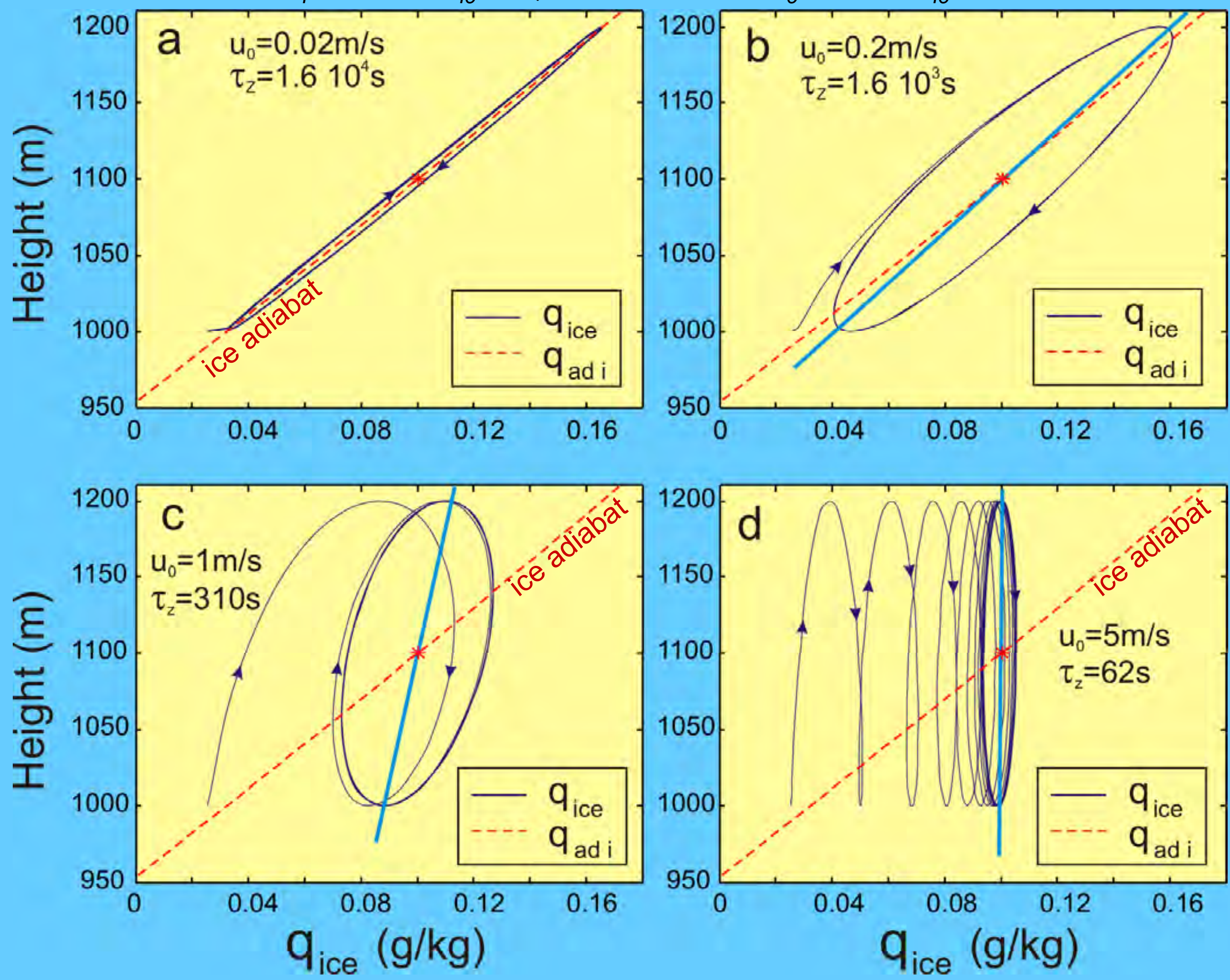
$$u^*_{Z\,\min}(N_i, r_{i0}, T_0)$$

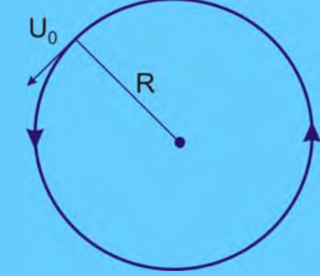
$$\Delta Z^*_{\max}(N_i, r_{i0}, T_0)$$

$$\Delta Z_C(T_0)$$

HARMONIC OSCILLATIONS

N_i=1000l^{-1}, r_{i0}=20μm, ΔZ=200m, T_0=-10C, S_{i0}=1.01





$$\bar{r}_{im} = \left(\frac{3q_{im}}{4\pi\rho_i N_i}\right)^{1/3}$$

$$q_{im} = q_{i0} + \delta q + \beta_i \frac{\Delta Z}{2}$$

$$T_m = T_0 + \delta T + \gamma_i \frac{\Delta Z}{2}$$

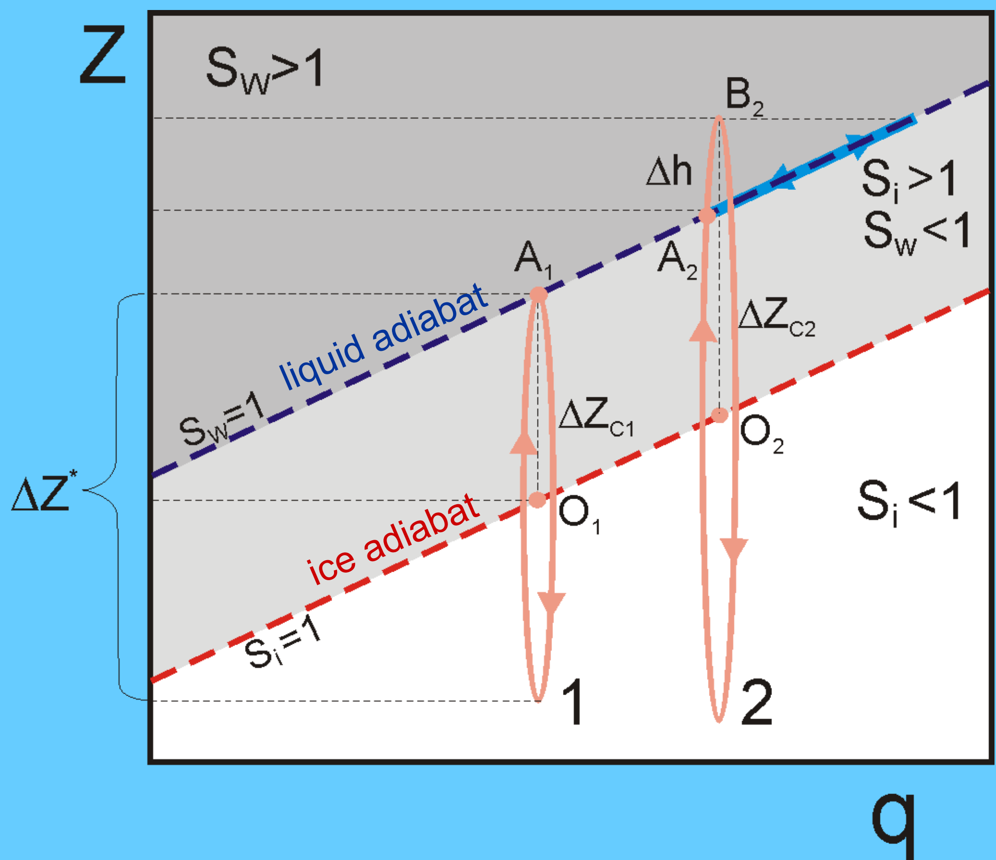
$$\bar{u}_z = \frac{2u_0}{\pi}; \qquad \tau_z \sim \frac{\Delta Z}{u_z}$$

For all four cases

$\tau_{ph} \approx 220$s

During vertical harmonic oscillations of an adiabatic cloud parcel, the microphysical parameters (e.g. particle sizes, condensed water, supersaturation, etc.) approach a limit-cycle.

HARMONIC OSCILLATIONS



$$\Delta Z > 2\Delta Z_c$$

$$u_z = u_0\sqrt{1 - 4\frac{\Delta Z_c^2}{\Delta Z^2}}$$

$$u_0^* = \frac{u_z^*\Delta Z}{\sqrt{\Delta Z^2 - 4\Delta Z_c^2}}$$

$$u_0^* = \frac{k_0 b_m (E_w - E_i) N_i \bar{r}_m \Delta Z}{E_i\sqrt{\Delta Z^2 - 4a_w^2 \ln^2\left(\frac{E_w}{E_i}\right)}}$$

$$u_0 > u_0^*$$

$$\Delta Z > 2\Delta Z_c$$

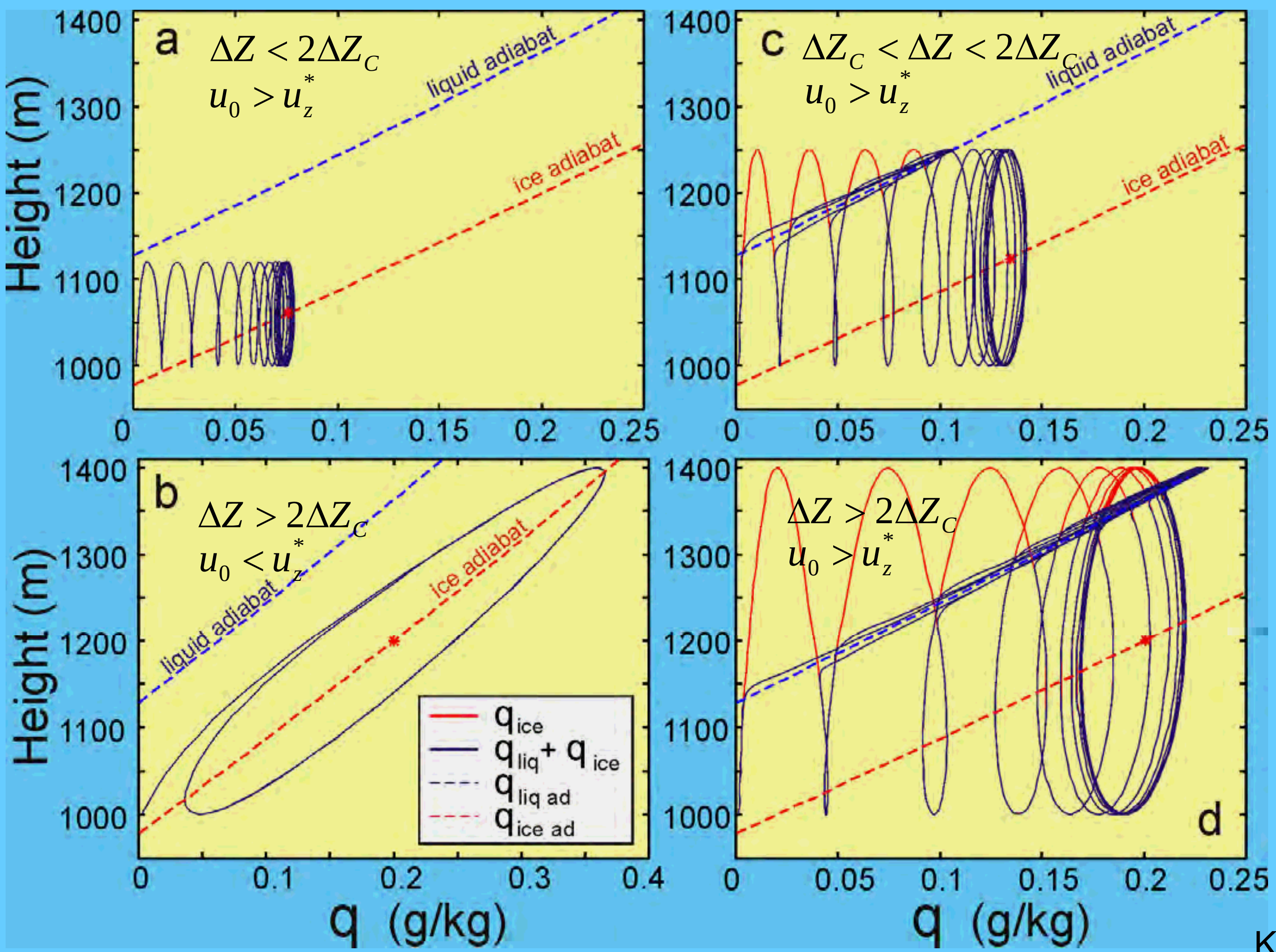
Necessary and sufficient conditions for the indefinitely long maintenance of mixed phase during *harmonic oscillations*.

HARMONIC OSCILLATIONS

modeling of the activation of liquid water in ice cloud during vertical harmonic oscillations.

N_{ice}=50l^{-1}, r_{i0}=20μm, S_{i0}=1.01, T_0=-10C, ΔZ_C=153m.

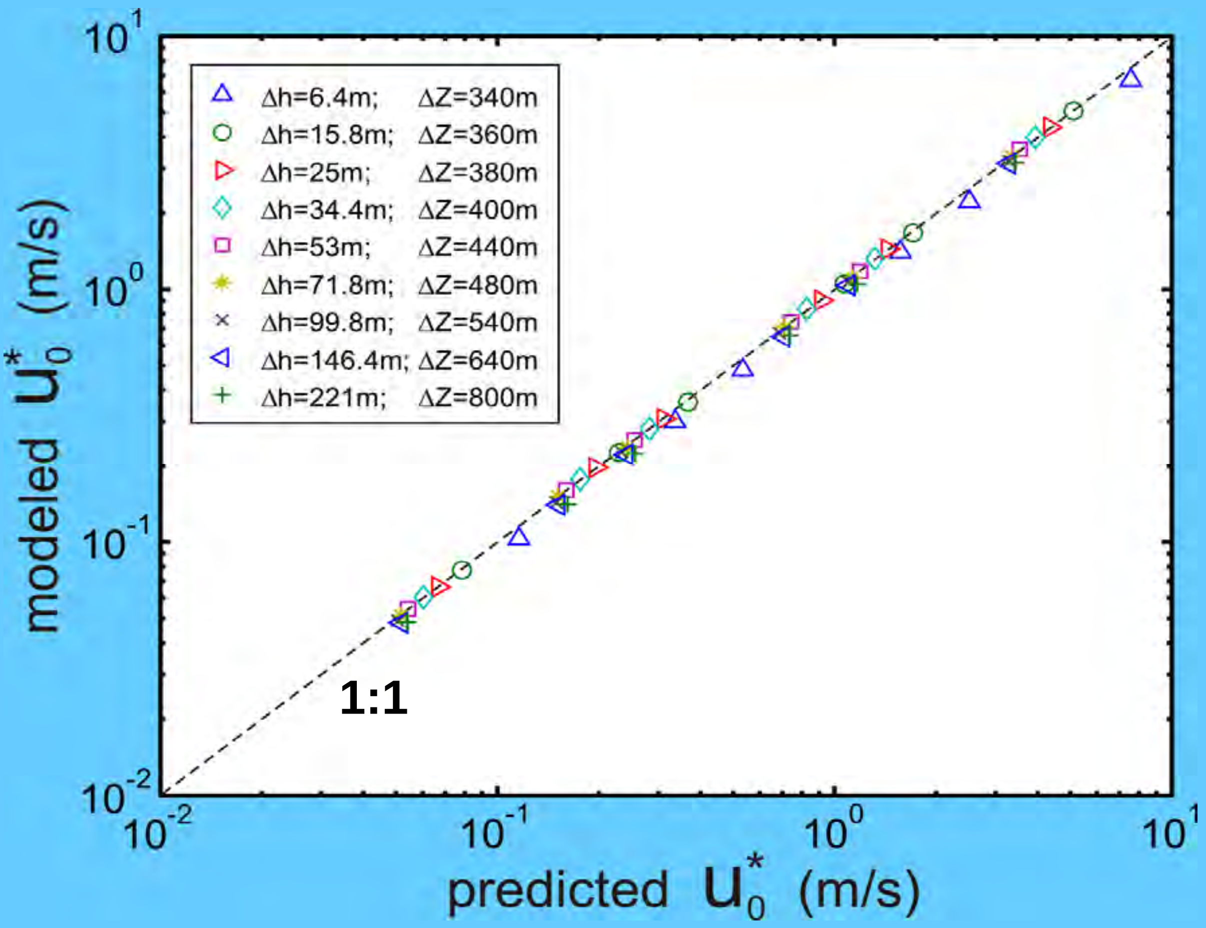
(a) ΔZ=125m, u_0=0.5m/s, u_0*=0.08m/s; ΔZ_C=153m
(c) ΔZ=250m, u_0=1m/s, u_0*= 0.10m/s; ΔZ_C=153m
(b) ΔZ=400m, u_0=0.05m/s, u_0*= 0.12m/s; ΔZ_C=153m
(d) ΔZ=400m, u_0=1m/s; u_0*= 0.12m/s ΔZ_C=153m



Korolev and Field, 2007, JAS

HARMONIC OSCILLATIONS

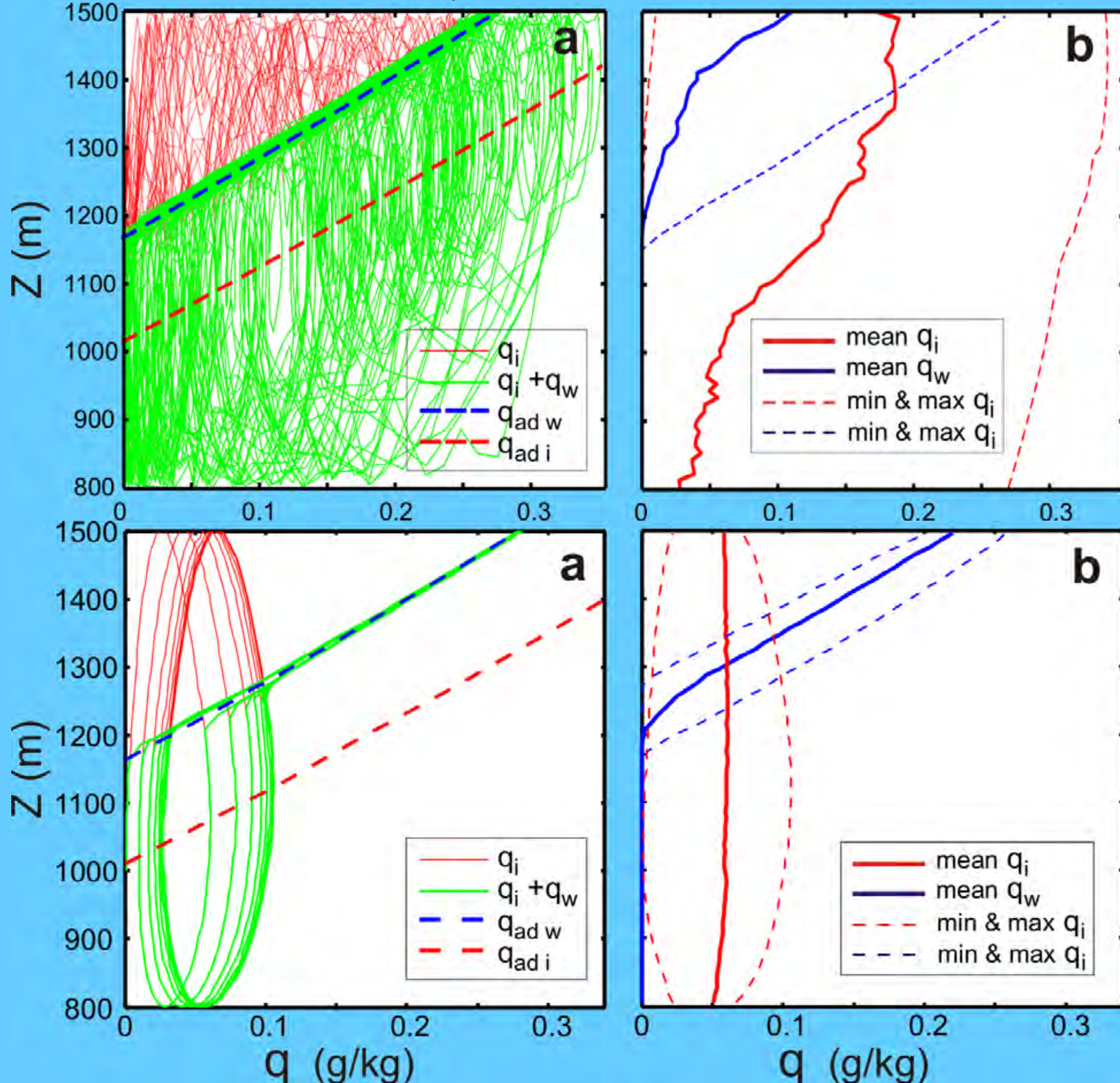
Comparison of the theoretical and modeled threshold velocity U^*_0
T_0=-10C, 340m<ΔZ<800m; 0.05m/s<u_0<6m/s;
20μm<r_{i0}<200μm; 50ℓ$^{-1}$ <N_i< 5000ℓ$^{-1}$



$$u_0^* = \frac{k_0 b_m (E_w - E_i) N_i \bar{r}_m \Delta Z}{E_i \sqrt{\Delta Z^2 - 4a_w^2 \ln^2\left(\frac{E_w}{E_i}\right)}}$$

Random vertical fluctuations

N_{ice}=50ℓ$^{-1}$; T_{top}=-14°C; T_{base}=-8°C; σ_u=0.75m/s

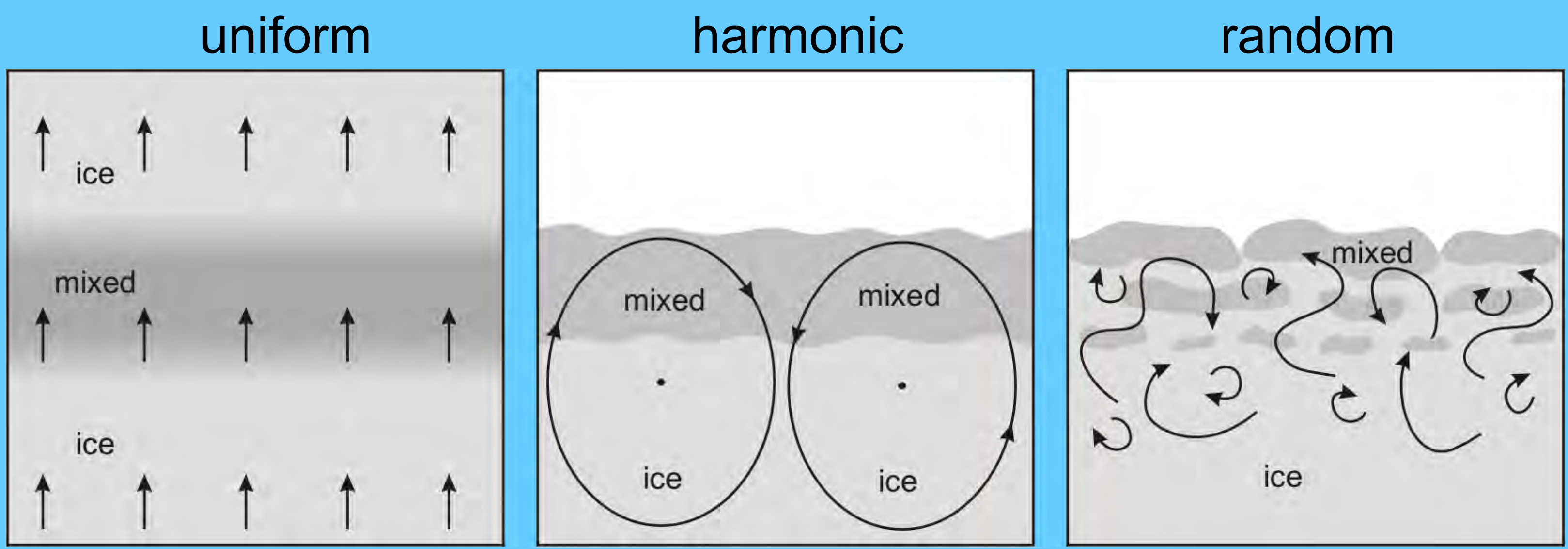


Lower sun pillar, subsun

Lower sun pillar, subsun

Lower sun pillar, subsun

The effect of dynamics of the formation of mixed phase



$$\tau_{mix\ ph} \sim \frac{\Delta Z}{u_Z}$$

$$\tau_{mix\ ph} \to \infty$$

$$\tau_{mix\ ph} \to \infty$$

The conditions required for the maintenance of mixed phase can be used for the parameterization and forecast of mixed phase clouds in numerical models.