WRF 4DVAR

1. WRF 4DVAR project
2. Formulation
3. Status

WRF 4DVAR project

Supported by AFWA

The team: Dale, Hans, John, Qingnong, Wei H.

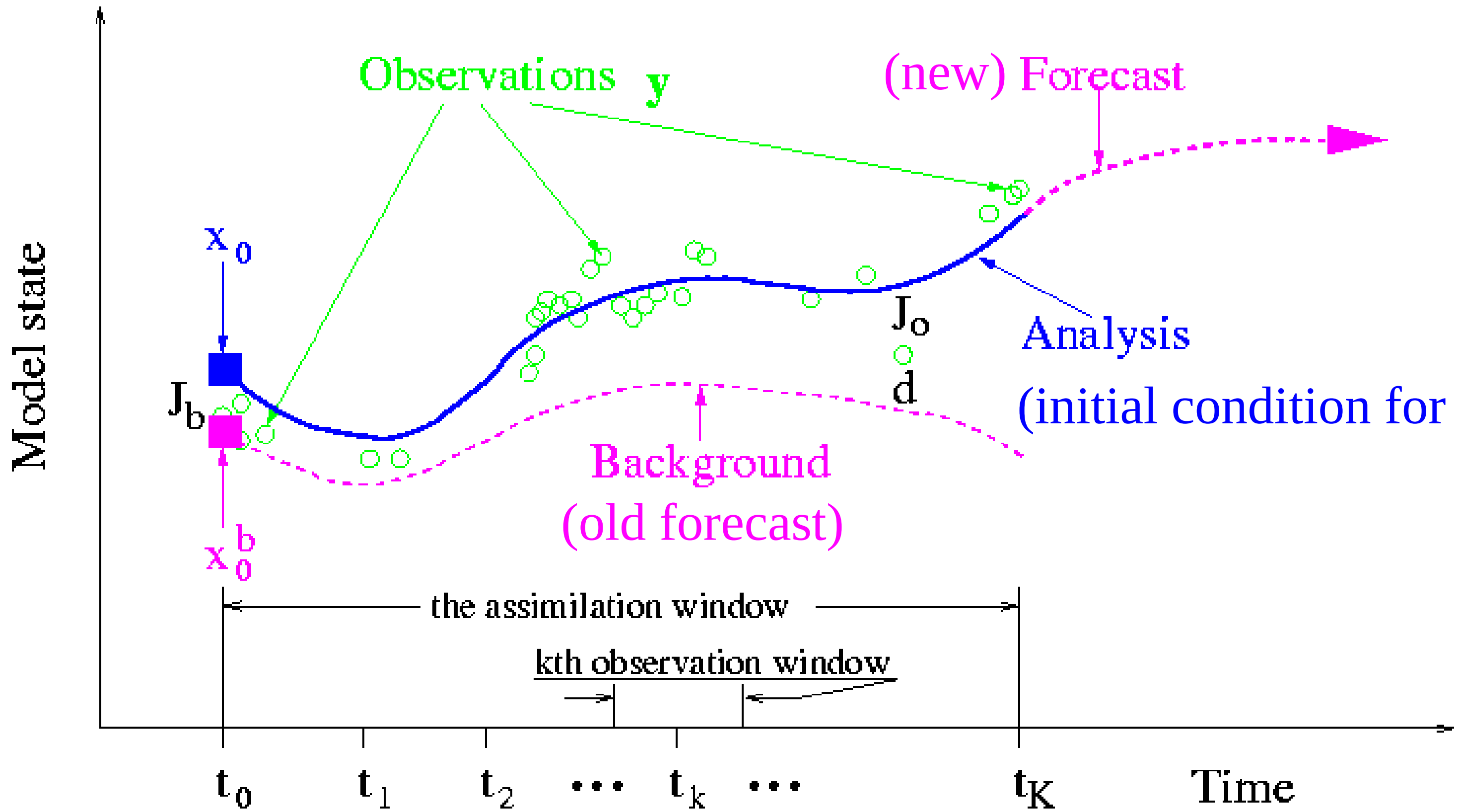
Schedule

- FY04: prepare. (wrf model, simplified model, testing TAF on wrf subroutines.)
- FY05: construct. (4DVAR framework, basic (dry) wrf TL and AD components, initial experiments.)
- FY06: refine. (more physics, parallel code, extensive testing.)

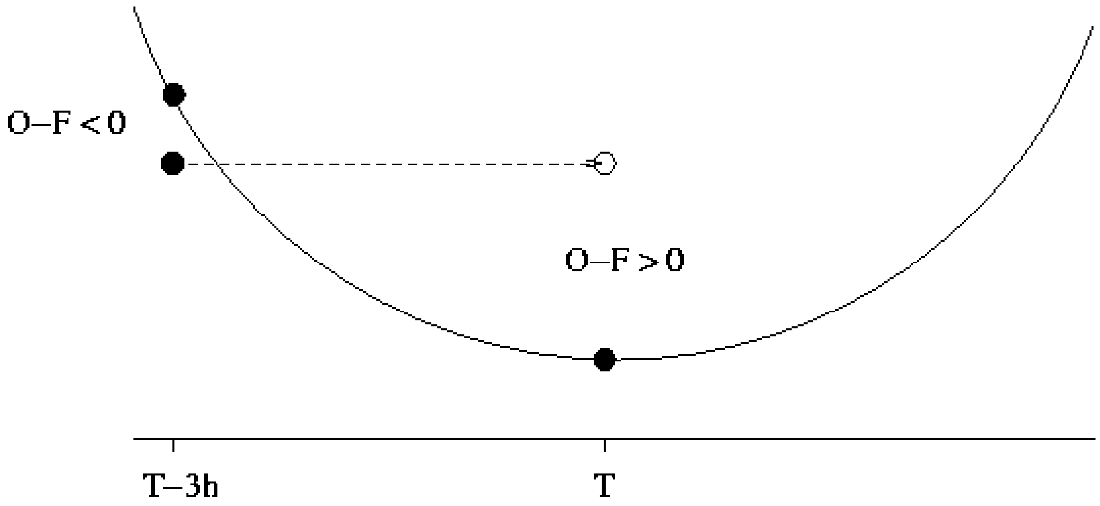
Variational methods

$$J = \frac{1}{2}\left\{\left(\mathbf{x}-\mathbf{x}_0^b\right)^T \mathbf{B}^{-1}\left(\mathbf{x}-\mathbf{x}_0^b\right) + \left[H(M(\mathbf{x}))-\mathbf{y}\right]^T \mathbf{R}^{-1}\left[H(M(\mathbf{x}))-\mathbf{y}\right]\right\}$$

$\delta\mathbf{x}$



FGAT: First Guess at Appropriate Time



Algorithms

4DVAR:

$$J' = \mathbf{B}^{-1}\delta\mathbf{x} + \mathbf{M}^T\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\mathbf{M}\delta\mathbf{x} + \mathbf{M}^T\mathbf{H}^T\mathbf{R}^{-1}\left\{H\left[M(\mathbf{x})\right] - \mathbf{y}\right\}$$

3DVAR: $M = \mathbf{M} = \mathbf{M}^T = \mathbf{I}$

$$J' = \mathbf{B}^{-1}\delta\mathbf{x} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\delta\mathbf{x} + \mathbf{H}^T\mathbf{R}^{-1}\left[H(\mathbf{x}) - \mathbf{y}\right]$$

FGAT: $\mathbf{M} = \mathbf{M}^T = \mathbf{I}$, but $M \neq \mathbf{I}$

$$J' = \mathbf{B}^{-1}\delta\mathbf{x} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\delta\mathbf{x} + \mathbf{H}^T\mathbf{R}^{-1}\left\{H\left[M(\mathbf{x})\right] - \mathbf{y}\right\}$$

Important issues

- *H* observation operator, including the tangent linear operator **H** and the adjoint operator $\mathbf{H}^T$.
- *M* forecast model, including the tangent linear model **M** and adjoint model $\mathbf{M}^T$.
- **B** background error covariance (N*N matrix).
- **R** observation error covariance which includes the representative error (K*K matrix).

The background error constraint

We look for a variable transform: $\chi = \mathbf{U}\,\delta\mathbf{x}$
with $\mathbf{B}^{-1} = \mathbf{U}^T\mathbf{U}$ so that $2J_{\mathbf{B}}(\chi) = \chi^T\chi$, $J'_{\mathbf{B}}(\chi) = \chi$

The outer-loop/multi-incremental formulation

Start with (*0*th loop): $\mathbf{x}^0 = \mathbf{x}^{\mathbf{b}}$
For the *n*th loop (note: n is neither iteration nor timestep):

$$J'_{\delta\mathbf{x}^n} = \mathbf{B}^{-1}(\delta\mathbf{x}^n + \mathbf{x}^{n-1} - \mathbf{x}^{\mathbf{b}}) + \sum_{k=0}^{K} \mathbf{M}_k^{\mathbf{T}}\mathbf{H}_k^{\mathbf{T}}\mathbf{R}^{-1}[\mathbf{H}_k\mathbf{M}_k\delta\mathbf{x}^n + H_k(M_k(\mathbf{x}^{n-1})) - \mathbf{y}_k]$$

$$\mathbf{x}^n = \mathbf{x}^{n-1} + \delta\mathbf{x}^n$$

Implementation

Use U-transform; group obs to K windows

$$\boldsymbol{\chi} = \mathbf{U}\,\delta\mathbf{x} \quad \text{and} \quad \mathbf{x}^{n-1}-\mathbf{x}^{\mathbf{b}} = (\mathbf{x}^{n-1}-\mathbf{x}^{n-2}) + (\mathbf{x}^{n-2}-\mathbf{x}^{n-3}) + \dots + (\mathbf{x}^{0}-\mathbf{x}^{\mathbf{b}})$$

$$J'_{\chi^n} = \chi^n + \sum_{i=1}^{n-1}\chi^{\mathbf{i}} + \mathbf{U}^{-\mathbf{T}} \sum_{k=1}^{K}\mathbf{M}_k^{\mathbf{T}}\mathbf{H}_k^{\mathbf{T}}\mathbf{R}^{-\mathbf{1}}[\mathbf{H}_k\mathbf{M}_k\mathbf{U}^{-\mathbf{1}}\chi^n + H_k(M_k(\mathbf{x}^{n-1})) - \mathbf{y}_k]$$

$$\mathbf{x}^n = \mathbf{x}^{n-1} + \mathbf{U}^{-\mathbf{1}}\chi^{\mathbf{n}}$$

WRFVDA structure

Outer-loop: innovation; wrf_nl

$$J'_{\chi}{}^{n} = \chi^n + \sum_{i=1}^{n-1} \chi^{\mathbf{i}} + \mathbf{U}^{-T} \sum_{k=1}^{K} \mathbf{M}_k{}^{T} \mathbf{H}_k{}^{T} \mathbf{R}^{-1}[\mathbf{H}_k \mathbf{M}_k \mathbf{U}^{-1}\chi^n + H_k(M_k(\mathbf{x}^{n-1})) - \mathbf{y}_k]$$

Inner-loop:

vtox

wrf_tl

xtoy

Jo_grad_y

xtoy_ad

wrf_ad

vtox_ad

Step I. Prepare WRFVDA (start from WRF3DVAR)

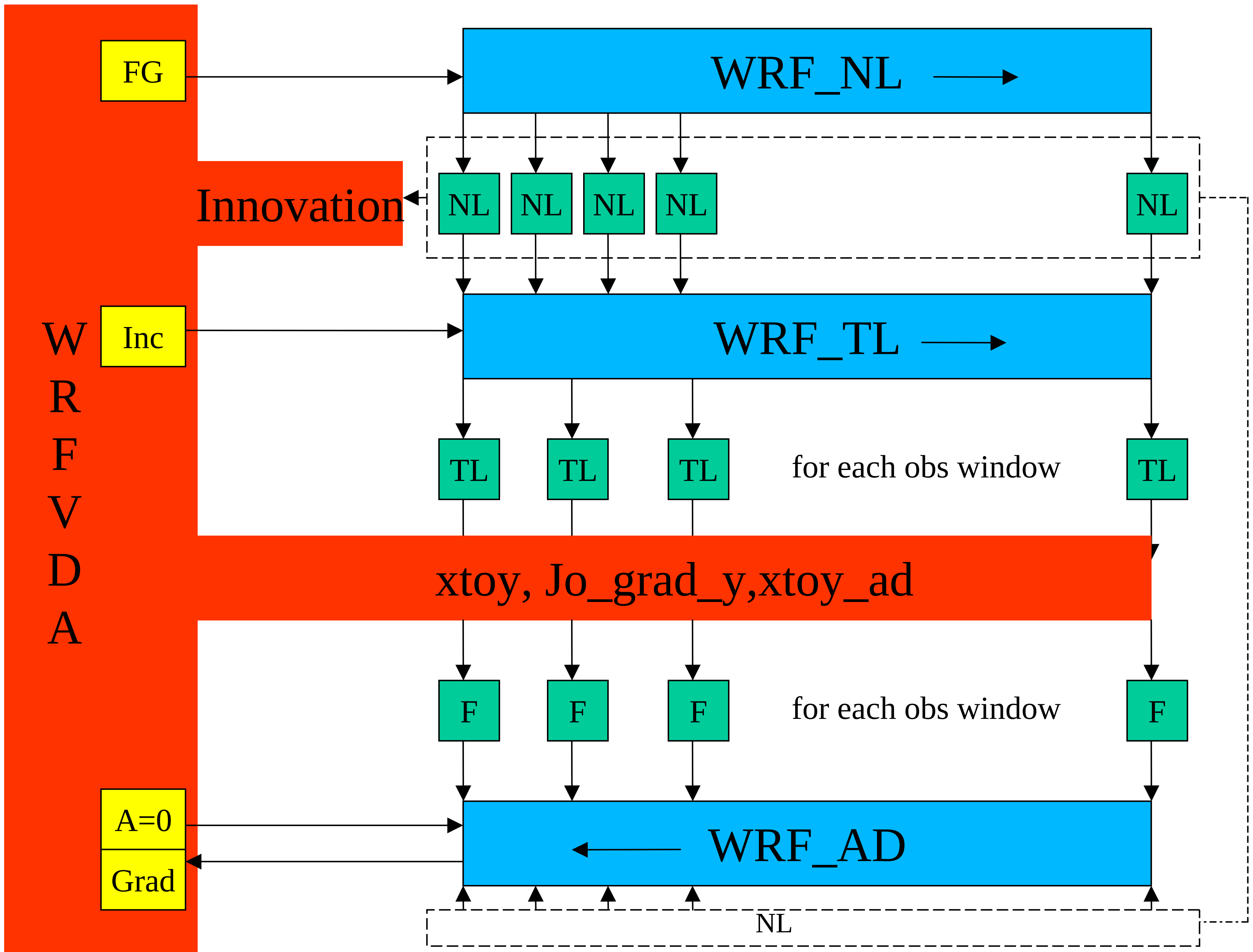
Subroutine	Comments	Current status
innovation	Break H to H_k	Ready due to FGAT
wrf_nl	Exists	Start with a system call
vtox	The same as in 3DVAR	Ready
wrf_tl	New development	Start with I
xtoy	Break $\mathbf{H}$ to $\mathbf{H}_k$	Done
Jo_grad_y	Can be applied for all obs win	Ready
xtoy_ad	Break $\mathbf{H}^T$ to $\mathbf{H}^T{}_k$	Done
wrf_ad	New development	Start with I
vtox_ad	The same as in 3DVAR	Ready

Stdout from wrfvda.exe

```
start outerloop          1
 1. for 4dvar compute the guess x_g
 ---------------------------------------------------
  wrf nonlinear model run ...
  full resolution model states at time           0 s
  full resolution model states at time        3600 s
  full resolution model states at time        7200 s
  full resolution model states at time       10800 s
 2. calculate innovation [y-H(x_g)]
 ---------------------------------------------------
 3. interpolate x_g to low resolution grid
 ---------------------------------------------------
  read fullres and write lowres at time          0 s
  read fullres and write lowres at time       3600 s
  read fullres and write lowres at time       7200 s
  read fullres and write lowres at time      10800 s
 4. minimise the cost function J
 ---------------------------------------------------
  innerloop          0
d    jo_grad_y
e    xtoy_ad, write forcing for nobwin: 4   to file:  75
     xtoy_ad, write forcing for nobwin: 3   to file:  74
     xtoy_ad, write forcing for nobwin: 2   to file:  73
     xtoy_ad, write forcing for nobwin: 1   to file:  72
f    wrf adjoint model run ...
        read forcing at time  10800 s  read from file: 75
        read forcing at time   7200 s  read from file: 74
        read forcing at time   3600 s  read from file: 73
        read forcing at time      0 s  read from file: 72
g    vtox_adj
```

```
 innerloop          1
a    vtox
b    wrf tangent linear model run ...
        wrf_tl at time              0 s  write to file:  76
        wrf_tl at time           3600 s  write to file:  77
        wrf_tl at time           7200 s  write to file:  78
        wrf_tl at time          10800 s  write to file:  79
c    xtoy, read xa from tl ourput for nobwin: 1 from file: 76
     xtoy, read xa from tl ourput for nobwin: 2 from file: 77
     xtoy, read xa from tl ourput for nobwin: 3 from file: 78
     xtoy, read xa from tl ourput for nobwin: 4 from file: 79
d    jo_grad_y
e    xtoy_ad, write forcing for nobwin:        4 to file:  75
     xtoy_ad, write forcing for nobwin:        3 to file:  74
     xtoy_ad, write forcing for nobwin:        2 to file:  73
     xtoy_ad, write forcing for nobwin:        1 to file:  72
f    wrf adjoint model run ...
        read forcing at time  10800 s  read from file: 75
        read forcing at time   7200 s  read from file: 74
        read forcing at time   3600 s  read from file: 73
        read forcing at time      0 s  read from file: 72
g    vtox_adj
 5. update the latest analysis
 ---------------------------------------------------
 6. no request to diagnose w increment
 ---------------------------------------------------
 7. write out diagnostics
 ---------------------------------------------------
 8. interpolate ana inc to the original grid
    put lowres inc back to fullres
 ---------------------------------------------------
 9. write out analysis and analysis increments
 ---------------------------------------------------
…
start outerloop          2
```

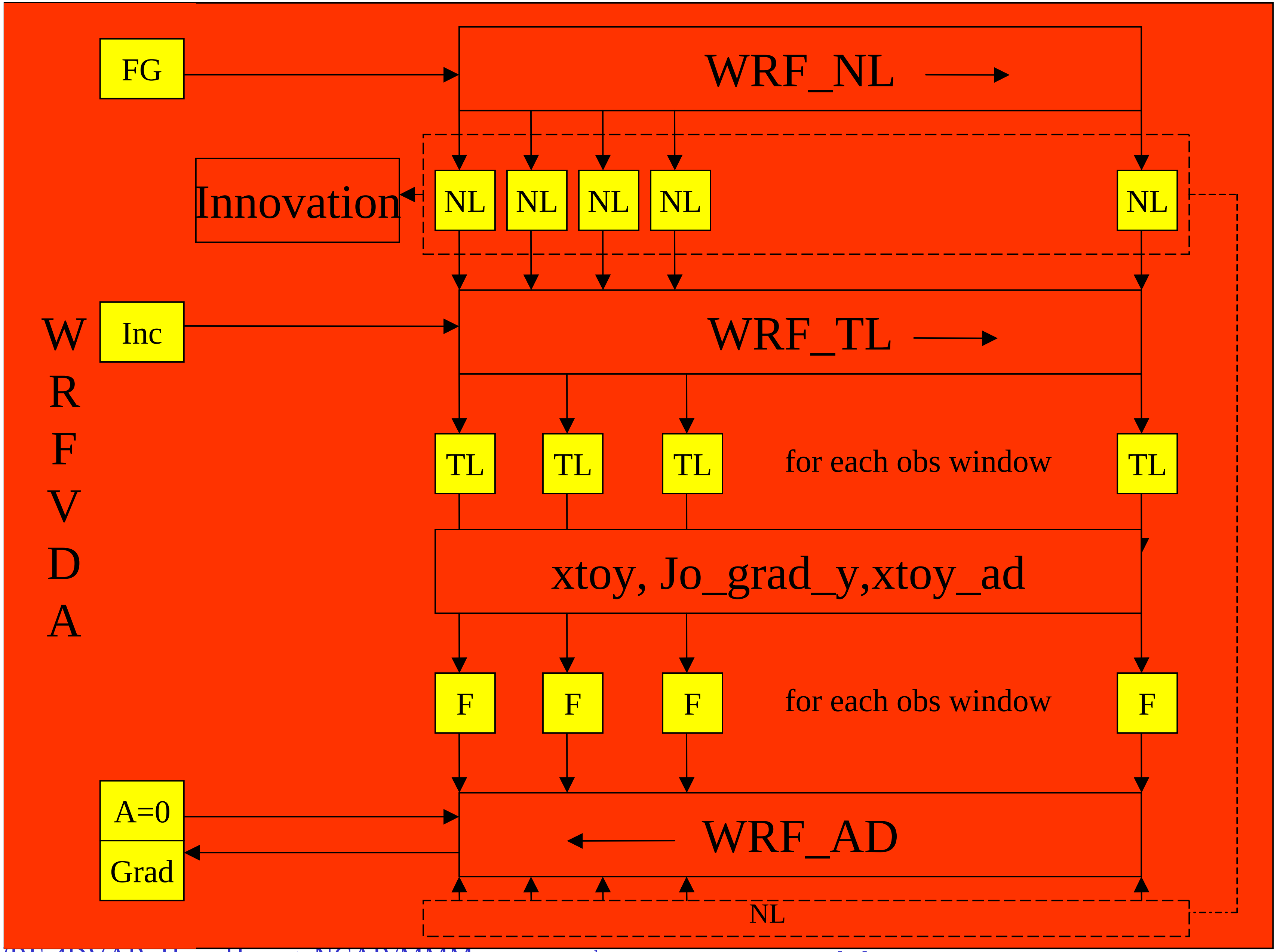
Step II. Use separate executables, communicate through I/O



Develop WRF_SN, WRF_TL, WRF_AD

- Started with the latest WRF_NL, v_2_0_3_1, a highly simplified version has been derived, WRF_SN.
- Testing WRF_SN.
- Generating WRF_TL and WRF_AD from WRF_SN using TAF.

Step III. Integrate HIRVDA



Step IV. Improve the performance

- Multi-resolution
- MPP
- More physics
- Start single observation experiments
- Start real data experiments

Noise control J_c using digital filters

$$J_C = \frac{1}{2}(\mathbf{x}_{N/2} - \mathbf{x}^{DF}_{N/2})^T C^{-1} (\mathbf{x}_{N/2} - \mathbf{x}^{DF}_{N/2})$$

$$\mathbf{x}_{N/2} - \mathbf{x}^{DF}_{N/2} = \mathbf{x}_{N/2} - \sum_{n=0}^{N} f_{\mathbf{n}}\, \mathbf{x}_{\mathbf{n}} = \sum_{n=0}^{N} h_{\mathbf{n}}\, \mathbf{x}_{\mathbf{n}} = \sum_{n=0}^{N} h_{\mathbf{n}}\, M_n(\mathbf{x_0})$$

$$J'_C = \sum_{k=N}^{0} h_{\mathbf{k}}\, \mathbf{M}_{\mathbf{k}}^{\mathbf{-T}} C^{-1} \sum_{n=0}^{N} h_{\mathbf{n}}\, M_n(\mathbf{x_0})$$

$$\chi = \mathbf{U}(\mathbf{x_0} - \mathbf{x_g})$$

$$J'_C = \mathbf{U}^{\mathbf{-T}} \sum_{k=N}^{0} h_{\mathbf{k}}\, \mathbf{M}_{\mathbf{k}}^{\mathbf{-T}} C^{-1} \sum_{n=0}^{N} h_{\mathbf{n}}\, [M_n(\mathbf{x_g}) + \mathbf{M_n} \mathbf{U}^{\mathbf{-1}} \chi]$$