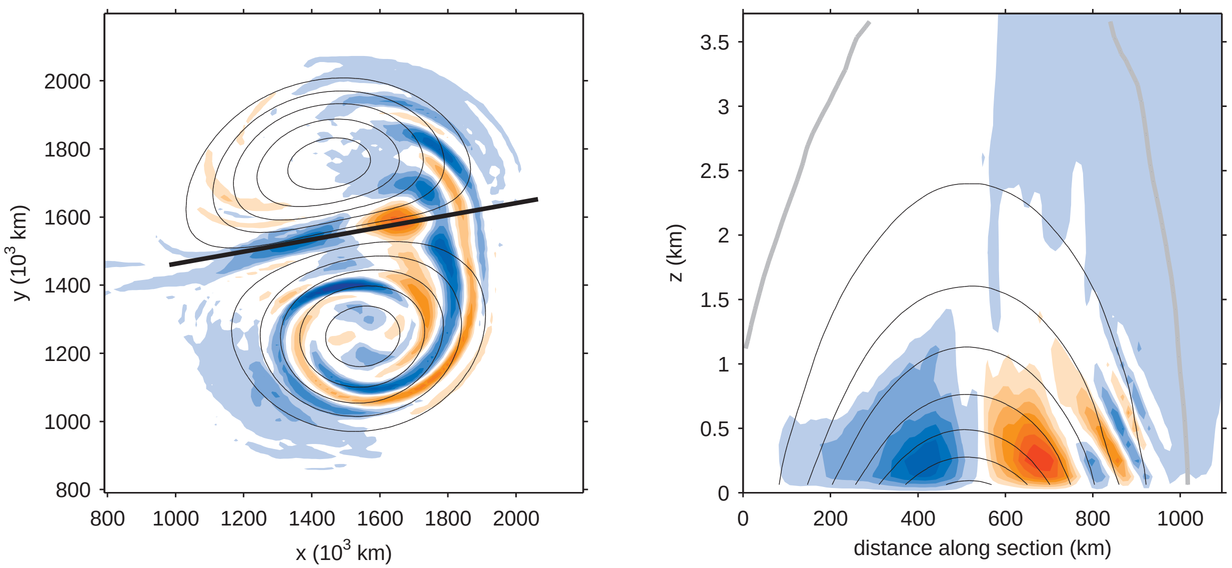
Mechanisms for spontaneous gravity-wave generation within a dipole vortex



Snyder et al., 2007: *J. Atmos. Sci.*, **64**, 4417-4431.

- Chris Snyder, NCAR, Boulder CO, US
- Riwal Plougonven, LMD/IPSL, Paris, France
- David Muraki, Simon Fraser, Vancouver BC, Canada

Overview

Interaction of balanced motions and gravity waves

Motivation

- ▷ quantify, parameterize source of waves
- ▷ path to dissipation from energy in balanced motion
- ▷ classic problem of geophysical fluid dynamics

Dipole = simplification of localized, baroclinic jet ("jet streak")

- ▷ observed scenario for wave generation (e.g. Uccellini and Koch 1987)

Simulations of a Dipole Vortex

Consider Boussinesq, rotating, stratified flow

- rigid, flat lower boundary
- doubly periodic domain on an f-plane

Initial conditions from quasigeostrophic (QG) dipole

- exact, steadily propagating solution under QG dynamics
- uniform interior PV, dipole (warm/cold) of θ at lower boundary
- $\max(u) = 10\,\mathrm{m\,s^{-1}}$

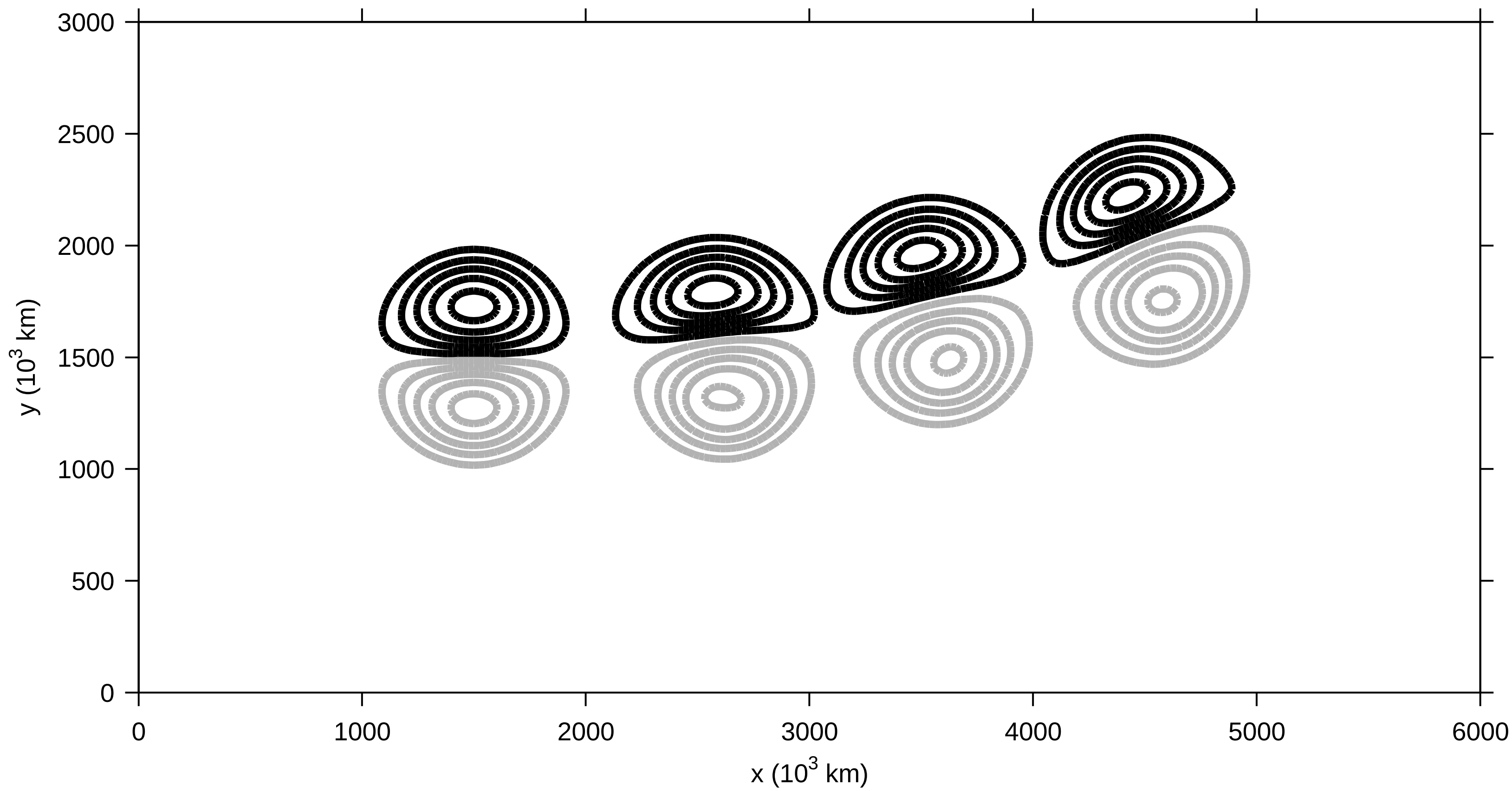
Simulate numerically with nonhydrostatic, compressible model

- $3000\,\mathrm{km} \times 3000\,\mathrm{km} \times 15\,\mathrm{km}$ domain

See Snyder et al. (2007 *JAS*)

Simulations of a Dipole Vortex

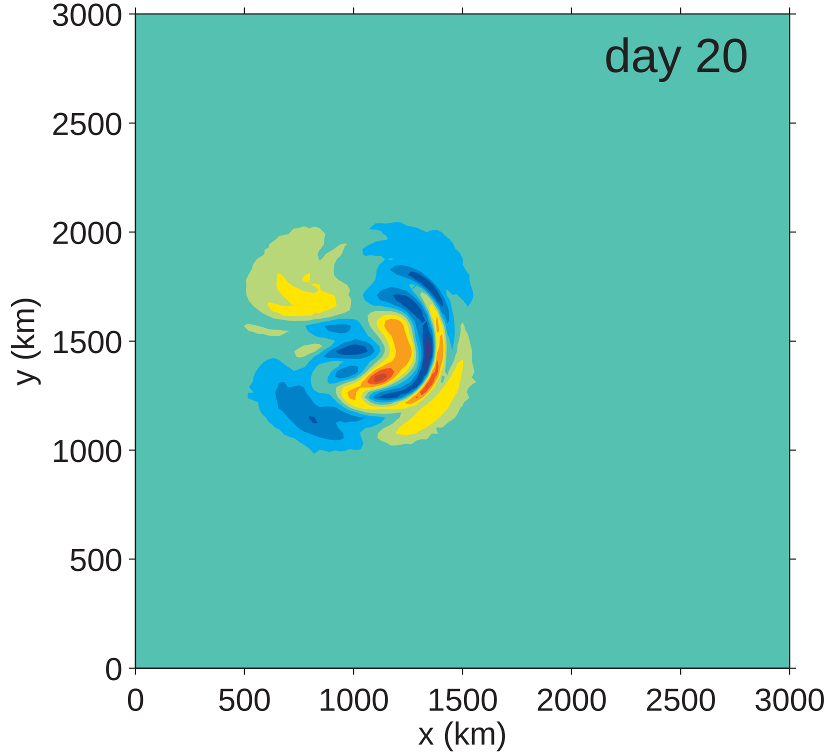
▷ surface θ at $t = 0,\ 12.5,\ 25,\ 37.5$ days



... $\sim$ steady propagation along curved path

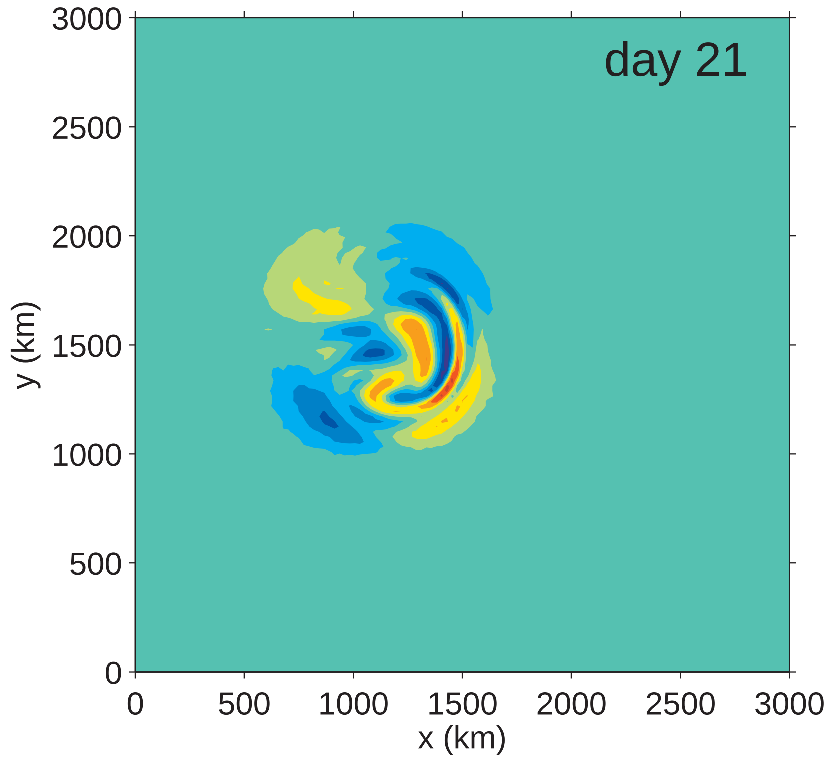
Simulations of a Dipole Vortex

- w at lowest model level



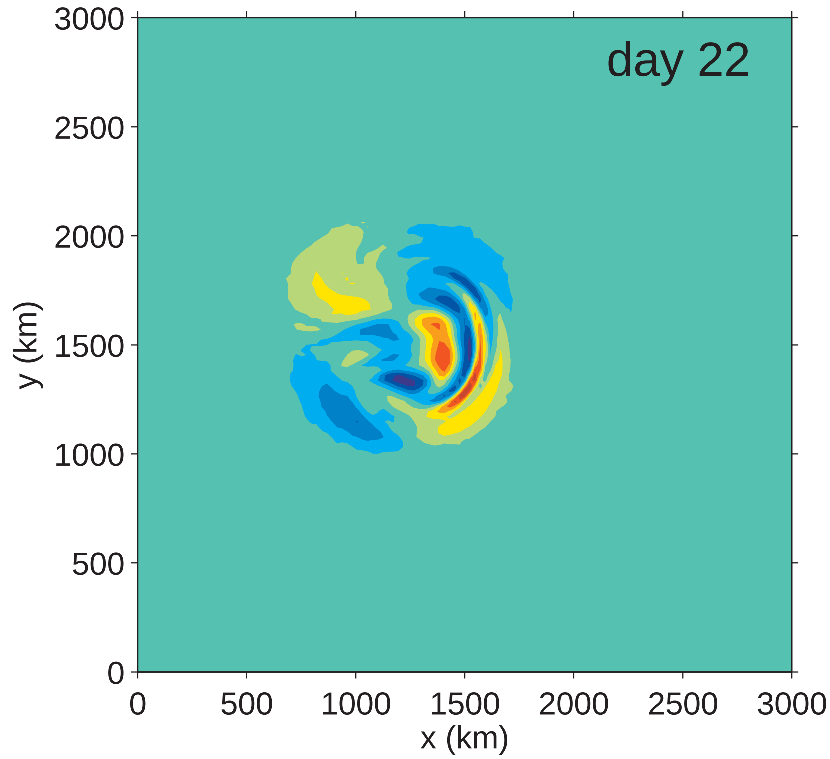
Simulations of a Dipole Vortex

- w at lowest model level



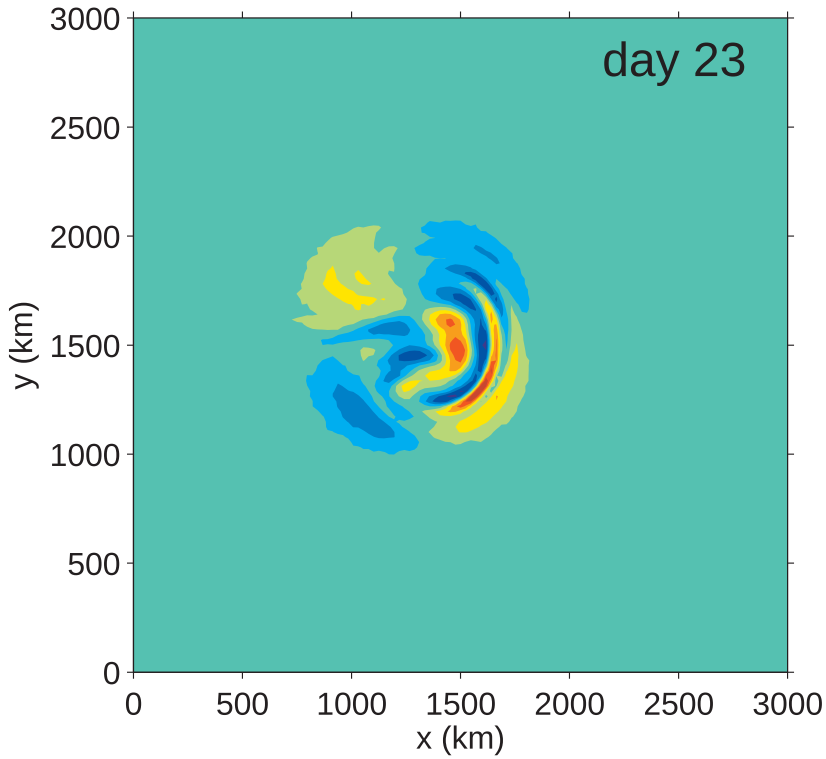
Simulations of a Dipole Vortex

- w at lowest model level



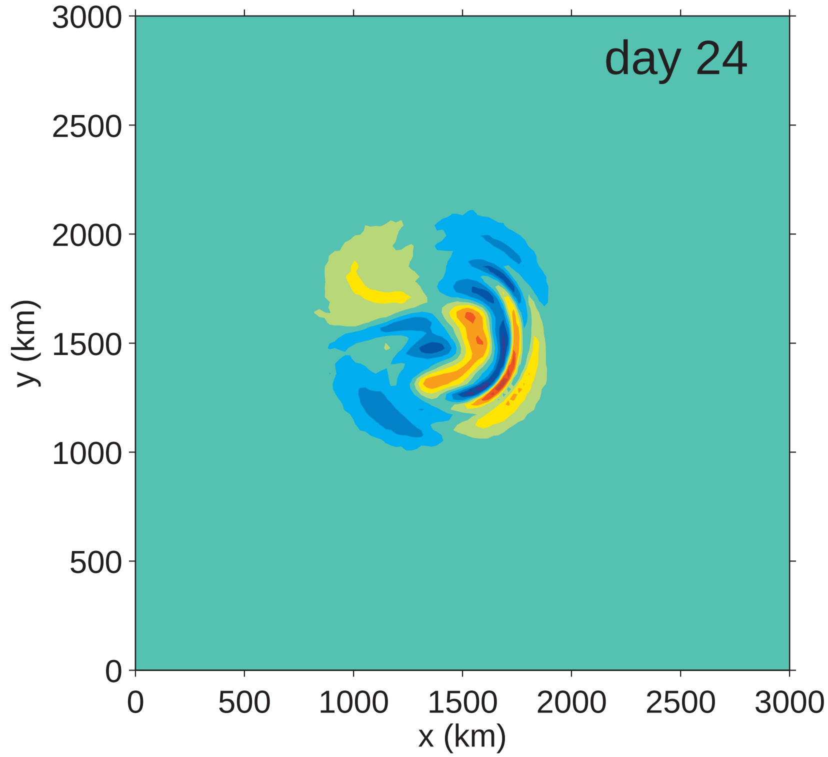
Simulations of a Dipole Vortex

- w at lowest model level



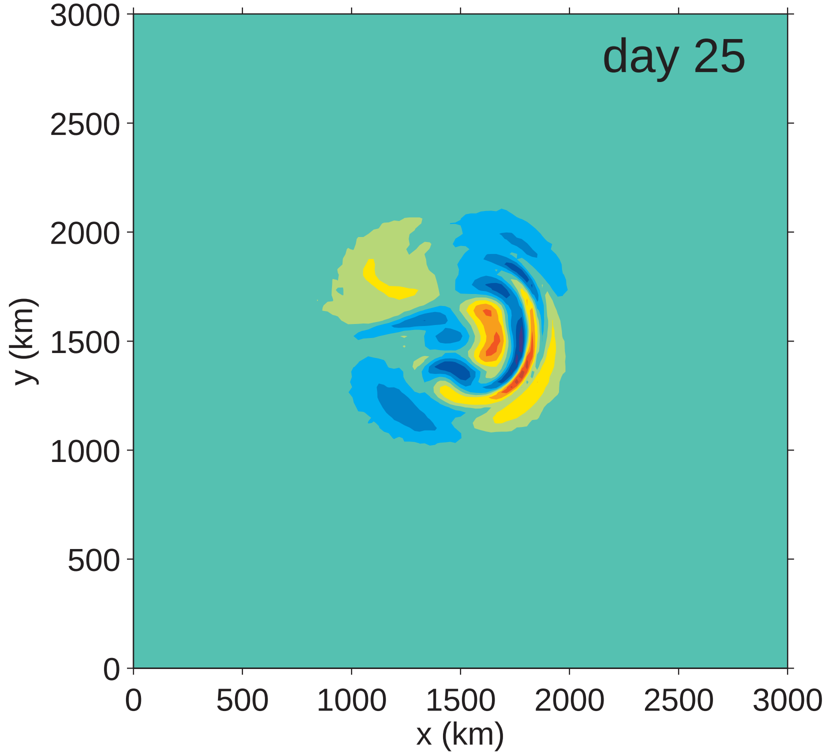
Simulations of a Dipole Vortex

- w at lowest model level



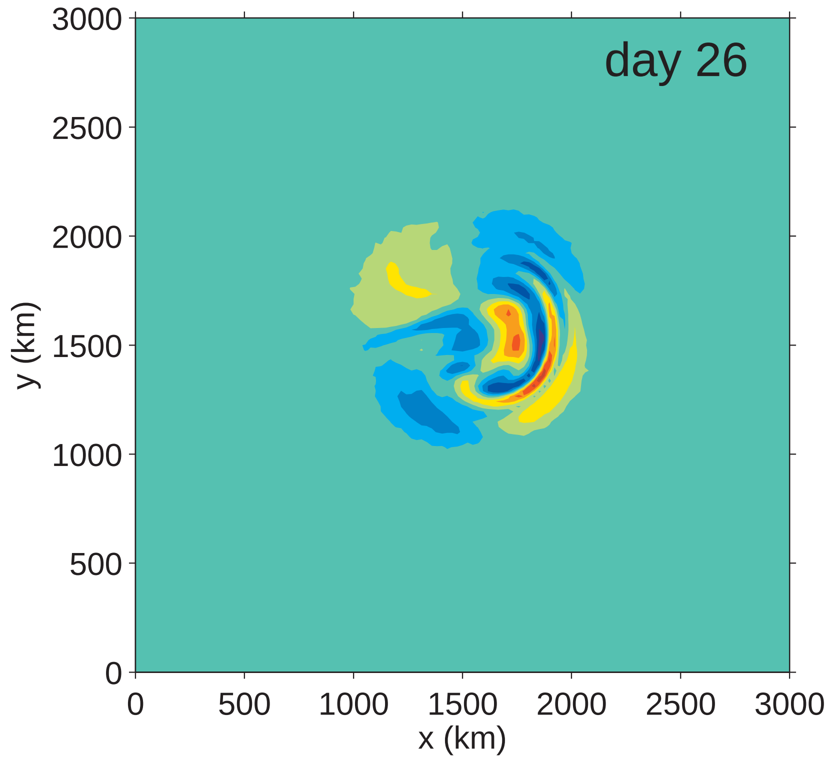
Simulations of a Dipole Vortex

- w at lowest model level



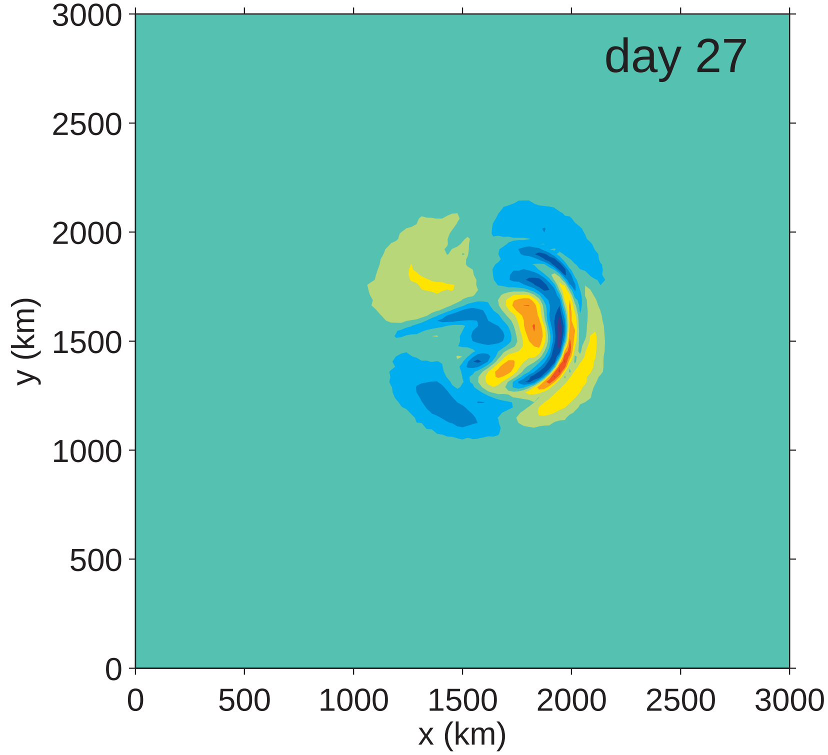
Simulations of a Dipole Vortex

- w at lowest model level



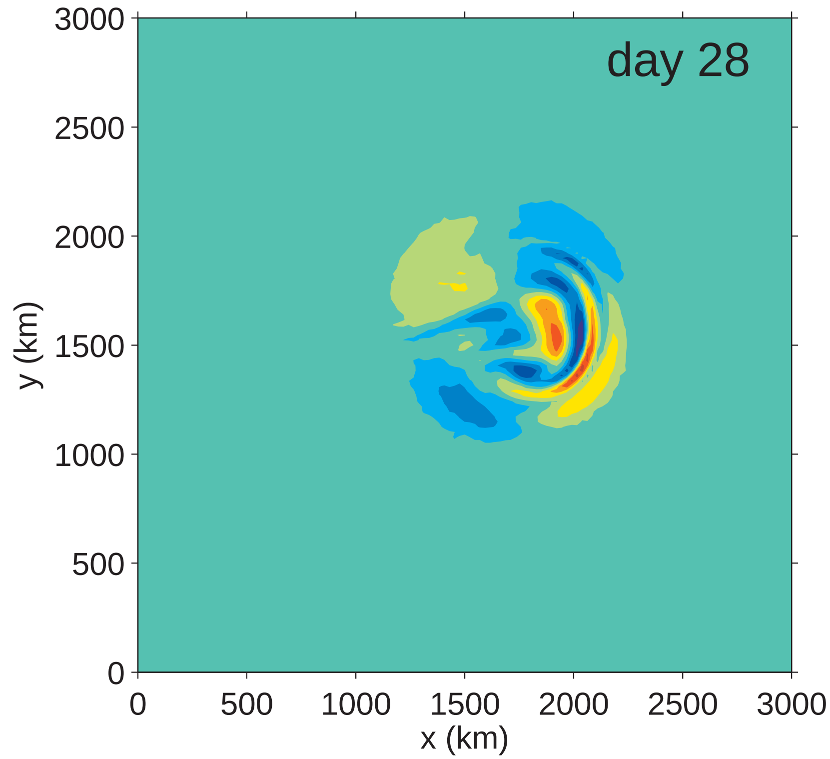
Simulations of a Dipole Vortex

- w at lowest model level



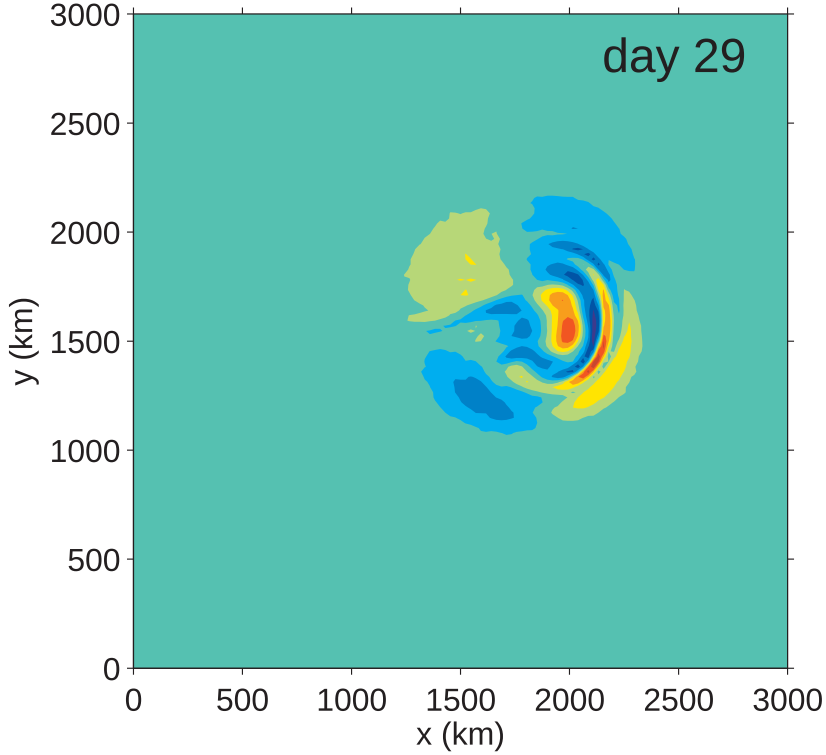
Simulations of a Dipole Vortex

- w at lowest model level



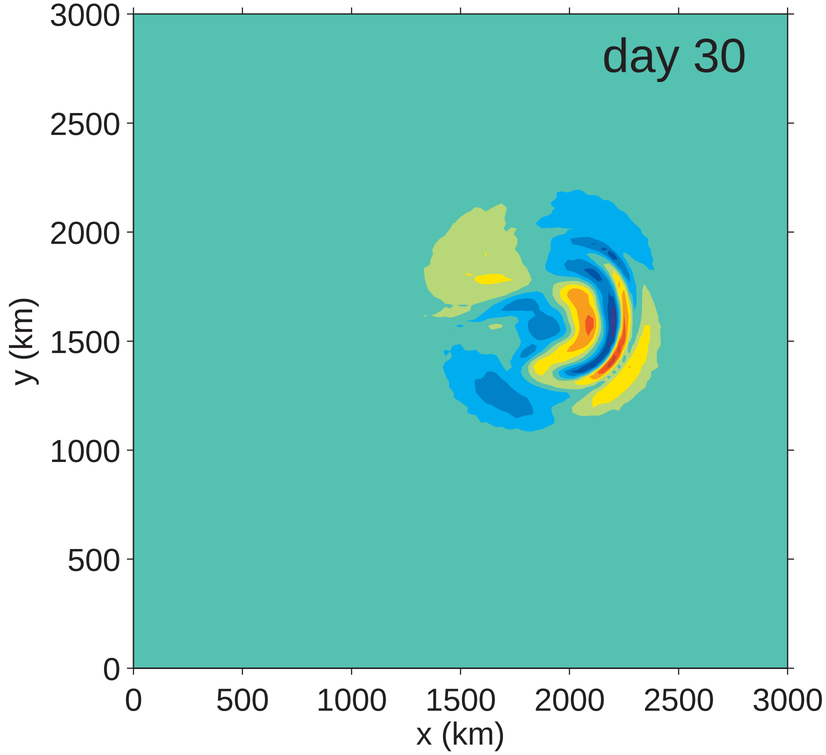
Simulations of a Dipole Vortex

- w at lowest model level



Simulations of a Dipole Vortex

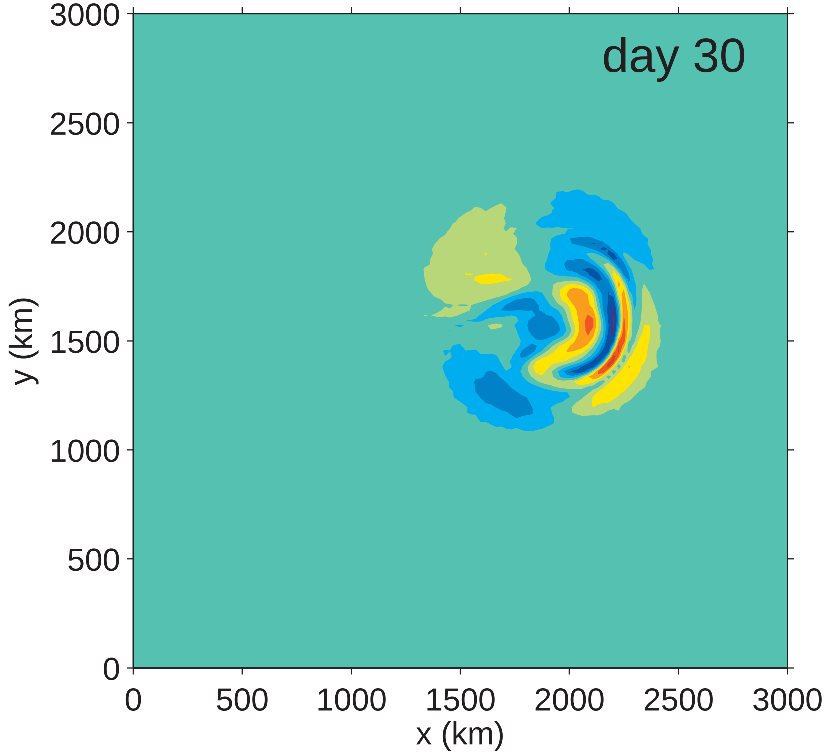
- w at lowest model level



... inertia-gravity waves downstream of jet core, stationary w.r.t. dipole

Simulations of a Dipole Vortex

- w at lowest model level



... inertia-gravity waves downstream of jet core, stationary w.r.t. dipole
(In u, v and θ, waves are weak relative to dipole.)
(Waves have upward group velocity.)

Theory: Small Perturbations from QG Dipole

- evolution eqn. for state $\mathbf{s}(x, y, z, t)$,

  $$\partial \mathbf{s}/\partial t = N(\mathbf{s})$$

- assume $\tilde{\mathbf{s}}$ is approximate (balanced) solution such that $\mathbf{s}' = \mathbf{s} - \tilde{\mathbf{s}}$ is small over some time interval. Here, $\tilde{\mathbf{s}}$ is QG dipole.
- then

  $$N(\mathbf{s}) \approx N(\tilde{\mathbf{s}}) + L(\mathbf{s}'), \quad \text{where } L = \partial N/\partial \mathbf{s}$$

- finally, evolution of $\mathbf{s}'$ is approximated by the forced, linear eqn.,

  $$(\partial/\partial t - L)\, \mathbf{s}' = N(\tilde{\mathbf{s}}) - \partial \tilde{\mathbf{s}}/\partial t$$

- forcing given by "residual tendency" from balanced solution

Theory: Small Perturbations from QG Dipole

Mechanisms for wave generation

- again, evolution of $\mathbf{s}'$:

$$(\partial/\partial t - L)\,\mathbf{s}' = N(\tilde{\mathbf{s}}) - \partial\tilde{\mathbf{s}}/\partial t$$

- instabilities: growing, homogeneous solutions to l.h.s. operator
- wave emission: forced solutions, where forcing depends only on $\tilde{\mathbf{s}}$

Numerical Solutions for $\mathbf{s}'$

Linearize primitive equations about QG dipole

- work in frame of reference in which QG dipole is steady
- tangent-linear version of nonlinear model
- steady forcing computed from QG dipole

Solutions valid only for limited time

- as long as QG dipole approximates full primitive-equation solution
- scaling estimate suggests 3–5 days

Forced, Linear Equations

$$\left(\frac{\partial}{\partial t} + \tilde{\mathbf{v}} \cdot \nabla\right)\mathbf{v}' + \mathbf{v}' \cdot \nabla\tilde{\mathbf{v}} + f\mathbf{k} \times \mathbf{v}' + \nabla\phi' - \frac{g\theta'}{\theta_0}\mathbf{k} = (F_u, F_v, F_w)$$

$$\left(\frac{\partial}{\partial t} + \tilde{\mathbf{v}} \cdot \nabla\right)\phi' + \mathbf{v}' \cdot \nabla\tilde{\phi} + c_s^2\nabla \cdot \mathbf{v}' = F_\phi$$

$$\left(\frac{\partial}{\partial t} + \tilde{\mathbf{v}} \cdot \nabla\right)\theta' + \mathbf{v}' \cdot \nabla\tilde{\theta} = F_\theta$$

Residual Tendencies

To compute forcings, begin with ageostrophic winds. These are determined by QG dipole,

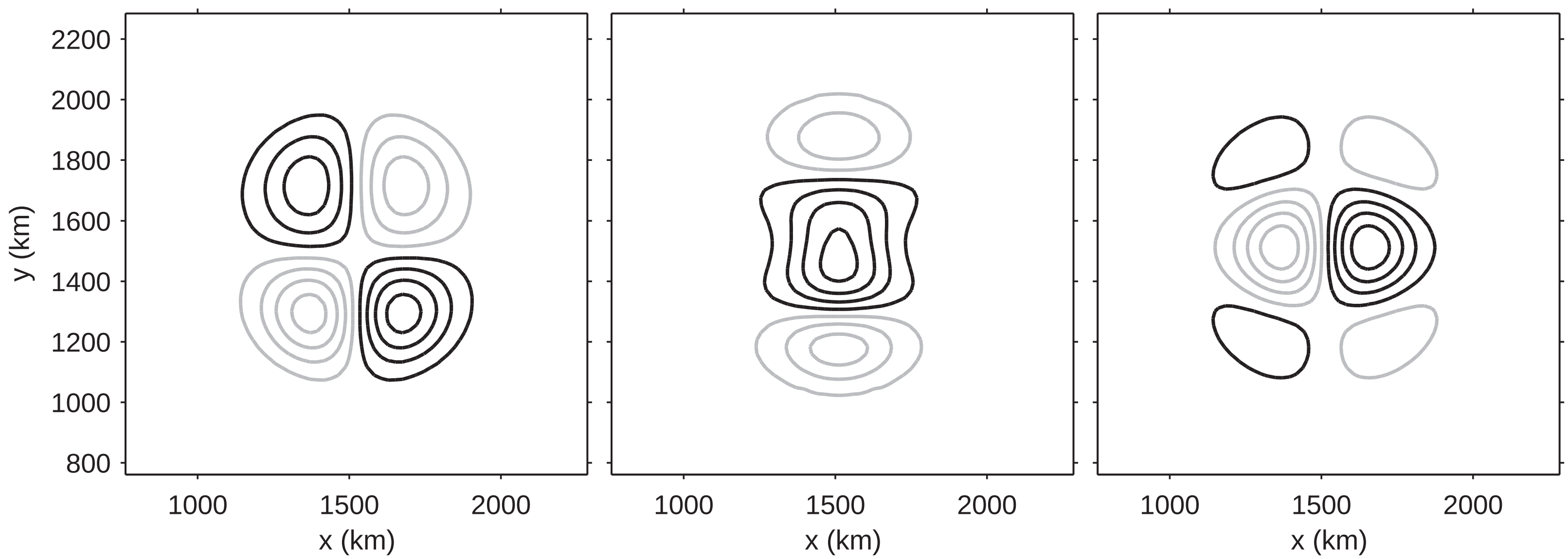
$$\begin{aligned}
\tilde{u}_a &= f^{-1}(\partial/\partial t + \tilde{\mathbf{v}}_g \cdot \nabla)\tilde{v}_g, \\
-\tilde{v}_a &= f^{-1}(\partial/\partial t + \tilde{\mathbf{v}}_g \cdot \nabla)\tilde{u}_g, \\
\tilde{w} &= -(g/\theta_0)N^{-2}(\partial/\partial t + \tilde{\mathbf{v}}_g \cdot \nabla)\tilde{\theta}.
\end{aligned}$$

Residual tendencies are then

$$\begin{aligned}
F_u &= -\tilde{\mathbf{v}}_a \cdot \nabla\tilde{u} - \tilde{\mathbf{v}}_g \cdot \nabla\tilde{u}_a, \\
F_v &= -\tilde{\mathbf{v}}_a \cdot \nabla\tilde{v} - \tilde{\mathbf{v}}_g \cdot \nabla\tilde{v}_a, \\
F_\theta &= -\tilde{\mathbf{v}}_a \cdot \nabla\tilde{\theta}.
\end{aligned}$$

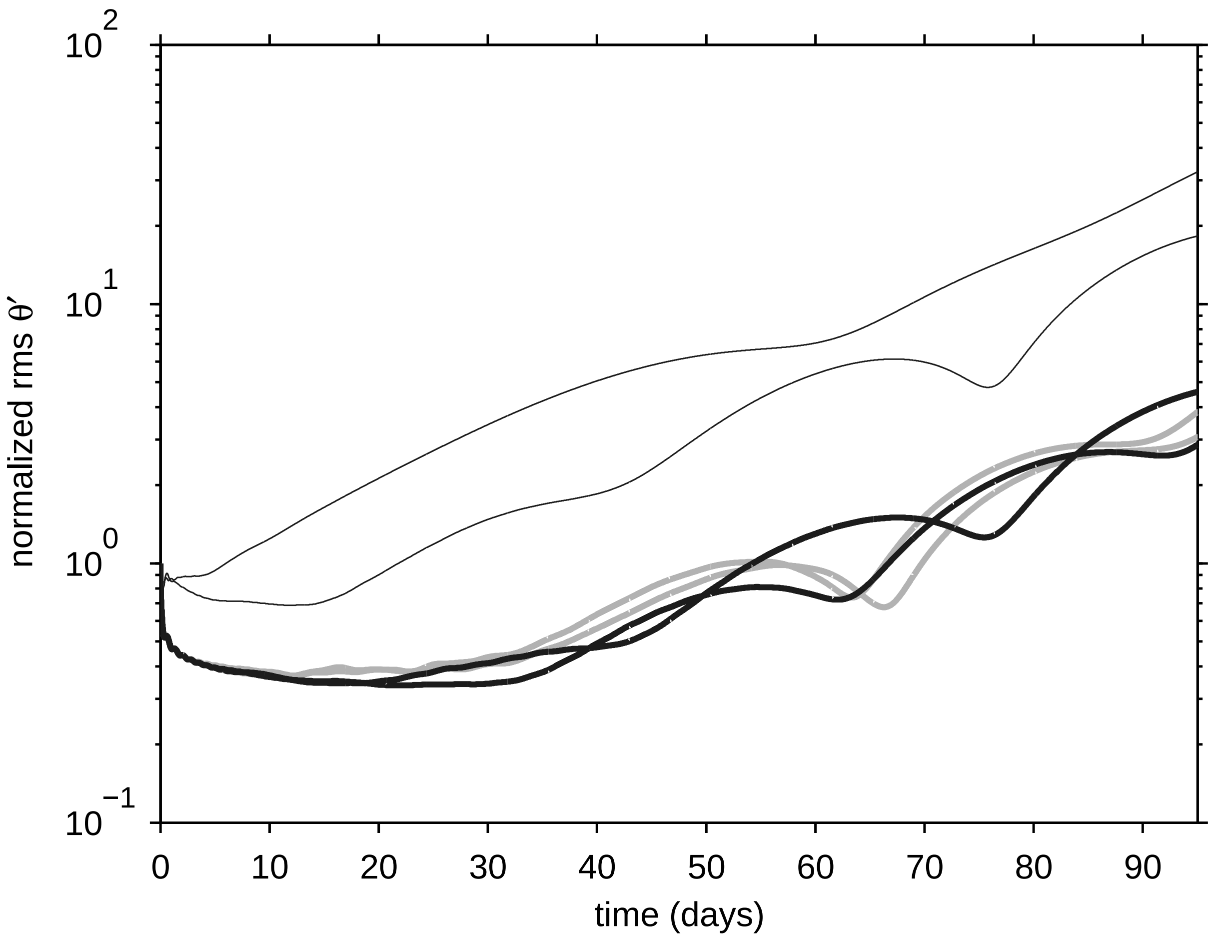
Numerical Solutions for $\mathbf{s}'$

- forcing: residual tendencies from QG dipole for u, v and θ at lowest model level.



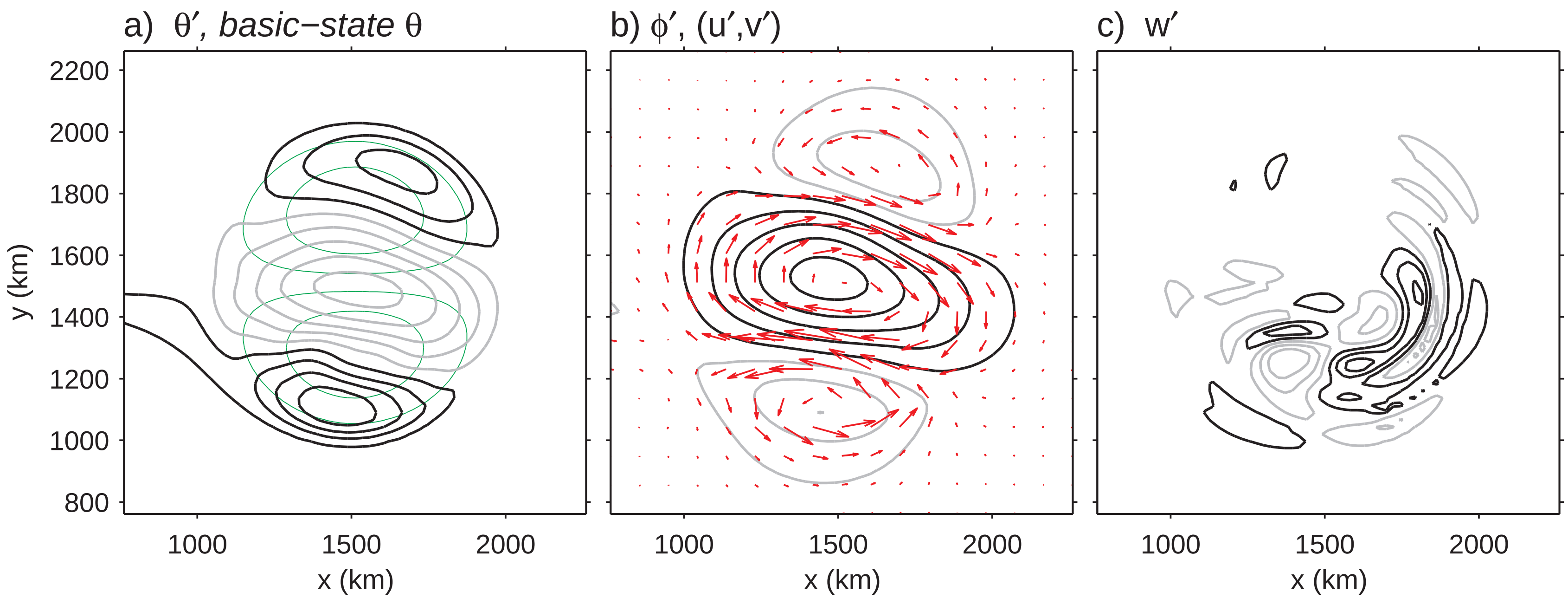
Instability of QG Dipole?

- Try various ICs: white noise in θ', also larger-scale perturbations
- perturbations grow, but too slowly to explain wave



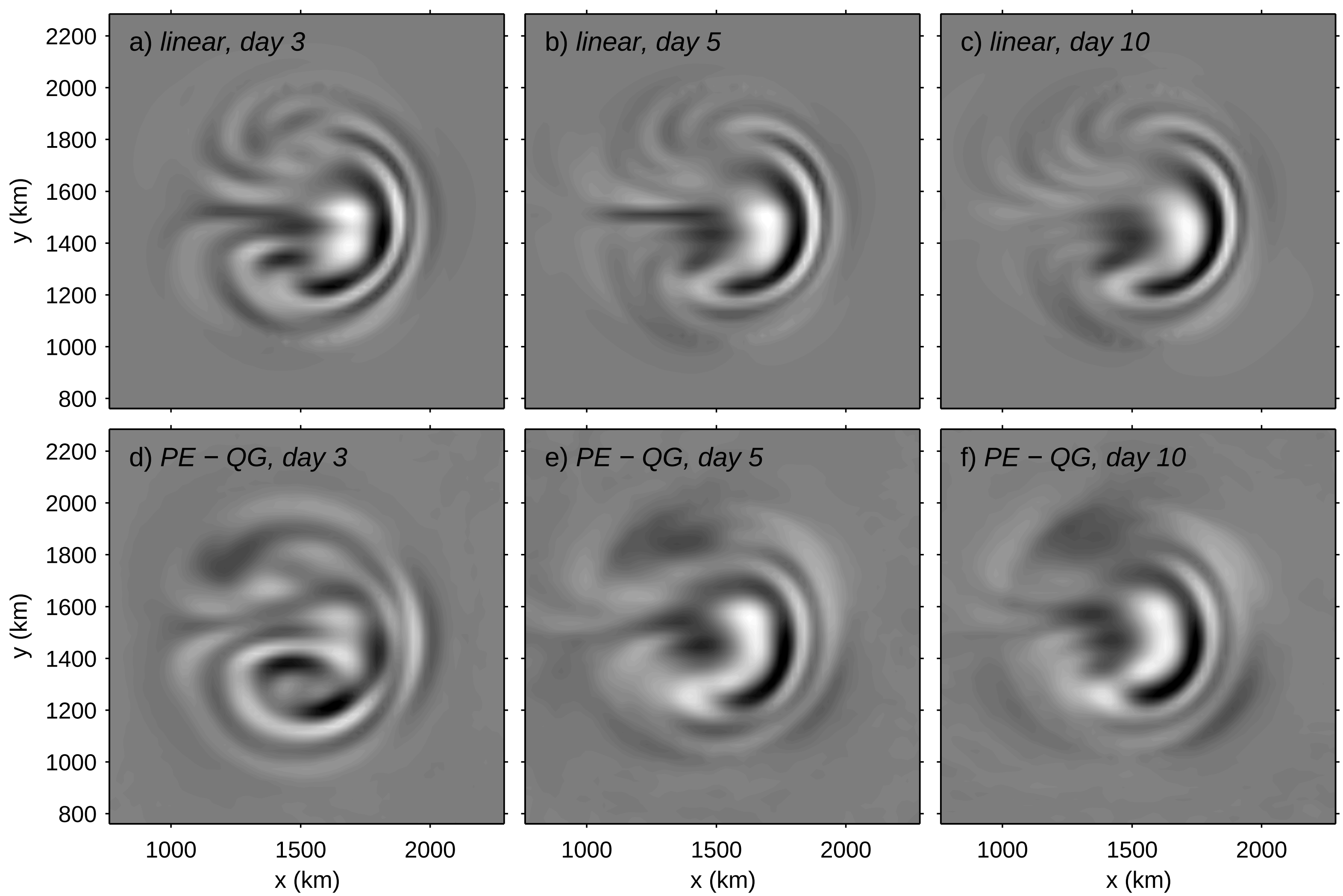
Instability of QG Dipole?

- growing perturbations are balanced, with scale comparable to dipole



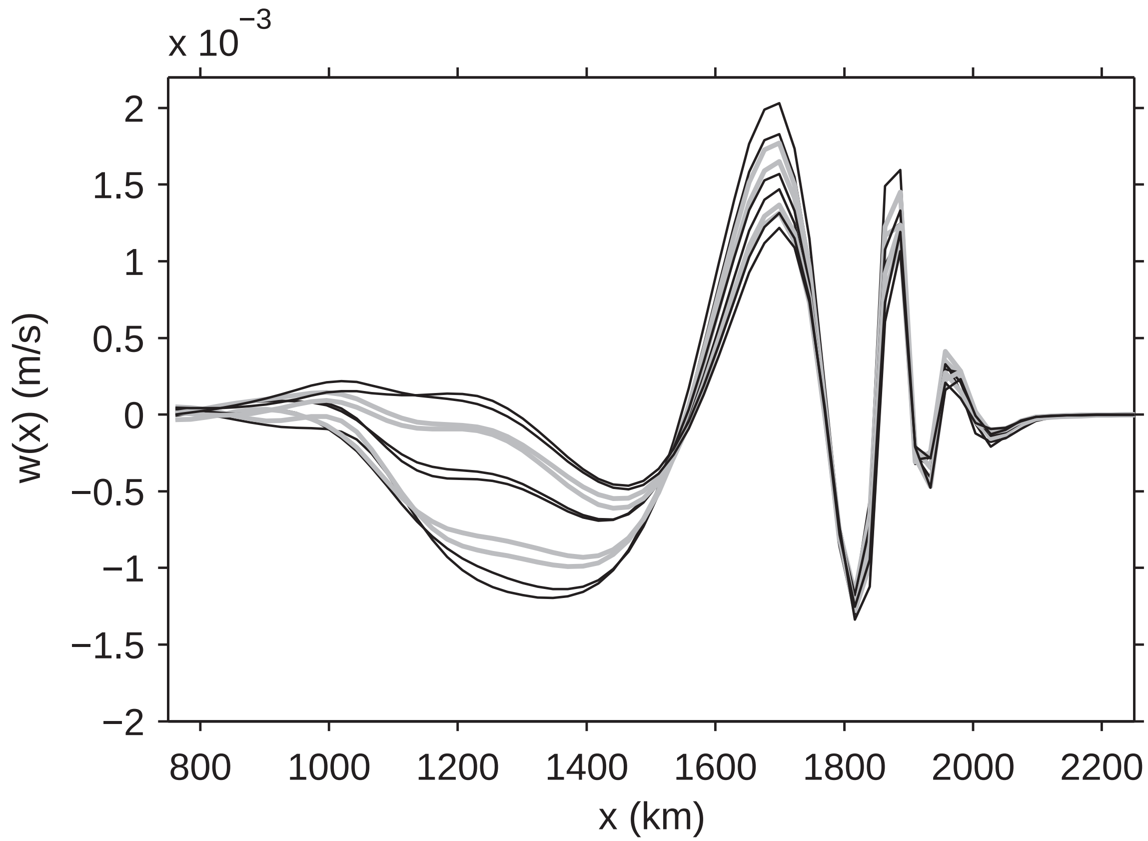
Forced solutions

- w' from forced solution (top row) and PE − QG (bottom)
- forced solution has waves: $\sim$ steady by day 5, structure similar to that of PE − QG



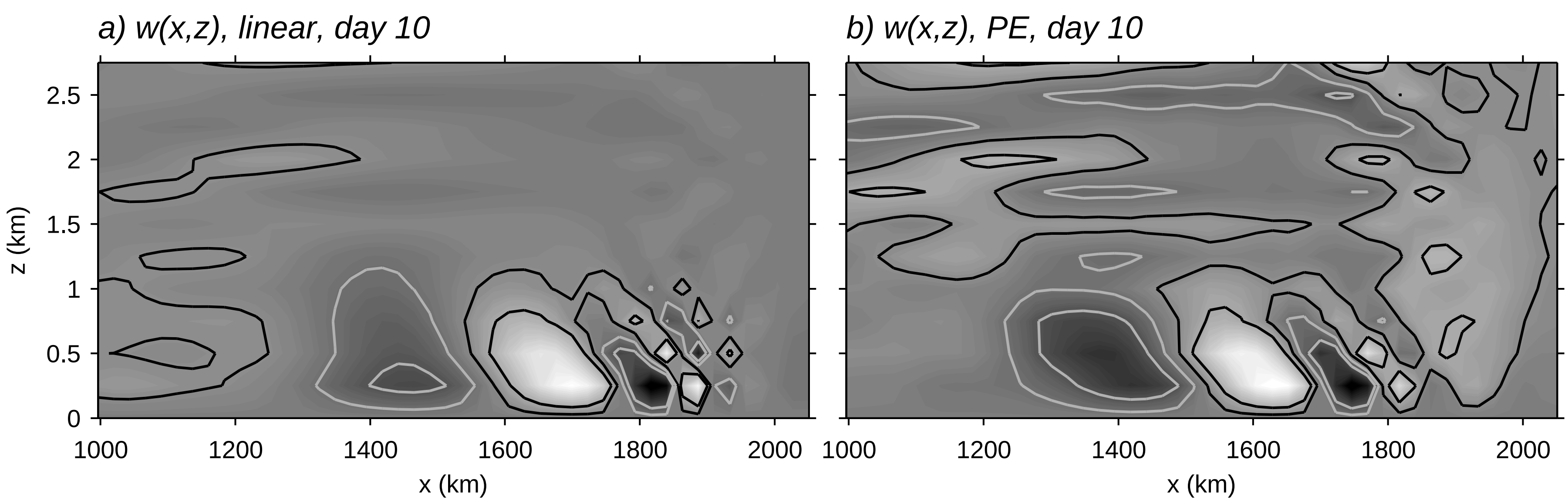
Forced solutions (cont.)

- $w'(x)$ every 12 h from 5 d to 9.5 d, at lowest level and $y = 1500\,\mathrm{km}$
- wave phase $\sim$ steady
- slow growth of amplitude, esp. up/down couplet in center of dipole



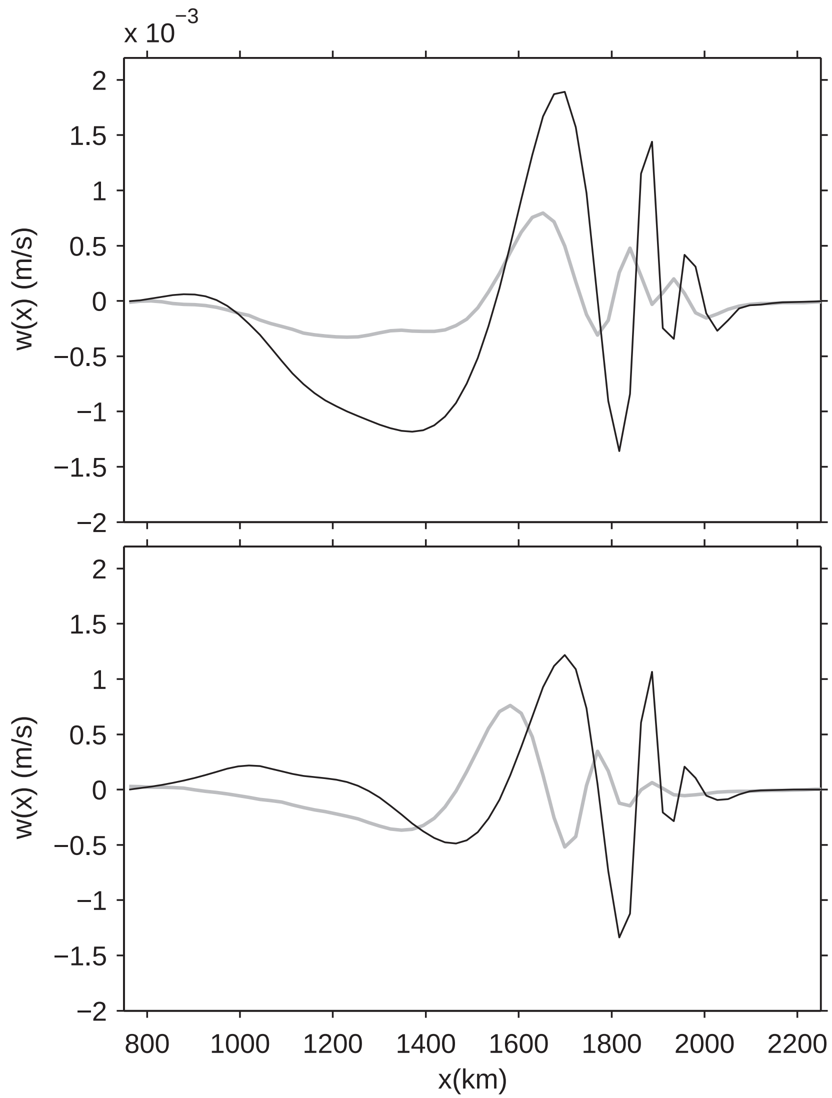
Forced solutions (cont.)

- vertical section of w' from forced solution (left) and PE (right)
- upward group velocity; structure again similar to that of PE



Forced solutions (cont.)

- $w'(x)$ (black) and PE w (gray) at day 5 and day 10
- amplitude of linear solution substantially too large (2–3 times)



Discussion

Heuristic explanation of wave properties

- for dipole, $R = U/fL << 1$, $B = NH/fL = O(1)$
- non-dimensional dispersion relation for inertia-gravity waves
$$R^2(\omega - ku - lv)^2 = 1 + B^2(k^2 + l^2)/m^2$$
- at least one of ω and k must then be $O(R^{-1})$; waves are either fast or small scale compared to the dipole
- since dipole is nearly steady, almost no fast waves excited
- stationary, small-scale waves require balanced flow relative to dipole; akin to mountain waves

Comparison with Lighthill (1952) and Ford et al. (2000)

- in Lighthill/Ford, waves are *large scale* compared to balanced flow
- also expect waves that are fast if Ford theory extended to $R << 1$
- linear operator is that for flow at rest; waves with $\omega << 1$ not allowed

Summary

Waves as forced, linear response to balanced flow

- numerical solutions capture stationary behavior and phase of waves
- amplitude overestimated by factor > 2