

Richardson Preconditioners for Elliptic Problems in Numerical Models of Weather and Climate

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Numerical models for integrating evolutionary partial-differential equations of weather and climate (W&C) often involve solution of elliptic boundary value problems (EBVPs) at each time step of the model. Solutions of such EBVPs amount to inverting a sparse N x N matrix, with N being a total number of nodes discretizing the W&C model domain. With numerical weather prediction aiming at horizontal resolutions O(1)~km globally, N can be on the order of 10⁹. The favored methods for handling such EBVPs are the matrix-free Krylov-subspace algorithms. Notwithstanding their virtues of simplicity and robustness, Krylov algorithms lose their efficacy for stiff problems characterized by the multiplicity of scales. To aid their performance, the vanilla schemes incorporate operator preconditioning that amounts to swiftly inverting an onerous part of the governing EBVP, and thus reducing its stiffness.

This talk presents a suite of preconditioners steaming from the Richardson's (1910) seminal idea of augmenting EBVP with a transient problem converging to this EBVP in a pseudo-time $\tau \rightarrow \infty$. Exploiting this idea for mixed explicit-implicit time-stepping schemes leads to a suite of stationary (as opposed to variational) iteration solvers, including the many classical algorithms. The high-performance all-scale EULAG model [J. Comput. Phys. 263 (2014) 185-205], with a flexible three-dimensional decomposition of MPI tasks, is armed with the preconditioners akin to standard alternative direction implicit algorithms, extended to optional permutations of parallel tridiagonal inversions. Given the availability of the recent article [J. Comput. Phys. 463 (2022) 111296], the talk only highlights key theoretical aspects of the EBVPs in W&C models, while focusing on the relative efficacy gains attainable with the developed preconditioners in simulations of atmospheric flows across scales from planetary to cloud and laboratory.

Thursday, 18 May 2023, 2:00pm Refreshments 1:45pm Please also join colleagues for refreshments and informal discussion after the seminar until 3:30pm NCAR-Foothills Laboratory, 3450 Mitchell Lane

FL2-1022, Large Auditorium

Seminar will also be live webcast

https://operations.ucar.edu/live-mmm Participants may ask questions during the seminar via Slido.

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