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The organization of moist convection by internal gravity waves

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ABSTRACT

A two-dimensional, Boussinesq approximated model is used to investigate the grouping of moist convection and its connection with internal gravity waves. The effect of moisture has been taken into account as a flow-dependent heating. The model sensitivity to parameters has been studied systematically. Numerical simulations are started from a state of rest with a conditionally unstable stratification. Distinct individual convective cells appear soon after small random perturbations are introduced. These cells later become organized into groups, as the equilibrium state of the model is approached asymptotically in the form of an oscillation. The oscillation of the condensational heating generates gravity waves, which are embedded in the subsidence of the convective cells and thus propagate in a stably stratified environment. The gravity waves modulate the generation of new convective cells which in turn introduce new heat sources, giving rise to additional gravity waves, and so on. If the lapse rate of the reference temperature profile is unstable, the oscillation of the condensational heating will persist for a long time and more updrafts will appear. If the dissipation is weak, the waves can propagate a long distance without being significantly damped. In these situations, strong nonlinear interactions between waves and moist updrafts are possible which leads to the grouping of convective clouds.

1. Introduction

In the atmosphere, cumulus clouds often occur in groups, which have a spatial scale much larger than the individual cloud cells and maintain their identity over many cloud lifetimes. The members of each group are much more closely spaced than the average spacing over the cloud population. Plank (1969) has performed an aircraft photographic survey of Florida cumulus populations in summer. In a typical diurnal cycle, clouds are relatively small and spaced uniformly during the first two hours of moist convection in the morning; these individual clouds tend to clump together and group structures form around midday (1030–1300 local time); finally the group structures decrease in size and eventually dissipate as the stratification of the atmosphere stabilizes in the late afternoon or in the evening. The evolution of moist convection in the above study can be assumed to be independent of the synoptic situation and surface features. Therefore, there must exist some mechanism which is responsible for organizing individual convective cells into convective clusters.

In most theories, grouping of clouds is a result of the interaction between the motion field, temperature field and moisture anomalies. Moisture therefore appears to be vital for the grouping process. In the numerical study by Hill (1974), the main factor governing the development of a multi-cell cloud has been found to be the incorporation of smaller surrounding moisture anomalies into the domain of a well-developed cumulus cloud. López (1978) has pointed out that the strong dynamic forcing provided by evaporative downdrafts formed in raining clouds can initiate new clouds nearby. Using a stochastic

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model, Randall and Huffman (1980) have found mesoscale groups of cumulus clouds by assuming that cumulus clouds tend to moisten the near environment.

Using a two-dimensional cumulus convection model with random forcing from below, Van Delden and Oerlemans (1982) found that the organization of clouds takes place through the competition between adjacent updrafts in which latent heat release plays an important role, and through a strong dynamical forcing provided by sharp downdrafts at the cloud edges which favours the formation of new clouds in the vicinity. Although Van Delden and Oerlemans claimed that gravity waves are not needed to generate organized patterns, the cloud cover evolution shown in Fig. 1 and Fig. 2 of their paper do reveal some wave-like patterns.

Huang (1988) has performed a study of moist convection with a Boussinesq approximated model, where the effect of moisture is only taken into account as a flow-dependent heating. In numerical integrations with a constant reference static stability three regimes of moist convection were found: first a growing stage with distinct individual convective cells; later a grouping stage in which individual cells collapse and become organized; and finally a stage with only one updraft in the model domain. When a variable static stability is used to simulate a diurnal cycle, the time evolution of the updraft pattern has some similarities to that observed by Plank (1969), although a direct comparison is difficult due to the two-dimensionality of the numerical model. As in the study by Van Delden and Oerlemans (1982), wave-like patterns in the motion field have also been described by Huang (1988), see Fig. 5 and Fig. 7 of that paper, but not very much attention was given to this phenomenon. The main purpose of this paper is to extend the previous work, i.e., to investigate the interaction of moist convection and internal gravity waves. In particular, we will try to investigate under which conditions small scale convective cells can become organized into mesoscale cloud groups.

In the ascending branch of a moist convective cell, condensation takes place. The condensational heating generates gravity waves in a stably stratified environment. When clouds are formed, the convectively generated gravity waves have been observed from visible and infrared satellite pictures (Erickson and Whitney, 1973). These waves may propagate horizontally and, under certain conditions, intensify the old convective systems and initiate new ones (Balachandran, 1980). Vertically propagating waves excited by cumulus clouds have also been observed by Kuettner, et al. (1987) through flight measurements and simulated by Clark et al. (1986) with a two-dimensional numerical model. These waves in turn feed back on the cumulus clouds by changing their spacing and intensity. The gravity waves generated by convective clouds should therefore play a role in the organization of moist convection. The numerical results by Van Delden and Oerlemans (1982) and Huang (1988) also point in this direction.

In this study, we use a simple numerical model in which the parameter sensitivity of the model can easily be investigated, and the grouping mechanism is present. Grouping structures like those in more complicated models are obtained in the model simulations. Even if the model ignores many physical processes, it has the advantage of isolating the particular mechanisms we are concerned with and giving qualitative explanations to some phenomena observed in reality. The model and the numerical scheme are presented in Section 2. The numerical results are shown and discussed in Section 3. Finally, conclusions are given in Section 4.

2. The model

We consider a two-dimensional flow between horizontal boundaries and we keep the temperatures at the top and bottom constant, thus fixing the lapse rate of the reference temperature profile. When convection sets in, the temperature profile will deviate from the reference one as convection stabilizes the flow towards a convectively neutral stratification. The model properties are influenced by the height of the lifting condensation level as it determines how strong the condensational heating can be and where the heating maximum will appear (see Huang and Källén, 1986). However, in this paper we are mostly interested in the grouping of convection and the gravity waves generated by condensational heating, and as far as the grouping and

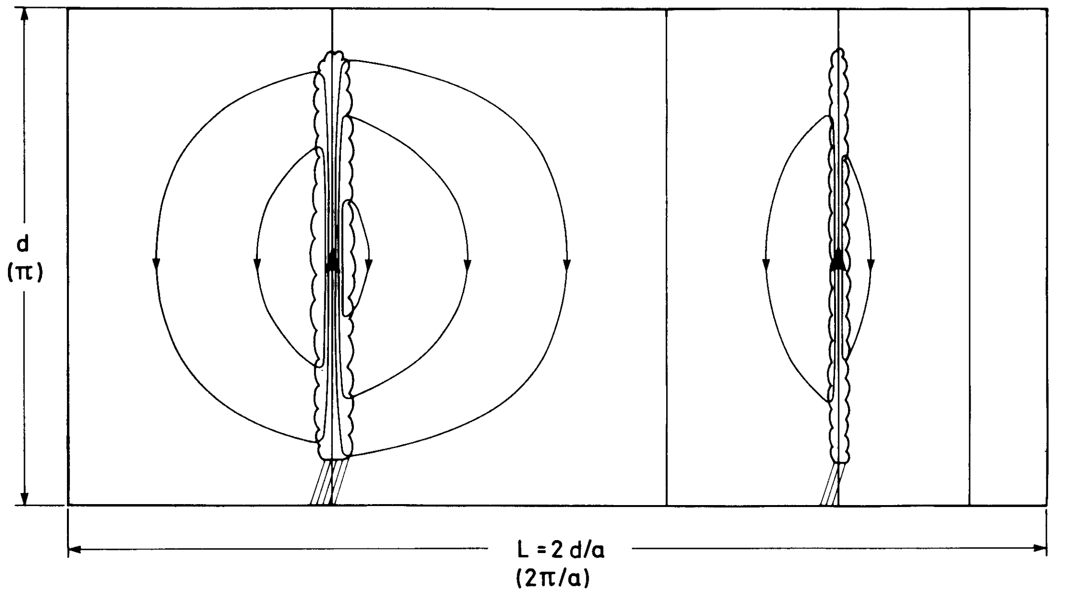

Fig. 1. The model structure. The contours in the figure are streamlines and the arrows indicate the direction of the motion. The condensational heating is indicated by "clouds".

the waves are concerned, the qualitative conclusions of this study will not be changed by assuming the lifting condensation level to be at the surface. We assume all condensed water to rain out instantly so that condensational heating takes place in the ascending region but no evaporational cooling exists in the descending region. Horizontally the motion is assumed to be periodic and therefore waves propagating out of the model domain through one side-wall will re-enter through the other side. In designing the numerical integrations, we try to choose the length of the model domain such that during the time interval we choose to study the model properties are not influenced by the periodic boundary conditions. To make the physical system as simple as possible, rotational effects are neglected, the dissipation terms are represented by constant eddy diffusivity coefficients, and the Prandtl number is set to unity. The model structure is schematically given in Fig. 1, where condensational heating takes place in the ascending branches of convective cells and the contour lines are streamlines with arrows indicating the direction of motion. Both dimensional and non-dimensional units are given in the figure.

As basic model equations we use the Boussinesq approximated vorticity equation and the thermodynamic equation in a two-dimensional (y, z) plane. The non-dimensional form is adopted for algebraic convenience and the scaling scheme can be found in Huang and Källén (1986). The model equations may be written:

$$\partial \nabla^2 \Psi / \partial t = -J(\Psi, \nabla^2 \Psi) + \partial \theta / \partial y + \nabla^2(\nabla^2 \Psi), \tag{2.1a}$$

$$\partial \theta / \partial t = -J(\Psi, \theta) + \eta(\varepsilon + \delta) \partial \Psi / \partial y + \nabla^2 \theta, \tag{2.1b}$$

where Ψ is the streamfunction and $\partial \Psi / \partial y = w$ gives the vertical velocity, θ is the temperature deviation from a linear reference temperature profile which is defined through the boundary conditions, $J(a, b)$ is the Jacobian with respect to y and z, and finally, η, ε and δ are defined as:

$$\eta = g \alpha d^4 (\Gamma_d - \Gamma_m) / \pi^4 \nu^2, \tag{2.2}$$

$$\varepsilon = -\frac{\Gamma_d - \Gamma}{\Gamma_d - \Gamma_m}, \tag{2.3}$$

$$\delta = \begin{cases} 0 & \text{for } w \leqslant 0 \\ 1, & \text{for } w > 0 \end{cases} \tag{2.4}$$

where Γ is the lapse rate of the reference temperature profile, Γ_d is the dry adiabatic lapse rate, Γ_m is the moist adiabatic lapse rate, g is the gravity acceleration, α is the thermal expansion coefficient, d is the height of the model domain and ν is the eddy diffusivity. The expression in (2.2) gives the condensational heating coefficient, as $\eta\delta w$ represents the diabatic heating due to condensation (Huang and Källén, 1986). In the definition of η, the gravity acceleration and the thermal expansion coefficient are assumed constant ($g = 9.8$ m s^{-2}, $\alpha = 1/300$ K^{-1}). The parameters which are free to vary are d, the height of the model domain and ν, the eddy diffusivity. Furthermore, d and ν always appear together in the form of a dissipation time scale, or diffusion time scale d^2/ν (Krishnamurti, 1975). In this sense, the change in η is due to the change in dissipation time scale in our formulation. In order for the control parameters of the model to be independent of each other, we choose a non-dimensional number ε instead of choosing Rayleigh number like in Huang and Källén (1986). The change in ε is only due to the change in Γ, the lapse rate of the reference temperature profile, because the moist and the dry adiabatic lapse rates can be kept constant ($\Gamma_d = 10$ K km^{-1}, $\Gamma_m = 5$ K km^{-1}) without loosing model generality. The reference stratification is absolutely stable if $\varepsilon < -1$, conditionally unstable if $-1 < \varepsilon < 0$ and absolutely unstable if $\varepsilon > 0$.

We expand Ψ and θ as Fourier series which are truncated in order to solve the problem numerically. The truncated series are defined as follows:

$$\Psi = \sqrt{2} \sum_{n=1}^{N} [\Psi_{2n-1} \sin any + \Psi_{2n} \cos any] \sin z, \tag{2.5a}$$

$$\theta = \theta_0 \sin z + \theta_s \sin 2z + \sqrt{2} \sum_{n=1}^{N} [\theta_{2n} \sin any + \theta_{2n-1} \cos any] \sin z, \tag{2.5b}$$

where a is the aspect ratio defined as $2d/L$, L being the length of the model domain. In (2.5) we take N horizontal components into account but assume that the convection cells extend through the entire vertical model domain (see Fig. 1). As pointed out by Arakawa and Schubert (1974), a very important mission of cumulus clouds is to relieve the conditional instability. In this sense, models of cumulus convection should take into account the modification of the model stratification. This brings another important nonlinear effect into the problem, i.e., the interaction between moist convection and the mean stratification. In the truncated system (2.5), we include in the temperature deviation one more term (θ_s) which has a sin $2z$ structure in the vertical and is independent of y. Physically, this term describes the modification of the mean stratification due to convective overturning processes. A discussion about the truncation (2.5) is given in Huang (1988).

The model equations are solved by a transform method, i.e., the condensational heating is computed in the physical domain, while the time integrations are performed in spectral space and Fourier transformations are conducted twice at each time step to transform variables back and forth between the physical domain and spectral space. Formally we may write the transformation of the condensational heating, $\delta\eta w$, using the same truncation form as that for θ:

$$\delta\eta w = Q_0 \sin z + Q_s \sin 2z + \sqrt{2} \sum_{n=1}^{N} [Q_{2n} \sin any + Q_{2n-1} \cos any] \sin z. \tag{2.6}$$

Inserting (2.5) and (2.6) into the basic equations (2.1a, b), and using the orthogonal properties of the trigonometric functions, we obtain the model equations in spectral space as follows:

$$\mathrm{d}\Psi_m/\mathrm{d}t = b(m)\,\theta_m/[1 + b(m)^2] - [1 + b(m)^2]\,\Psi_m, \tag{2.7a}$$

$$\mathrm{d}\theta_m/\mathrm{d}t = b(m)[\eta\varepsilon + \theta_s]\,\Psi_m - [1 + b(m)^2]\,\theta_m + Q_m, \tag{2.7b}$$

$$\mathrm{d}\theta_0/\mathrm{d}t = -\theta_0 + Q_0, \tag{2.7c}$$

$$\mathrm{d}\theta_s/\mathrm{d}t = -\sum_{k=1}^{2N} b(k)\,\Psi_k\,\theta_k - 4\theta_s Q_s, \tag{2.7d}$$

where $m = 1, 2, \ldots, 2N$, and

$$b(m) = \begin{cases} -am/2 & m \text{ even} \\ a(m+1)/2 & m \text{ odd.} \end{cases} \tag{2.8}$$

From (2.7), we see that there is no wave–wave interaction in the spectral model and therefore the model physics is simplified even further due to the truncation used in (2.5) and (2.6). This is acceptable because, as pointed out by Sheu et al. (1980), condensational heating is a dominating mechanism in organizing convective cells when moisture processes are involved. To integrate the spectral model (2.7) in time we use a leapfrog scheme with a filter:

$$\bar{F}(t) = F(t) + \gamma[F(t-\Delta t) + F(t+\Delta t) - 2F(t)], \tag{2.9}$$

where $F(t)$ can be any variable in (2.7), Δt is the time step for the numerical integration and γ is a constant ($\gamma = 0.05$) in all experiments. The maximum wave number N ($N = 30$) and the aspect ratio a ($a = 1/20$) are chosen in such a way that the model results do not depend on the horizontal spectral truncation and the cyclic boundary conditions very much.

The assumption of a constant moist adiabatic lapse rate and the neglect of the impact of moist processes upon the dissipation are the major differences between the present model and the model used in the previous study of Huang (1988). In this study, the direct destabilizing effect of the condensational heating upon the mean stratification is excluded ($Q_s = 0$). However, these simplifications remove a number of parameters from the parameter sensitivity study (e.g., the specific humidity, the surface temperature, etc.) and, as we are going to show in Section 3, retains the same grouping characteristics of moist convection as in Huang (1988).

In order to compare the numerical results easily, we will present all our results in non-dimensional units. The model characteristics are determined by two non-dimensional numbers, which represent two different physical processes, i.e., the dissipation (η) and the reference stratification (ε). A small random forcing [$O(10^{-2})$ in non-dimensional units] is added on the right-hand side of every equation of (2.7) at the first two time steps as initial perturbations. If the model stratification is stable the small perturbations are dissipated, otherwise they are amplified and convection will appear.

3. Results

3.1. An example of grouping

We first investigate the results obtained by numerically integrating the model equations from a state of rest. In Fig. 2, we show the time evolution of the vertical velocity at the mid-level, $w(y, \pi/2, t)$. As has been found in Huang (1988), three distinct regimes are observed following the numerical integration. In the first stage, the mean kinetic energy of the model increases rapidly and a preferred mode with $l/\pi \approx 5$ is observed at the end of this stage, l being the characteristic wavelength in non-dimensional units. This preferred mode is characterized by relatively intense and narrow updrafts, which can be seen clearly at the end of this stage. The downdrafts are closely located around the updrafts but can hardly be observed from this figure. In fact, the downdrafts grow in scale as the intensity of the updrafts grows and no interaction between updrafts can be noticed until $t = 1$, when the second stage starts. The individual cells then become organized into groups, which are composed of many updrafts and have a larger spatial scale than individual cells. The grouping structures have the same features as those shown in Huang (1988), in which only the upward motion pattern (in a sense, the cloud pattern) was shown and the spatial scales of the groups were more clearly demonstrated. However, waves in the motion field are also displayed in the second stage, which would have escaped out attention if we were only concerned with "cloud patterns". In the final stage, which starts from about $t = 3$, only one updraft is left in the whole model domain as the model approaches its equilibrium stage. The waves are embedded in the subsidence of the main convective cell and cannot be shown either with the "cloud pattern". Due to dissipation in the model the wave amplitude decreases as waves propagate horizontally. As the equilibrium state of the model is approached, the amplitude of the heating oscillation decreases and so does the amplitude of the waves.

By this example, we show that the grouping structures found in more complicated numerical models (Van Delden and Oerlemans, 1982; Huang, 1988) also exist in our simplified model, i.e., the model includes the physical mechanism necessary to organize mesoscale clusters. As the simplifications made in deriving the present model exclude some physical processes which are considered to be crucial for grouping, like the moisture anomalies in Van Delden and Oerlemans (1982), other mechanisms must be

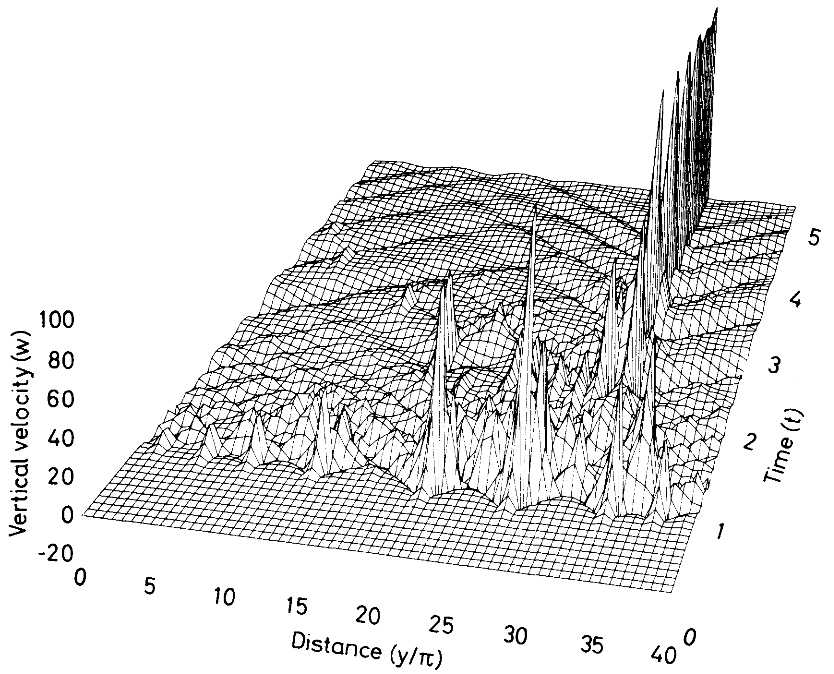

Fig. 2. Time evolution of the vertical velocity at the mid-level, $w(y, \pi/2, t)$. The initial values are set to zero and small random perturbations are introduced at the first two time steps in the numerical integration. The model parameters are: $\eta = 2956$, $\varepsilon = -0.5$.

examined in order to explain the grouping obtained in this model.

The cause of the cloud grouping as documented in this model (Fig. 2) must be connected with the condensational heating. When removing condensational processes in the model eqs. (2.7a–d) we have found that the model behaviour is very different and no grouping occurs. Individual convective cells found at the onset of convection remain almost unchanged throughout the time integration. Another important difference between a moist and a dry run is the asymptotic model stratification. In the dry case, the model will tend towards a dry adiabatic or neutral lapse rate while in the moist case the horizontally averaged stratification will be close to moist adiabatic. In the moist case this implies that the cloud environment can support gravity waves. This is not the case in the dry case with a nearly neutrally stratified environment.

The nonlinear nature of the condensational heating introduces an asymmetry with respect to ascending and descending motion. Regardless of the initial lapse rate of the reference temperature profile, it will approach a moist neutral stratification as convection proceeds. In Fig. 3, we show

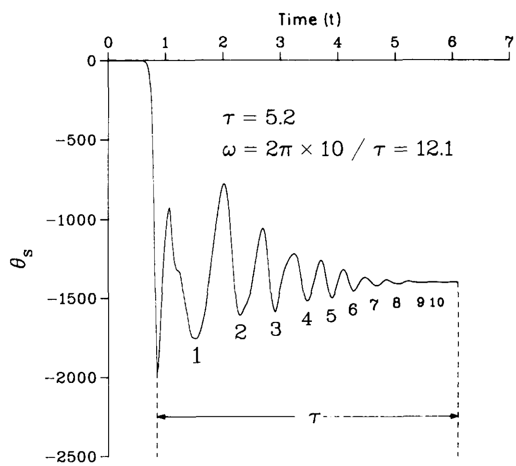

Fig. 3. Time evolutions of the modification of the mean stratification, θ_s. This is from the same numerical simulation described in Fig. 2.

the modification of the mean stratification of the model, θ_s, as a function of time for the numerical integration described in Fig. 2. When the variation in θ_s is less than 1 for a certain time period (0.5 non-dimensional time units), we consider the model to be in a steady state and stop the numerical integration. Two important features in Fig. 3 should be pointed out. First, θ_s is always negative, i.e., the modification of the mean stratification always acts as a stabilizing effect. The model stratification remains stable to downward motion and is therefore able to support gravity waves throughout the whole integration. Secondly, θ_s changes rapidly just before $t = 1$ and then approaches its steady state asymptotically in the form of an oscillation. The variation of θ_s may best be characterized by the oscillation time τ and the oscillation frequency ω. The oscillation time τ is defined as a time interval starting when the initial convection has become fully developed (the first minimum θ_s) and ending when steady state is reached. The oscillation frequency ω is obtained by counting the numbers of oscillation cycles during the time interval τ (see Fig. 3). Due to the model formulation, the oscillation of θ_s and the oscillation of the mean kinetic energy appear simultaneously (not shown). These oscillations are related to the oscillation of the condensational heating, which is proportional to the vertical velocity. One may also see this clearly by noting the updraft evolution in Fig. 2 and θ_s in Fig. 3 during the final stage of convection. The waves in Fig. 2 are closely related to the oscillation of the condensational heating or updrafts: no wave can be seen before $t = 1$ and the amplitude of the waves decreases as the amplitude of the oscillation of θ_s decreases.

We have demonstrated that it is possible for the model stratification to support waves and the oscillation of the condensational heating provide the wave generation mechanism. The waves propagate in subsidence regions and modulate the convective cells. When the wave amplitude is large enough to compensate the subsidence, new updraft will be formed and may grow further due to the conditionally unstable stratification. Since the wave amplitude decreases in space in the presence of dissipation, new updrafts tend to appear around pre-existing updrafts. However, it is also possible for new updrafts to appear far away from old updrafts. For instance, there are places in Fig. 2 where waves generated from two adjacent updrafts give rise to a new updraft in between. The new updrafts generate their own waves and so forth. This nonlinear interaction between waves and moist updrafts leads to the grouping of moist convection.

3.2. Oscillation of θ_s

The oscillation of θ_s in the model describes an interaction between the motion field and the stratification. When convection is vigorous, it transports more heat upwards and stabilizes more the stratification. When convection is weak, it transports less heat upwards and contributes less to the stabilization of the model stratification. Therefore the oscillation of θ_s is a result of the oscillation of the condensational heating or the oscillation of updrafts. On the other hand, the oscillation of θ_s also gives rise to an oscillation of the condensational heating due to the fact that stable stratification prohibits convection to continue while unstable stratification encourages convection to intensify. In the truncated system (2.7), the modification of the model stratification has been evenly distributed horizontally. No horizontal variation in the model stratification exists in spite of the locally placed condensational heating. In other words, the oscillation of θ_s is more or less affected by the model limitation.

In order to see how much does the grouping depend on the oscillation of θ_s, we have repeated the above integration but after convection has developed ($t = 0.81$), θ_s is kept constant ($\theta_s = -1399$, which is approximately the value at the end of the integration shown in Fig. 3). It turns out that this stratification is marginally unstable for moist updrafts. By doing this, we actually consider a quite different physical situation. There is no more interaction between the motion field and the stratification. The steady state of the model is obtained due to the final balance between the marginal unstable stratification and the dissipation.

In Fig. 4 we show the time evolution of the vertical velocity at the mid-level, $w(y, \pi/2, t)$. In this integration, the model state approaches an equilibrium state (not the one shown in Fig. 2) quicker than that in Fig. 2. The flow characteristics are very similar to those shown in Fig. 2 between $t = 0$ and $t = 3$, which suggests that the grouping structure in Fig. 2 remain unchanged no

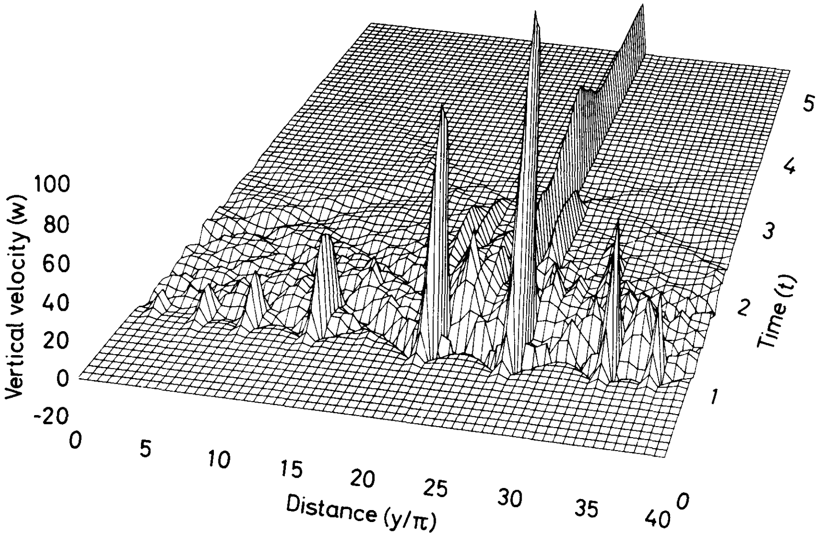

Fig. 4. Time evolution of the vertical velocity at the mid-level, $w(y, \pi/2, t)$. Small random perturbations are introduced at the first two time steps in the numerical integration. θ_s is kept constant ($\theta_s = -1399$) after convection has developed ($t = 0.81$). The model parameters are the same as those used in Fig. 2.

matter whether θ_s oscillates or not as long as the oscillation of the condensational heating behaves similarly during this period.

3.3. *An initial value problem*

From Fig. 2, we see that the grouping is affected by the interaction between individual convective cells. These cells stem from initial random perturbations, i.e., the initial state has no preferred spatial structure and the preferred mode is purely due to the interaction between the motion field and the condensational heating. However, as pointed out by Lilly (1960) and Kuo (1961), widely spaced clouds grow faster. Thus, a heating greater than the random forcing introduced at a single grid point will cause ascending motion at this point. The updraft will grow rapidly and its induced circulation will suppress all other possible developments of convective cells. No interaction exists between different cells in the beginning as there is only one updraft. In this situation one may ask if the grouping of the motion field will still be observed. In other words, we are confronted with an initial value problem: does the grouping effect depend on the initial state of the model?

In Fig. 5, we show the response of the model to a heating of $O(10^0)$ placed in the middle of the model domain at $t = 0$. The interaction between individual cells in the growing stage of the development has been excluded as a necessary cause of the grouping in this simulation. In comparison with Fig. 2, we see that the development of the updraft in this case is faster and its amplitude reaches a higher value. In order to see more detailed features, we provide y–w cross sections at different times up to $t = 1.5$ in Fig. 6. During the growing stage, the subsidence regions propagate away from the updraft. After the maximum vertical velocity has been reached, the intensity of the updraft decreases and the single updraft breaks into multiple cell structures. Then the oscillation of the condensational heating and the waves are observed and contribute to the organization of the individual cells into clusters. From $t = 1.5$, two clearly defined convective clusters are found. They are symmetric about the initial updraft which has died out almost completely. The clusters are maintained relatively unchanged in space for quite a long time until the final steady state with a single cell is approached (one of the two convective systems vanishes

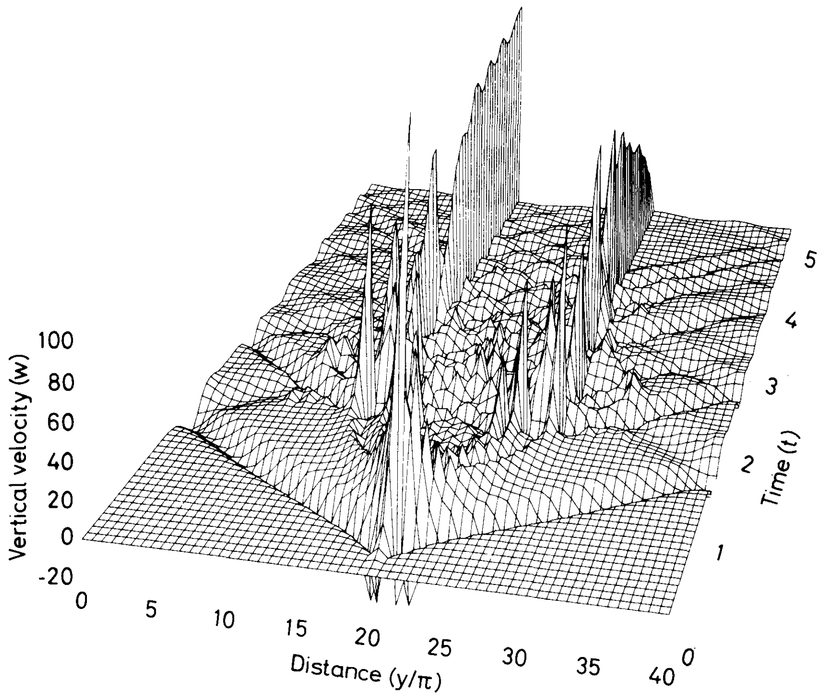

Fig. 5. Time evolution of the vertical velocity at the mid-level, $w(y, \pi/2, t)$. Small random perturbations are introduced at the first two time steps in the numerical integration. A heat source of $O(10^0)$ is placed at the centre of the model domain at $t = 0$ and turned off afterwards. The model parameters are the same as those used in Fig. 2.

gradually). We have also performed a number of numerical integrations with different horizontal structures of the initial perturbation. The evolution of the motion field of these integrations has been found similar to Fig. 5 after the grouping stage. The initial state has a pronounced influence on the convective pattern of the growing stage but not on the grouping stage and the final stage, although the time needed for the model to accomplish these processes depends on initial conditions.

From Fig. 5 and Fig. 6 we can see the effect of the periodic boundary condition clearly. The waves propagating away from updraft reach the boundaries at around $t = 1.5$ and then come into the model domain from the other side. These waves first meet the waves propagating in the opposite direction at around $t = 2.2$ and may further contribute to the grouping of the moist convection in the later stage of the numerical simulation. However, the cyclic boundary condition has no influence on the initial cell breaking and the organization of clusters, which appears before the influence originating from the boundaries reaches the region of interest. Under certain circumstances (one example was given in Fig. 8), the waves have been dissipated before they reach the boundaries and the cyclic boundaries have no influence on the model results.

3.4. *Examples of no grouping*

The presence of waves and the oscillation of the condensational heating cannot assure the grouping of moist convection to occur. In an early study with a similar numerical model by Van Delden (1985), the grouping stage was not shown, i.e., the motion field goes directly from the preferred mode with distinct updrafts to the final state with a single strong updraft, although this transition is slow. This type of solution has also been found in our model by using different parameter settings.

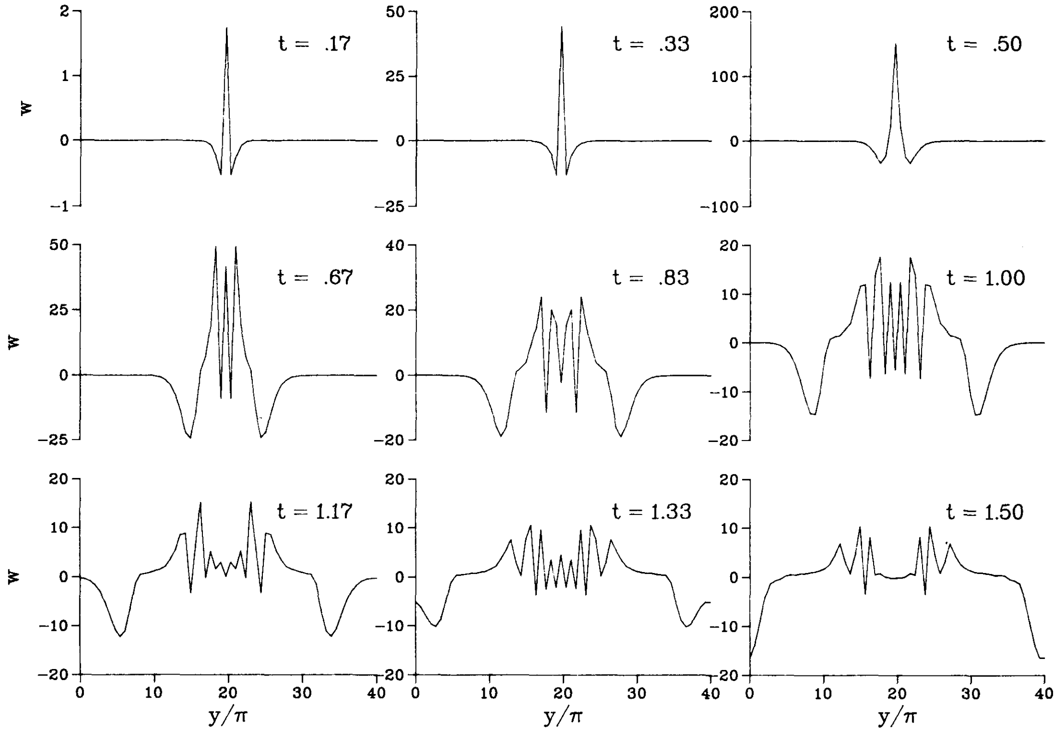

Fig. 6. The vertical velocity at the mid-level, $w(y, \pi/2, t)$, as a function of y at different times. This is a different view of the same numerical simulation presented in Fig. 5.

In Fig. 7, we show the numerical result with a more stable stratification ($\varepsilon = -0.9$), which is still conditionally unstable. The change in ε gives rise to a significant change in l, the average wavelength of the preferred mode of moist convection (here, $l/\pi \approx 10$). A close investigation of Fig. 7 also reveals waves propagating in the descending region during the transition from the growing stage of the preferred mode to the final stage of a steady state. However, as the environmental stratification is unfavourable for moist convection to develop, the amplitude of the waves triggered by the oscillation of the condensational heating cannot be large enough to initiate new convective cells. Furthermore, the larger l does not allow the waves originating from the existing updrafts to meet each other. Therefore, the interaction between internal gravity waves and moist updrafts is not strong enough to organize individual convective cells into groups. The gradual decrease in the number of convective cells is accomplished by the competition between updrafts, with the circulation of each cell suppressing the adjacent ones.

In Fig. 8, we show the result with a shorter dissipation time scale ($\eta = 185$), which can be obtained by either decreasing the height of the model domain d or increasing the eddy diffusivity ν. If we consider $d = 1$ km, $\eta = 185$ corresponds to $\nu = 95\ \mathrm{m^2\ s^{-1}}$ and $\eta = 2956$ (used in Fig. 2 and Fig. 7) corresponds to $\nu = 24\ \mathrm{m^2\ s^{-1}}$, which are acceptable values for atmospheric convection models (Krishnamurti, 1975; Asai and Nakasuji, 1982). The preferred wavelength l depends on the dissipation time scale ($l/\pi \approx 6$), but this dependence is not as significant as the lapse rate dependence. The grouping stage is also absent in this simulation as in Fig. 7, but the physical reason is different. Due to the large dissipation, the waves are damped out so efficiently that they cannot propagate far enough to interact strongly.

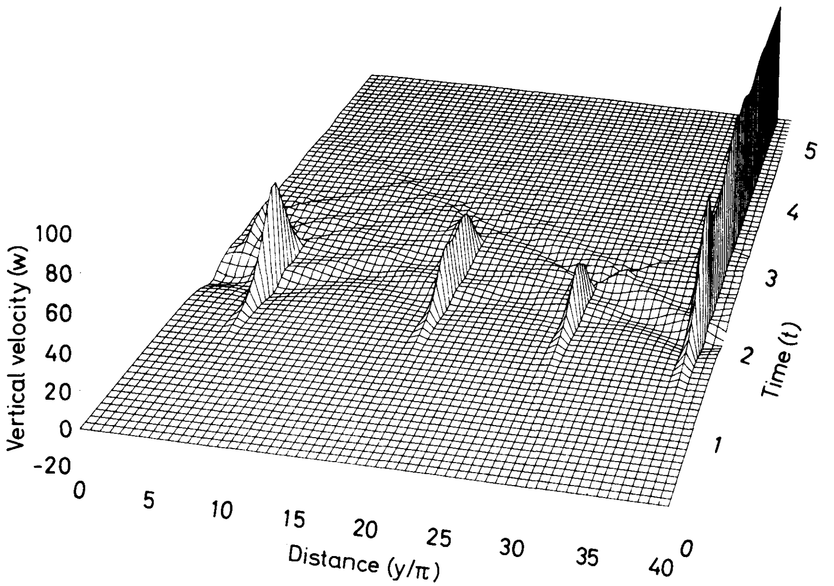

Fig. 7. Time evolution of the vertical velocity at the mid-level, $w(y, \pi/2, t)$. The initial values are set to zero and small random perturbations are introduced at the first two time steps in the numerical integration. The model parameters are: $\eta = 2956$, $\varepsilon = -0.9$.

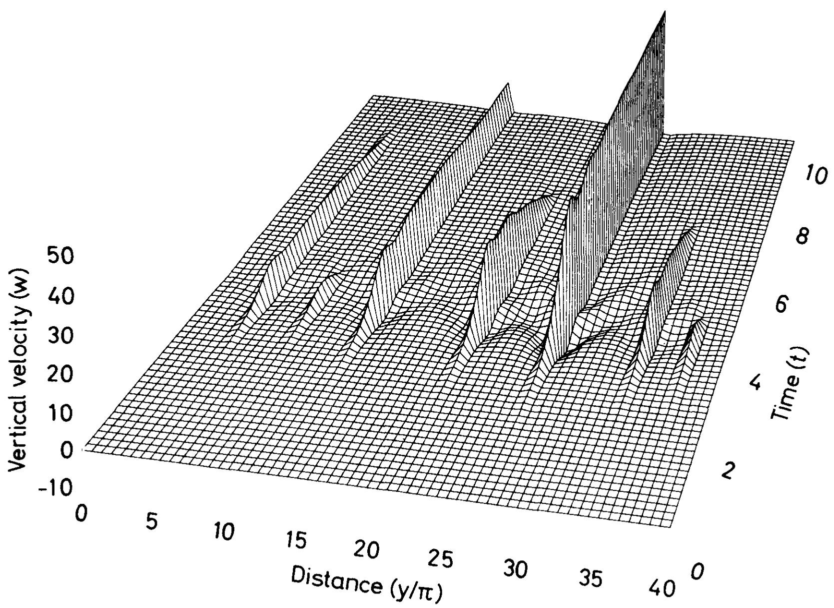

Fig. 8. Time evolution of the vertical velocity at the mid-level, $w(y, \pi/2, t)$. The initial values are set to zero and small random perturbations are introduced at the first two time steps in the numerical integration. The model parameters are: $\eta = 185$, $\varepsilon = -0.5$.

3.5. *Parameter sensitivity*

Due to the simplicity of the model formulation, parameter sensitivity can be explored exhaustively. A large number of numerical simulations have been performed systematically which reveal a separation in parameter space between parameter values giving rise to grouping structures and values giving a gradual evolution of the motion field with no apparent grouping. In Fig. 9, we show the results in parameter space, where each circle represents a model simulation. The simulations are classified in two groups, grouping and no grouping. A case belongs to the grouping category if the following criteria are fulfilled: (1) the spatial scale of the updraft pattern is more than 5 times greater than the scale of individual updrafts and (2) the updraft pattern persists more than 1 non-dimensional time unit in the numerical integration considered. The separation cannot be precisely defined in Fig. 9, but is roughly described by the following formula:

$$\eta(\varepsilon + 1 - a) = b. \tag{3.1}$$

The dashed curve in Fig. 9 has $a = 0.06$ and $b = 261$. The dashed curve can be extended to $\varepsilon > 0$, where the reference stratification is absolutely unstable. As long as moist processes are considered, the model stratification will approach a moist neutral lapse rate sooner or later and there is no big difference between cases where $\varepsilon > 0$ and cases where $\varepsilon < 0$. Table 1 and Table 2 give the parameter values used in the numerical integrations mentioned above and summarize the characteristics of the variation of θ_s. With a large η and a large ε, the numerical simulation generally gives rise to a long oscillation time τ and a high oscillation frequency ω, which allow the condensational heating to last longer and to generate more waves in a given time period. Both tables and Fig. 9 point out that a favourable situation for the organization of individual convective cells into convective clusters is a more conditionally unstable stratification (a large η) and a longer dissipation time scale (a large ε). Comparisons of Fig. 2 with Fig. 7 and Fig. 8 serve as examples of this conclusion.

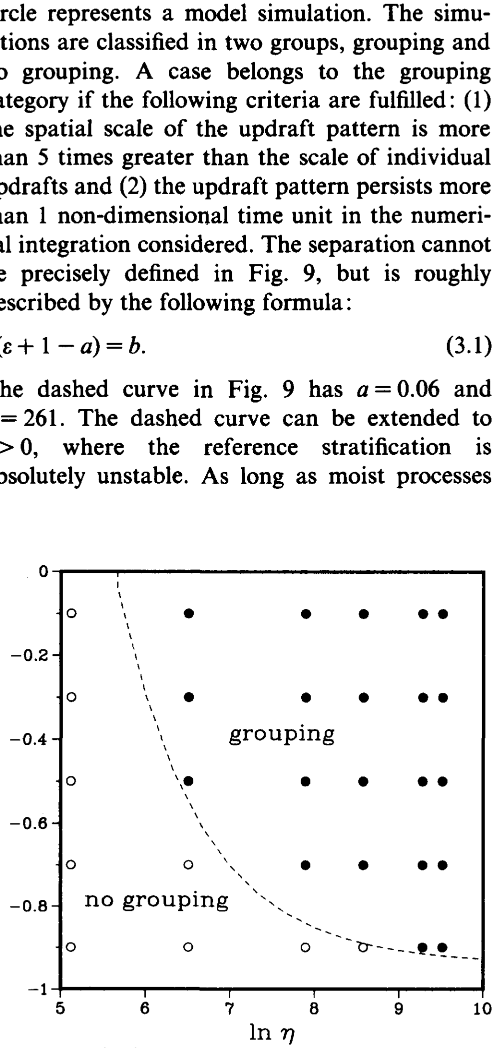

Fig. 9. Results from model sensitivity study. Each circle in the figure represents a model simulation. The filled circles indicate the numerical simulations in which grouping structures have been found. The dashed line is plotted according to eq. (3.1).

Table 1. *The oscillation time τ in non-dimensional units evaluated from numerical integrations*

τ (oscillation time in non-d units)

ε \ η	185	739	2956	5846	11824	14965
−0.1	1.9	3.9	3.5	4.1	4.9	4.5
−0.3	2.4	3.6	5.5	4.5	3.9	5.5
−0.5	2.0	3.4	5.2	6.2	4.4	6.8
−0.7	—	3.4	4.1	3.9	4.8	4.3
−0.9	—	—	2.0	2.0	1.9	2.8

An undefined τ (denoted by —) means that there is no oscillation in θ_s throughout the whole time integration in a case. Parameter values for ε and η for each case are also given in the table.

Table 2. *The oscillation frequency ω in non-dimensional units evaluated from numerical integrations*

ω (oscillation frequency, defined as $2\pi N/\tau$)

ε \ η	185	739	2956	5846	11824	14965
−0.1	6.5	27	23	31	42	47
−0.3	2.5	8.6	21	25	40	40
−0.5	3.0	5.5	12	20	29	35
−0.7	—	5.5	9.3	18	33	33
−0.9	—	—	3.1	6.3	17	9.1

An undefined ω (denoted by —) means that there is no oscillation in θ_s throughout the whole time integration in a case. Parameter values for ε and η for each case are also given in the table.

3.6. Phase speed of the waves

Propagating waves in a (dry) stable environment are clearly demonstrated in Figs. 2, 4, 5, 6, 7 and 8. The phase speed, C, can be evaluated by subjectively determining the wave pattern slope in t–y space. For instance in Fig. 2, the average slope between $t = 4$ and $t = 5$ is fairly well defined, giving $C = 48$ (non-dimensional units). A straightforward comparison can therefore be made between the phase speed obtained from the numerical simulations and that according to linear wave theory.

In order to use the linear dispersion relation, we have to linearize our model equations. One of the nonlinearities of the model is the nonlinear interaction between the motion field and the mean stratification. From Fig. 3 we see that θ_s changes a lot in the beginning of the numerical integration, and only after $t = 4$ is the change in θ_s ($\Delta\theta_s$, about 100) small compared with the final amplitude of $|\theta_s|$ (about 1500). We may use the time averaged θ_s in the spectral model (2.7a, b) to investigate the wave properties only in the period when $\Delta\theta_s/|\theta_s| \ll 1$. The condensational heating terms are nonlinear and have also been excluded. This does not mean that we are dealing with a dry model as the nonzero θ_s discussed above is implicitly a reflection of the effect of the condensational heating. Dropping the heating terms is acceptable because the waves propagate in a dry environment where the vertical velocity is negative. If it is positive anywhere a new cloud will develop and the linearized theory breaks down. We then get a linear system which gives the phase speed C_T, by the following dispersion relation:

$$C_T = \sqrt{-\frac{\eta\varepsilon + \theta_s}{1 + (an)^2}}, \tag{3.2}$$

where n is the average wave number over the period in which C is evaluated. Actually, n is evaluated by using the number of maxima, which is an overestimate when a pre-exist updraft breaks into smaller ones. In the example of Fig. 2, averaged over the time period from $t = 4$ to $t = 5$, we obtain $n = 8$ and $\theta_s = -1399$ giving $C_T = 49$ through (4.1).

We have also tried to relax the requirement $\Delta\theta_s/|\theta_s| \ll 1$ and compare C and C_T for the numerical simulation presented in Fig. 5.

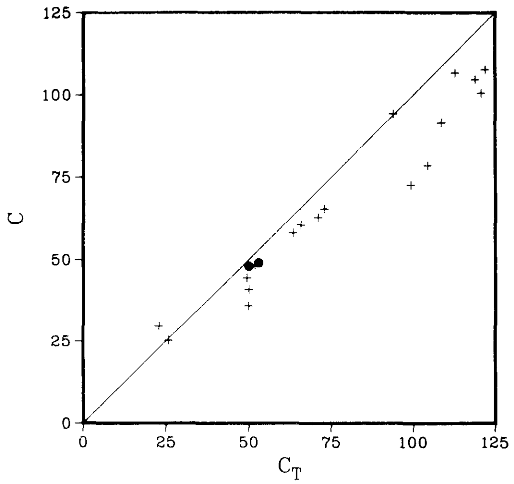

Fig. 10. Scatter diagram where one cross denotes a particular case. If the waves propagated according to the phase speed of the linear dispersion relation exactly, all crosses would lie on the straight line in the figure. The dots in the figure indicates the numerical simulation shown in Fig. 2 and Fig. 5, respectively.

Between $t = 0.4$ and $t = 1.4$, we see a very clear wave pattern, whose slope in t–y space gives $C = 49$. Averaged over this period, we obtain $n = 5$ and $\theta_s = -1561$ which gives $C_T = 52$. We see that C also in this case agrees fairly well with C_T in this case.

Proceeding in this way, we have tried to use the same model data as that used in Fig. 9. When a recognizable wave pattern slope is found in a numerical simulation, we select it as a case. In total, 20 cases are selected and the results are summarized in Fig. 10, which reveal a good correlation between the phase speed evaluated from numerical simulations (C) and that calculated according to linear gravity wave theory (C_T). We may therefore conclude that the waves simulated by numerical integrations are internal gravity waves and their phase speed may approximately be evaluated by the dispersion relation according to linear theory.

4. Conclusions

A two-dimensional, Boussinesq approximated model is used to investigate the grouping of moist convection. The effect of moisture has been taken into account as a flow-dependent condensational heating. The model physics is simplified considerably to isolate the dynamic mechanisms which we believe are most important for the grouping phenomenon. The model characteristics are determined by two non-dimensional numbers, which represent changes in the reference stratification and the dissipation time scale, respectively. The model sensitivity to these parameters has been studied systematically.

The model equations are numerically integrated from a state of rest with a conditionally unstable stratification. If the stratification is less unstable with regard to moist ascents or the dissipation time scale is short (a large eddy diffusivity and/or a shallow convective layer), the motion field goes from a preferred mode with distinct updrafts to a final state with a single strong updraft. This is consistent with similar model results by Van Delden (1985). If the stratification is unstable and the dissipation time scale is long, three distinct regimes of moist convection can be found in this simplified model as shown in Huang (1988). The individual cells which have developed in the growing stage collapse and become organized into convective clusters.

In the present model, moisture fields are not included explicitly. The grouping mechanisms related to moisture anomalies, evaporative downdrafts and so on have therefore been excluded in this study. The grouping of moist convection does, however, depend on a moist process—the condensational heating. Due to the condensational heating, the horizontally averaged stratification will tend towards a moist adiabatic lapse rate which is stable to dry motion and therefore supports internal gravity waves. After moist convection is fully developed, the model approaches an equilibrium state asymptotically in the form of oscillation. The oscillation of the condensational heating generates waves. If the lapse rate of the reference temperature profile is far from moist adiabatic lapse rate, the oscillation of the condensational heating will persist a long time and more updrafts will appear. If the dissipation time scale is long, the waves can propagate a long distance without being significantly damped. In these situations, strong nonlinear interactions between waves and moist updrafts are possible which leads to the grouping of moist convection. A more conditionally unstable stratification and a longer dissipation time scale form a favourable condition for the grouping of moist convection.

The requirement of a more unstable stratification for grouping is in agreement with the results of Bretherton (1987, 1988), even if a direct comparison cannot be made due to the different formulation of the mathematical models. In a linear instability analysis, Bretherton (1987) found multiple updraft eigenmodes only when the moist Rayleigh number N_c^2 is large enough, although these modes grow less rapidly than the single updraft and are unstable to perturbations of their cloud boundaries. His numerical integrations of fully nonlinear equations (Bretherton, 1988) indicated that closely spaced clouds may be obtained only when N_c^2 is 10 times greater than N_{c0}^2, which is the critical moist Rayleigh number for the onset of moist convection. These clouds were dissipated gradually on the diffusive time scale (the non-dimensional time scale in our notation). We have obtained a criterion for the grouping, i.e., $R_a = 261$, which is about 20 times greater than R_c, the critical Rayleigh number for the onset of convection. When delineating grouping and no grouping cases, we use 1 non-dimensional unit as a threshold for the time scale of the grouping.

The requirement of a longer dissipation time scale can be achieved by decreasing the eddy diffusivity or by increasing the height of the model domain. This is consistent with the numerical results by Van Delden and Oerlemans (1982) in which the height of the model is the decisive factor for grouping when the eddy diffusivity is kept constant. Only when convection is deep enough (more than 2 km), do cumulus clouds form into groups. In their model simulation, these groups have a horizontal scale 15 km while a typical cumulus size is 3 km.

In this study, we have demonstrated one of many possible mechanisms which organize small individual clouds into mesoscale clusters, i.e., the interactions between moist convective cells and gravity waves. It may be considered similar to the forced gravity wave mechanisms or wave–CISK (where CISK is the acronym for Convective Instability of the Second Kind), which also deals with convective cells and gravity waves (see Raymond (1987) for a more general discussion). A major difference between the two mechanisms can be pointed out immediately by the name of the wave–CISK, as CISK treats ensembles of convective cells rather than individual ones. In most wave–CISK models, the hydrostatic assumption is used and thermodynamic processes related to moist convection are all parameterized or ignored. Therefore, they are not able to show how small individual convective cells become organized as in the present study.

In a study on the response of a stratified atmosphere to a prescribed local heating, Lin and Smith (1986) showed that decaying waves move away from the heat source. The decay is rapid, especially when an unbounded fluid is considered. In our present model, the waves are also found to move away from the heat sources, which may themselves result from waves. Under certain circumstances, the waves can propagate a long distance without a significant decay. This may be due to the unrealistic boundary conditions and the severe truncation in the vertical direction (see (2.5) and (2.6)), which prevents the wave energy from radiating away vertically. However, Lindzen and Tung (1976) have shown that a structured atmosphere may also trap horizontally propagating waves. We may thus expect the same wave characteristics in such an atmosphere. A two-dimensional model with high resolution in the vertical will be developed to investigate the vertical propagation of the gravity waves, the lifting of the inversion layer, and their impact upon the organization of moist convection.

With a two-dimensional model, we can only study banded convective activities like the dryline convection (e.g., Sun, 1987). If the same vertical structure as the one used in this study is adopted, convectively induced gravity waves preserve their energy when propagating horizontally. However, in the real atmosphere grouping structures of cumulus cloud are often not bandlike and, for instance, cylindrical waves are observed from satellite pictures (Erickson and Whitney, 1973). The cylindrical waves do not preserve wave amplitude as they propagate. The wave activity is thus weakened in a three-dimensional situation, and the grouping may not appear with the same parameters as in the two-dimensional study. A three-dimensional model with simple vertical structures as used in this paper will be investigated next in order to get a better understanding of the spatial scale of grouping.

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