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DNS of MHD and Hall MHD turbulence

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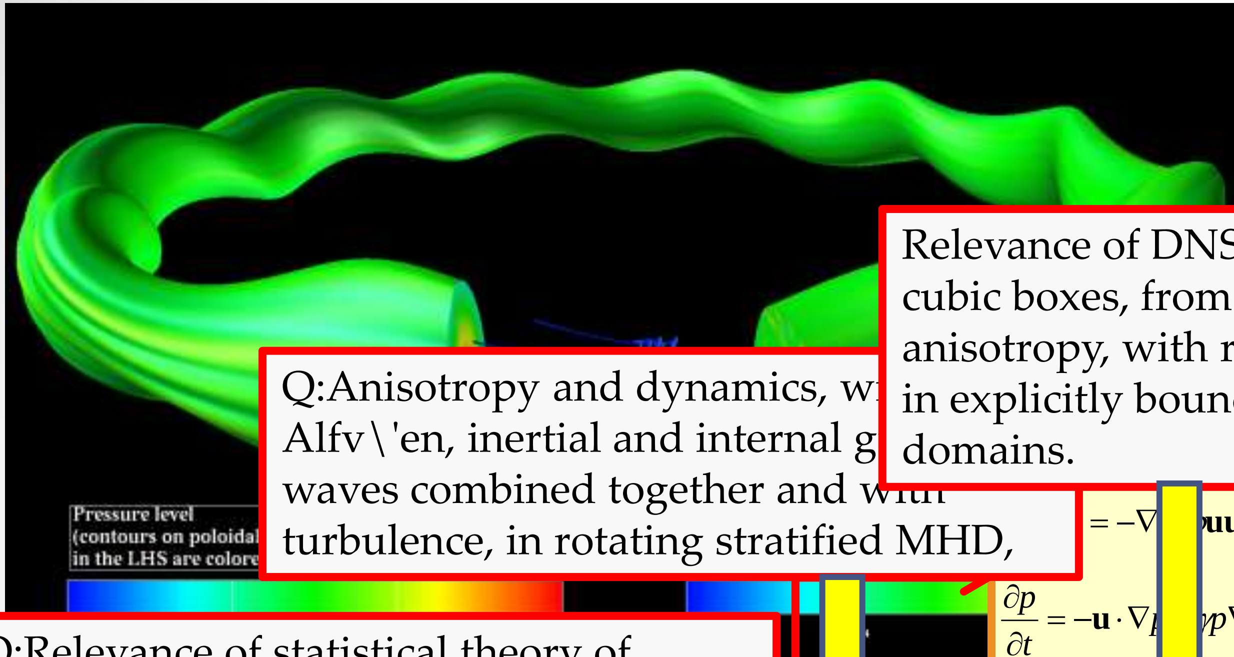
Why LES ? Workhorse for EM events



Relevance of DNS and LES in periodic cubic boxes, from isotropy to strong anisotropy, with respect to computations in explicitly bounded, e.g. sperical, domains.

Q:Anisotropy and dynamics, wi Alfv\'en, inertial and internal g waves combined together and with turbulence, in rotating stratified MHD,

$$= -\nabla\cdot(\rho\mathbf{uu} - \mathbf{BB}) - \nabla\left(p + \frac{1}{2}\mathbf{B}^2\right) - \nabla\cdot\mathbf{\Pi}^{gv},$$

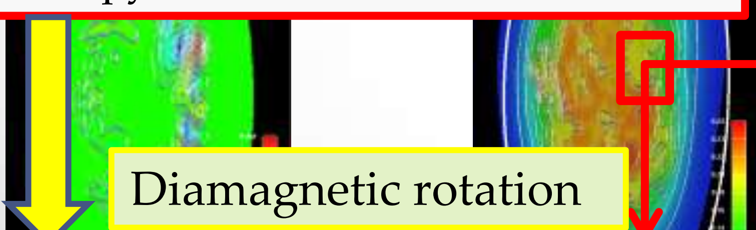
$$\frac{\partial p}{\partial t} = -\mathbf{u}\cdot\nabla p - \gamma p\nabla\cdot\mathbf{u} + \eta\mathbf{j}^2 - \nabla\cdot\left(-\kappa_{//}\nabla_{//}T - \kappa_{\perp}\nabla_{\perp}T + \kappa_{\wedge}\nabla_{\wedge}T\right),$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\left\{\left(\mathbf{u} - \frac{\varepsilon_H}{\rho}\mathbf{j}\right)\times\mathbf{B} + \frac{1}{\rho}\nabla p_e - \eta\mathbf{j}\right\}$$

Q:Relevance of statistical theory of homogeneous turbulence, towards very high Reynolds numbers, as generalized EDQNM, from isotropy to strong anisotropy,

Severe limitation eg. by whistler waves and kinetic Alfven waves

Instability & turbulence interaction



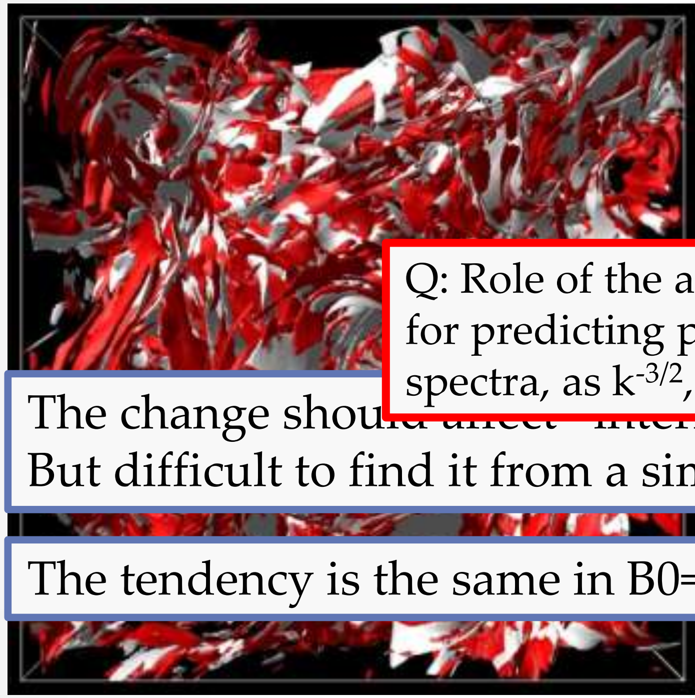
Diamagnetic rotation

LES for (local) turbulence ?

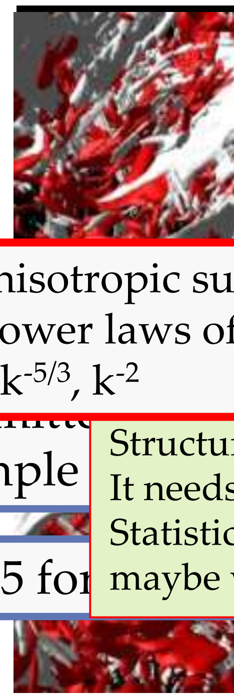
Locally turbulent state (in the sense of MHD):
Flattened pressure (**locally homogeneous, anisotropic**)
Rotation+ density&temperature gradient should come
Full EM events (not suitable for gyrokinetic simulations)

Hall MHD as a test model (periodic box computation is still helpful)

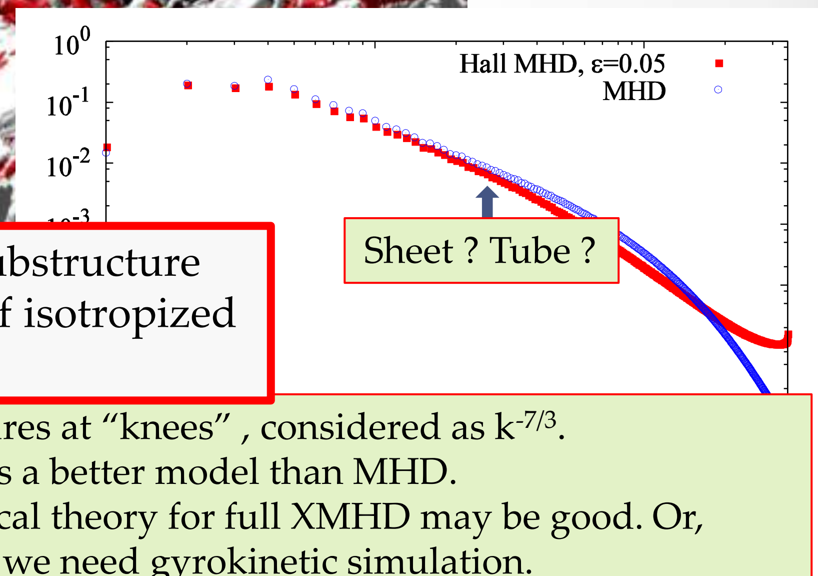
Non-MHD Effects: Local structures, MHD vs Hall MHD in freely decaying turbulence, B_0=0



MHD



Hall MHD



Sheet ? Tube ?

Q: Role of the anisotropic substructure for predicting power laws of isotropized spectra, as $k^{-3/2}$, $k^{-5/3}$, k^{-2}

The change should affect "intermitte
But difficult to find it from a simple

The tendency is the same in B0=5 for

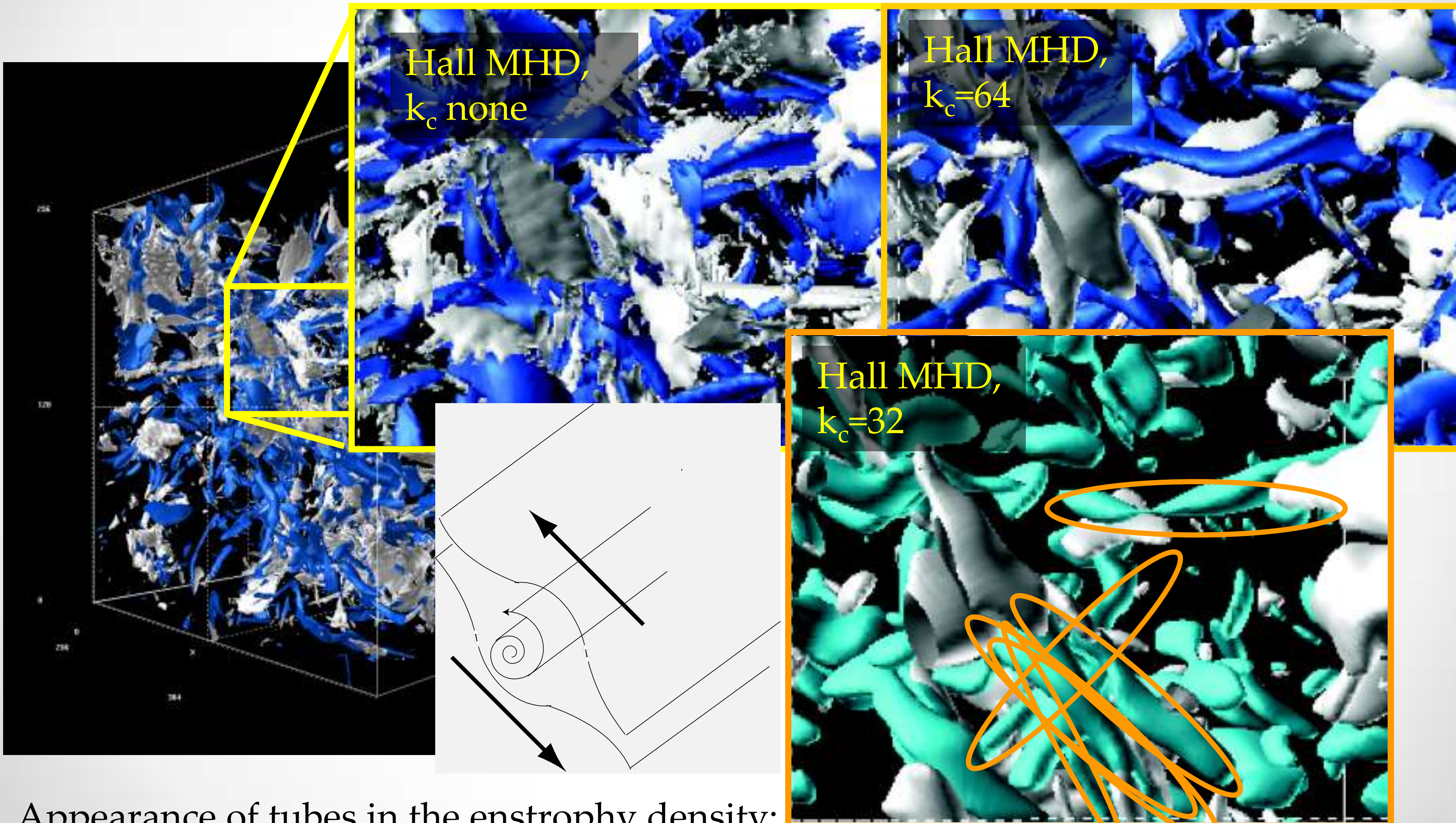
Structures at "knees" , considered as $k^{-7/3}$.
It needs a better model than MHD.
Statistical theory for full XMHD may be good. Or, maybe we need gyrokinetic simulation.

$\partial \mathbf{u}$ … $\cdot \mathbf{u} = 0,$

… $\mathbf{B} = 0.$

Turbulent structure is changed by two-fluid/FLR /kinetic effects.
(The situation is the same for anisotropic, B0 != 0 case , too.
To consider interaction of ballooning instability & turbulence, we need a SGS model which shows a good correspondence with DNS.

For SGS: coarse-grained enstrophy/current density fields can be tubular.



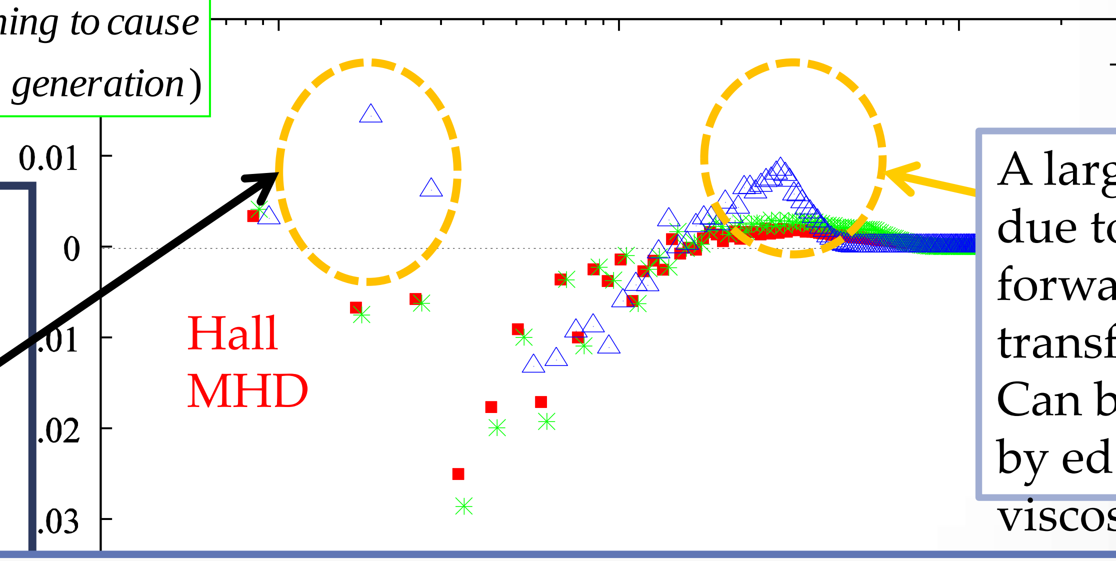
Appearance of tubes in the enstrophy density:
A possible change of dominant motions among scales. (It does not happen in MHD.)

Energy transfer is modified when coarser than the Taylor scale.

$$N^3 = 512^3,\ \varepsilon = 0.05.$$
$$R_\lambda^V \approx R_\lambda^M \approx 100$$

$$\begin{aligned}
\nabla\times[(\mathbf{u}-\varepsilon_H\mathbf{j})\times\mathbf{B}] = \\
-(\mathbf{u}-\varepsilon_H\mathbf{j})\cdot\nabla\mathbf{B} \quad & (\textit{advection}) \\
+\mathbf{B}\cdot\nabla(\mathbf{u}-\varepsilon_H\mathbf{j}) \quad & (\textit{stretching to cause} \\
& \textit{magnetic field generation})
\end{aligned}$$

$$[\mathbf{B}\cdot\nabla(\mathbf{u}-\varepsilon_H\mathbf{j})]$$



A strong backward transfer by the truncation: as it happens in a poorly resolved simulation.
No predict

A large bump due to lack of forward transfer:
Can be removed by eddy-viscosity-type

A smart SGS modeling approach is required to take some dispersive (not dissipative) nature of whistler and other waves.
Expectation to statistical theory. A**nisotropic MHD turbulence (w Alfven and/or rotation) theory and models should be established first.**

The residual part of the coarse-grained field is defined.

$$\mathbf{R}_V := \frac{\partial \mathbf{u}}{\partial t} - \frac{\partial \overline{\mathbf{u}}}{\partial t} = -\nabla \cdot \left[\left(\overline{\mathbf{u}}\,\overline{\mathbf{u}} - \mathbf{u}\mathbf{u} \right) - \left(\overline{\mathbf{B}}\,\overline{\mathbf{B}} - \mathbf{B}\mathbf{B} \right) \right], \quad \mathbf{W}_V := \nabla \cdot \left[\left(\Delta^2 \left(\frac{1}{2} \overline{S}_{ij}{}^2 + \frac{C_\lambda}{C_\nu} \overline{j}_i \overline{j}_i \right)^{1/2} \right) \overline{S}_{ij} \right],$$

$$\mathbf{R}_B := \left[\frac{\partial \mathbf{B}}{\partial t} - \frac{\partial \overline{\mathbf{B}}}{\partial t} \right]_{Induction} = -\nabla \times \left(\overline{\mathbf{u}} \times \overline{\mathbf{B}} - \mathbf{u} \times \mathbf{B} \right), \qquad \mathbf{W}_B := \nabla \times \left[\left(\Delta^2 \left(\frac{1}{2} \overline{S}_{ij}{}^2 + \frac{C_\lambda}{C_\nu} \overline{j}_i \overline{j}_i \right)^{1/2} \right) \overline{j}_i \right],$$

$$\mathbf{R}_H := \left[\frac{\partial \mathbf{B}}{\partial t} - \frac{\partial \overline{\mathbf{B}}}{\partial t} \right]_{Hall} = -\nabla \times \left[-\varepsilon_H \left(\overline{\mathbf{j}} \times \overline{\mathbf{B}} - \mathbf{j} \times \mathbf{B} \right) \right]$$

$$\mathbf{P}_\mathbf{V} := \mathbf{R}_V - \left(\mathbf{R}_V \cdot \mathbf{W}_V \right) \frac{\mathbf{W}_V}{|\mathbf{W}_V|}, \qquad \Rightarrow \Xi_V := \left\langle \mathbf{P}_\mathbf{V}{}^2 \right\rangle / \left\langle \mathbf{R}_V{}^2 \right\rangle,$$

$$\mathbf{P}_\mathbf{B} := \mathbf{R}_B - \left(\mathbf{R}_B \cdot \mathbf{W}_B \right) \frac{\mathbf{W}_B}{|\mathbf{W}_B|}, \qquad \Rightarrow \Xi_B := \left\langle \mathbf{P}_\mathbf{B}{}^2 \right\rangle / \left\langle \mathbf{R}_B{}^2 \right\rangle,$$

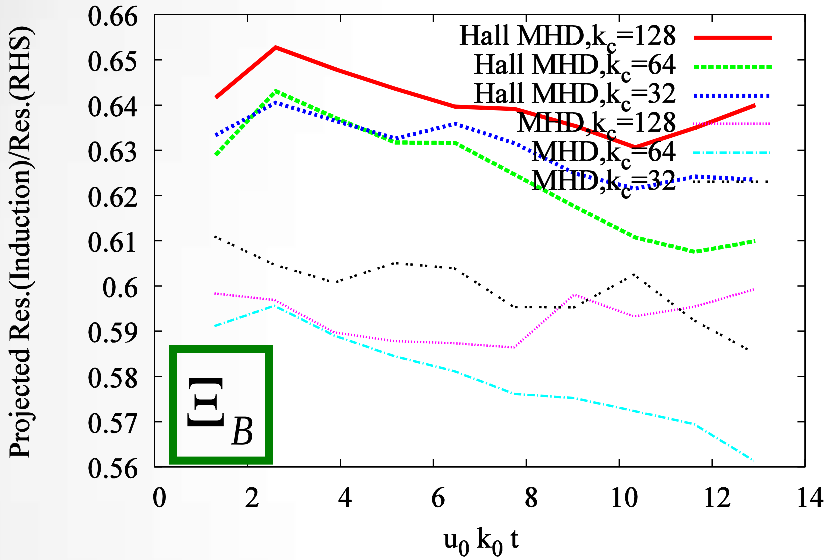
$$\mathbf{P}_\mathbf{H} := \mathbf{R}_H - \left(\mathbf{R}_H \cdot \mathbf{W}_V \right) \frac{\mathbf{W}_V}{|\mathbf{W}_V|} - \left[\mathbf{R}_H - \left(\mathbf{R}_H \cdot \mathbf{W}_V \right) \frac{\mathbf{W}_V}{|\mathbf{W}_V|} \right] \cdot \mathbf{W}_B \qquad \Rightarrow \Xi_B := \left\langle \mathbf{P}_\mathbf{H}{}^2 \right\rangle / \left\langle \mathbf{R}_H{}^2 \right\rangle.$$

A rough estimation how much the Smagorinsky-type diffusive model can cover the residual part of the low-pass-filtering operation will help our modeling.

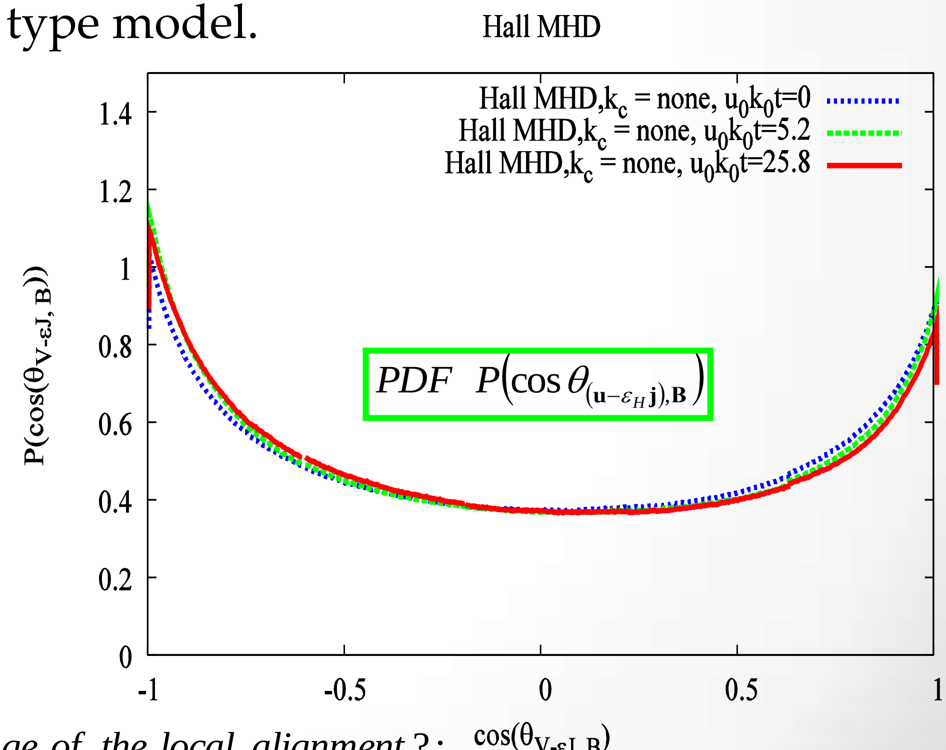
Miura and Araki, PPCF(2013)

The residual part of the coarse-grained field is estimated

SGS model by Hamba&Tsuchiya (2010) was used.



A rough estimation shows that about 60% of the norm remains to be modeled further in the Smagorinsky-type model.



By the Hall term, the model-efficiency of the induction is deteriorated than that in single-fluid MHD (55% <-> 65%).

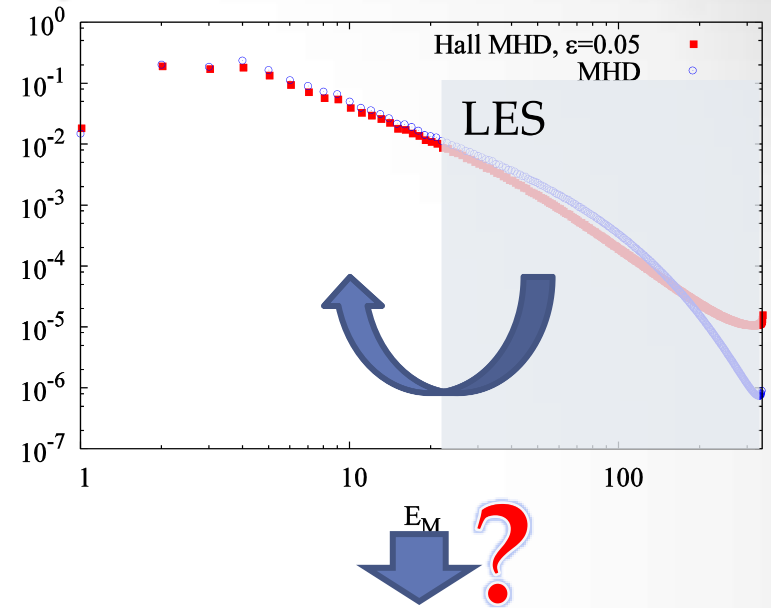
due to change of the local alignment ?:

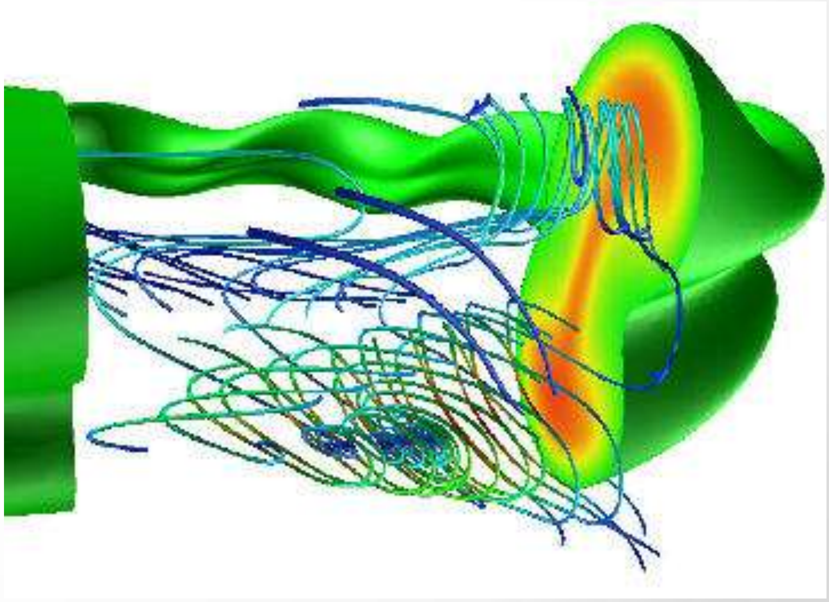
$$\mathbf{u} // \mathbf{B} \ (MHD) \Rightarrow (\mathbf{u} - \varepsilon_H \mathbf{j}) // \mathbf{B} \ (Hall \ \ MHD)$$

Summary

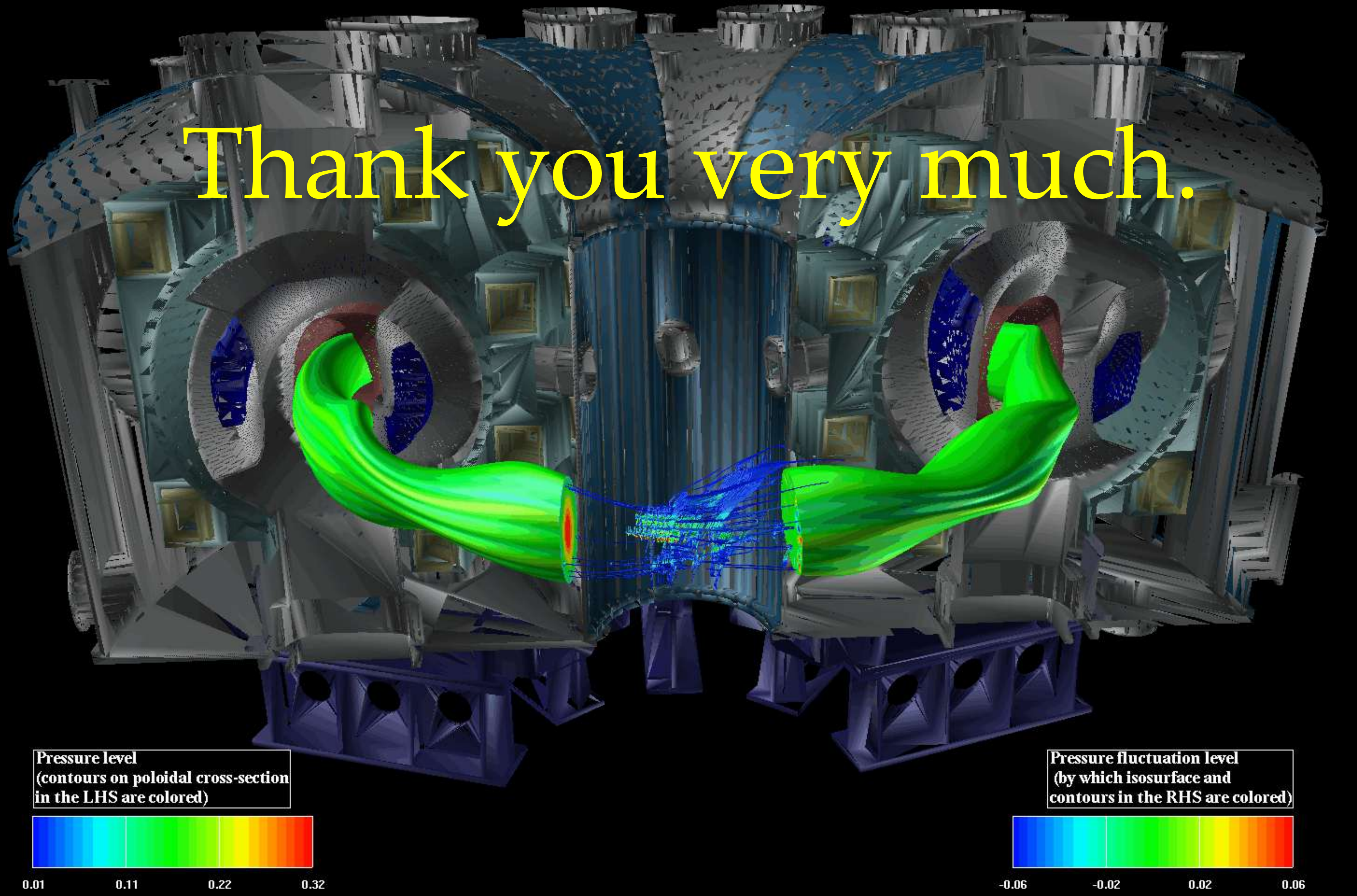
Approaches to turbulence by a fluid model

- **“Workhorse”** to study EM events with turbulence is required.
- An accuracy of a two-fluid or XMHD model for turbulence is limited, but useful for the purpose.
- Hall term and other extension of MHD can change turbulent structures drastically, even if they work only on high wave number region.
- Hall term and other extension of MHD can bring about fast dispersive waves and various aspects of anisotropy.
- LES can be a good solution to overcome the difficulty: removing fast waves and taking turbulence into account through SGS models.
- We need a SGS model for large $\nabla \rho$ and/or ∇T. Homogeneous turbulence can be helpful for turbulent region with a large magnetic island. Power-law, whether k^{-5/3} or k^{-7/3}, but anisotropic, makes a sense to constrain basic nature of the SGS model. Statistical theory will be also helpful on this context.
- SGS modeling man not be straightforward because of their non-dissipative natures .
- We need an LES which coincides with DNS data well so that EM events can be studied. (Mixtured model? Maybe.)



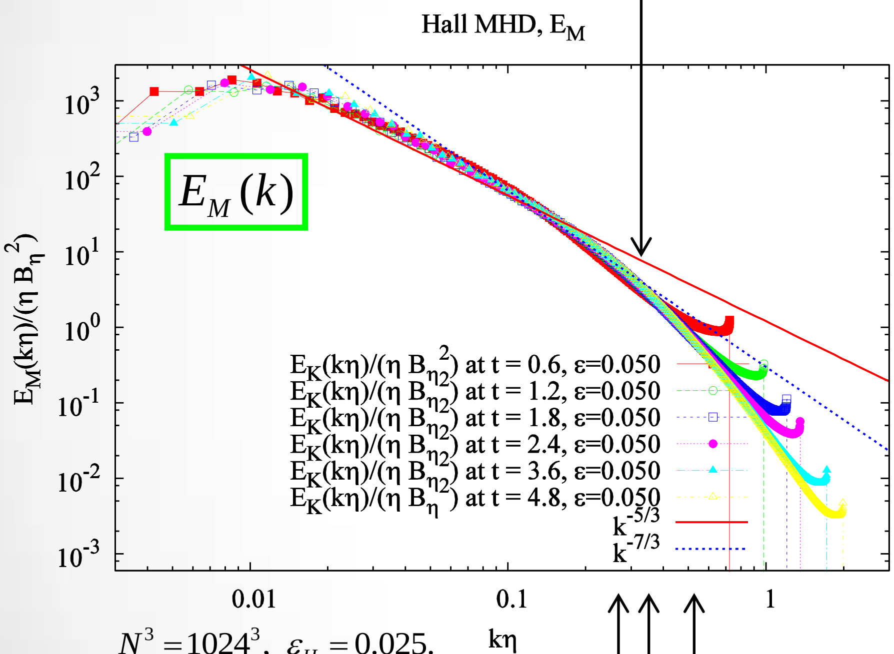


Thank you very much.



Spectral are likely $k^{-5/3}$ or $k^{-7/3}$

Taylor-scale $\lambda^M \approx 3\eta_K^M$

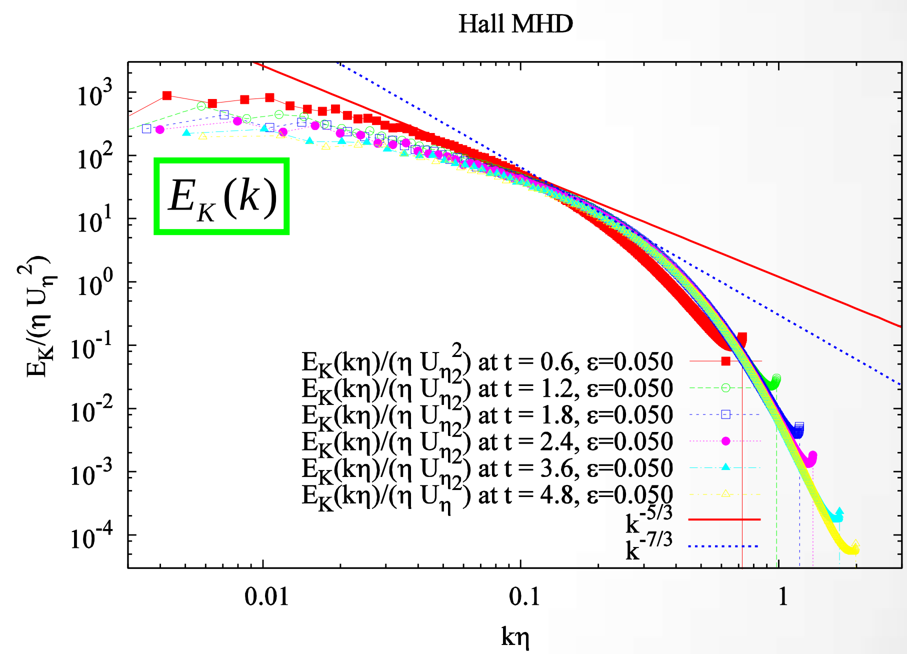


$N^3 = 1024^3,\ \varepsilon_H = 0.025.$

$1/\varepsilon_H = 20$

$k_c = 32$

$k_c = 64$



$N^3 = 1024^3,\ \varepsilon = 0.025.$

$R_\lambda^V \approx R_\lambda^M \approx 200 \Rightarrow R_L \approx O(10^4)$

Energy spectrum equations and transfer functions are defined as follows

$$\frac{\partial}{\partial t}\tilde{\mathbf{u}}_{\mathbf{k}} = F[-\mathbf{u}\cdot\nabla\mathbf{u} + \mathbf{j}\times\mathbf{B} - \nabla p] - \nu k^2 \tilde{\mathbf{u}}_{\mathbf{k}},$$

$$\frac{d}{dt}E_k^K(t) = \frac{\partial}{\partial t}\sum_{[\mathbf{k}]}\frac{1}{2}\left|\tilde{\mathbf{u}}_{\mathbf{k}}\right|^2 = T_k(t) - \nu k^2 \sum_{[\mathbf{k}]}\left|\tilde{\mathbf{u}}_{\mathbf{k}}\right|^2,$$

$$T_k^K(t) = \sum_{[\mathbf{k}]}\tilde{\mathbf{u}}_{\mathbf{k}} \otimes F[-\mathbf{u}\cdot\nabla\mathbf{u} + \mathbf{j}\times\mathbf{B} - \nabla p],$$

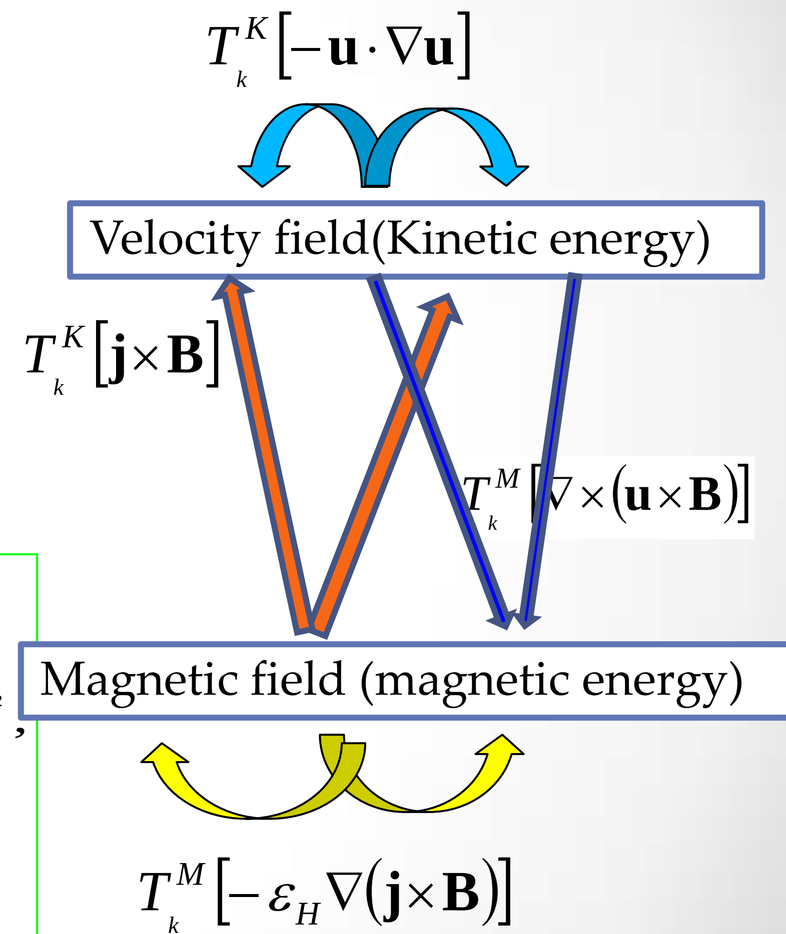
$$\Pi_k^K(t) = \sum_{k'\geq k} T_{k'}^K(t)$$

$$\frac{\partial}{\partial t}\tilde{\mathbf{B}}_{\mathbf{k}} = F[-\nabla\times[(\mathbf{u} - \varepsilon_H\mathbf{j})\times\mathbf{B}]] - \eta k^2 \tilde{\mathbf{B}}_{\mathbf{k}},$$

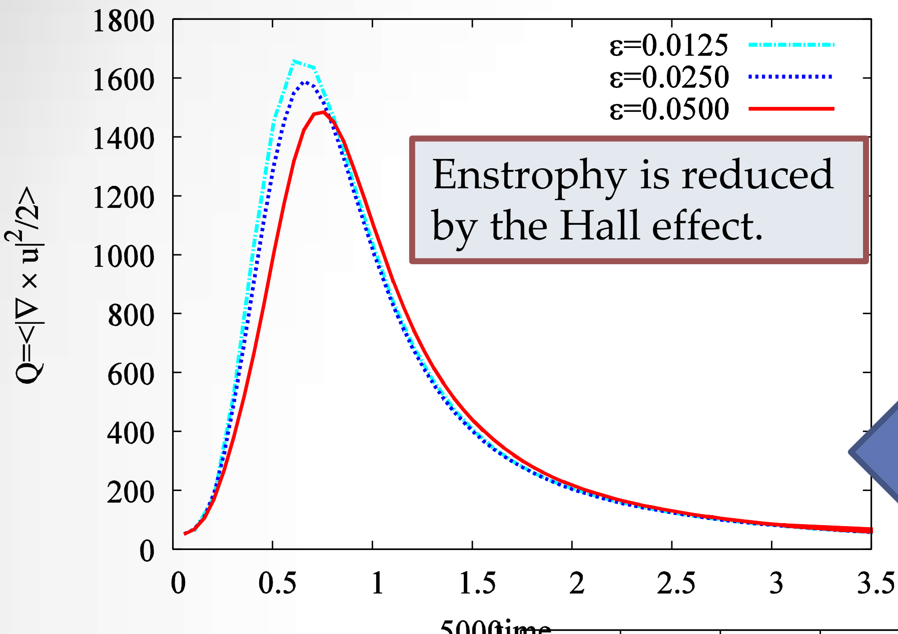
$$\frac{d}{dt}E_k^M(t) = \frac{\partial}{\partial t}\sum_{[\mathbf{k}]}\frac{1}{2}\left|\tilde{\mathbf{B}}_{\mathbf{k}}\right|^2 = T_k^M(t) - \nu k^2 \sum_{[\mathbf{k}]}\left|\tilde{\mathbf{B}}_{\mathbf{k}}\right|^2,$$

$$T_k^M(t) = \sum_{[\mathbf{k}]}\tilde{\mathbf{B}}_{\mathbf{k}} \otimes F[-\nabla\times[(\mathbf{u} - \varepsilon_H\mathbf{j})\times\mathbf{B}]],$$

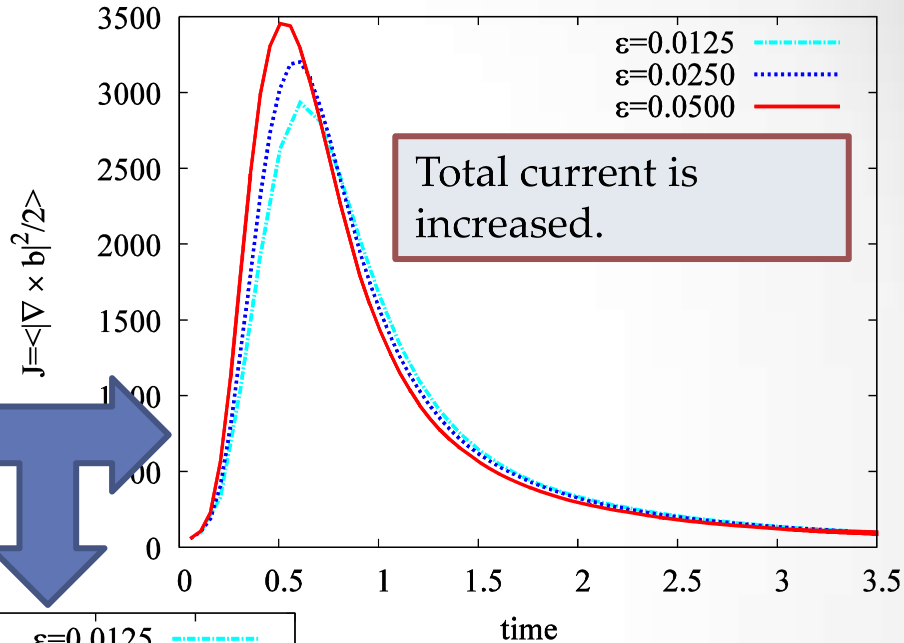
$$\Pi_k^M(t) = \sum_{k'\geq k} T_{k'}^M(t).$$



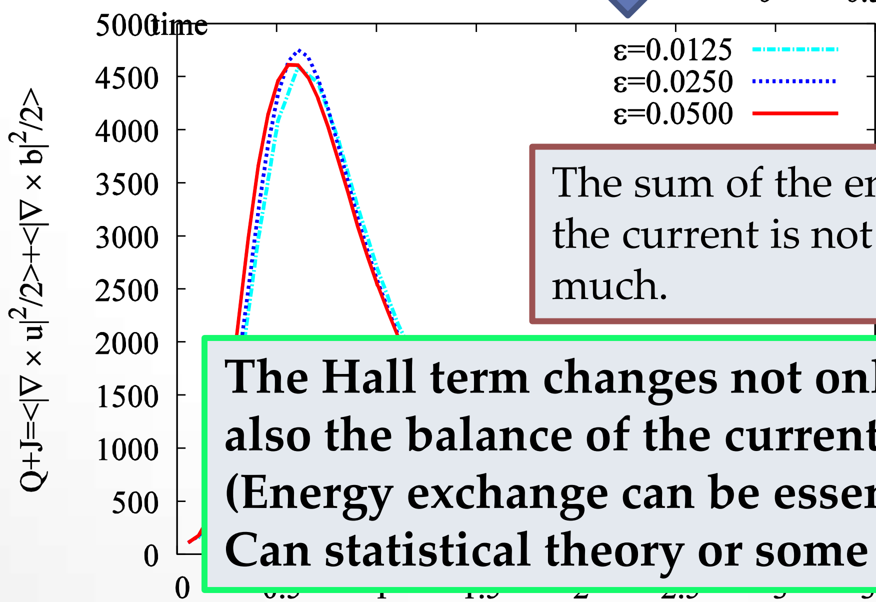
The Hall term controls the ratio of the current and the enstrophy



Enstrophy is reduced by the Hall effect.



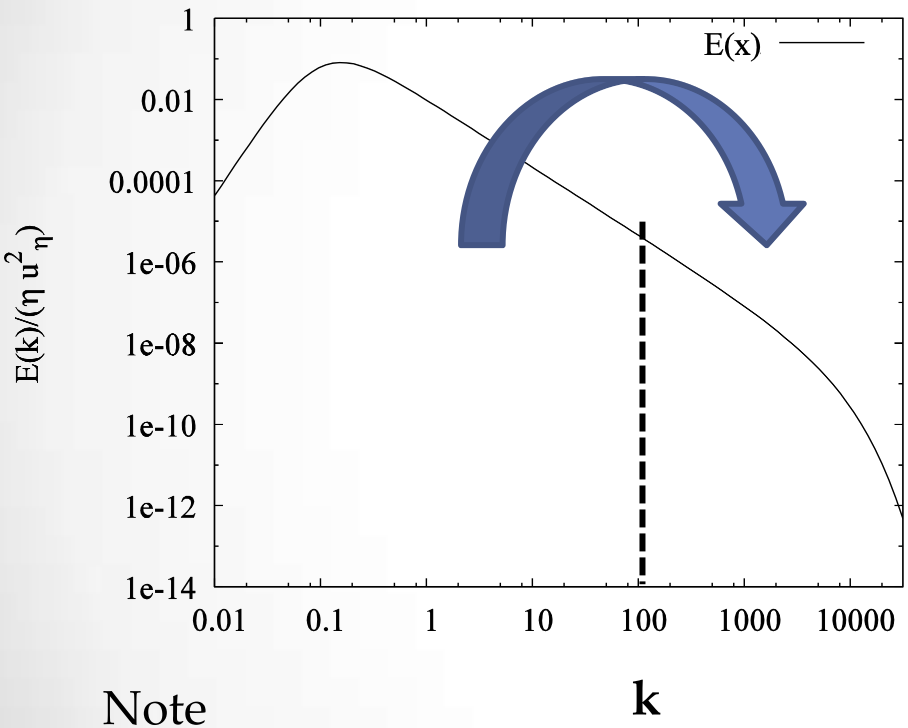
Total current is increased.

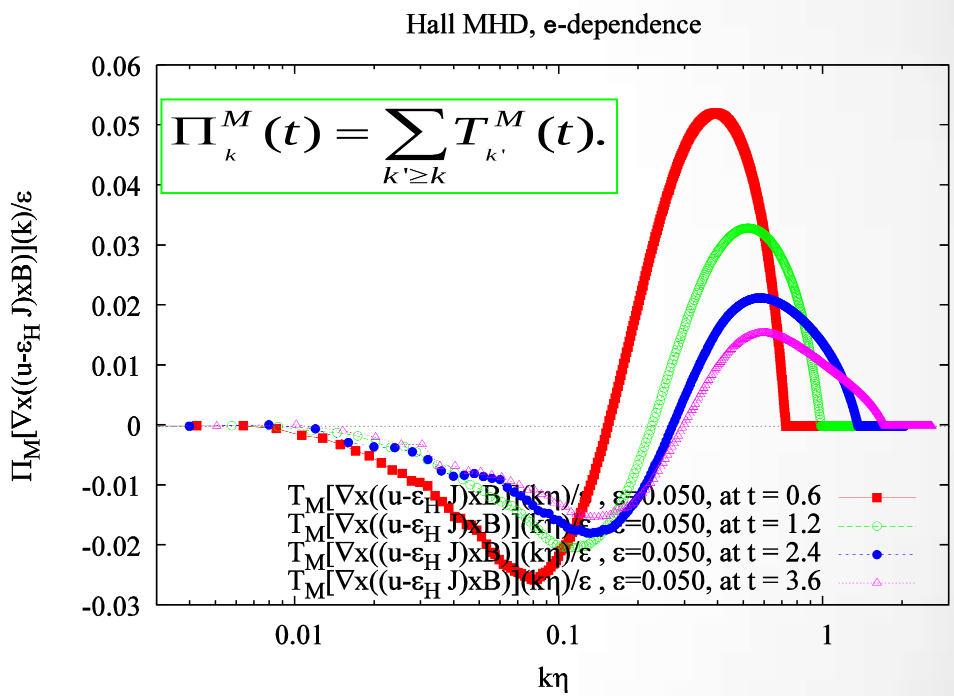


The sum of the enstrophy and the current is not changed very much.

The Hall term changes not only the magnetic field but also the balance of the current and the enstrophy.
(Energy exchange can be essential.)
Can statistical theory or some other model predict it ?

Energy flux functions represent the energy flow beyond the wave number.

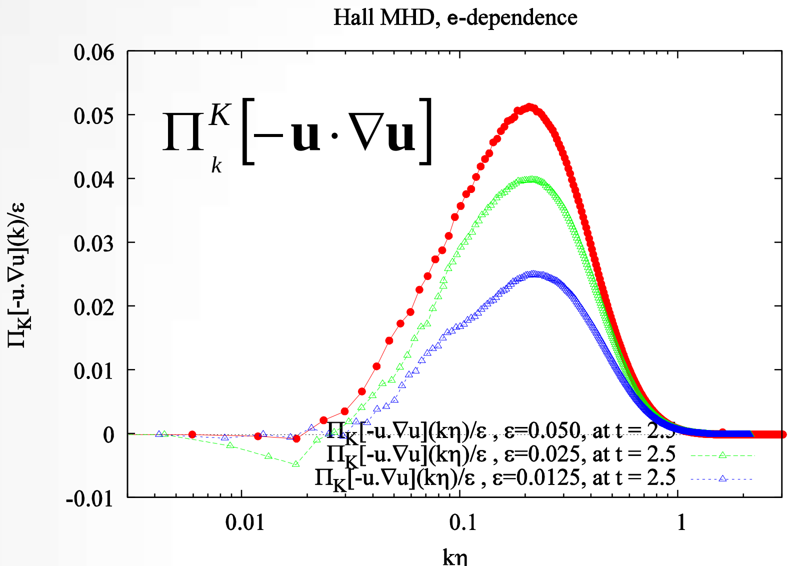


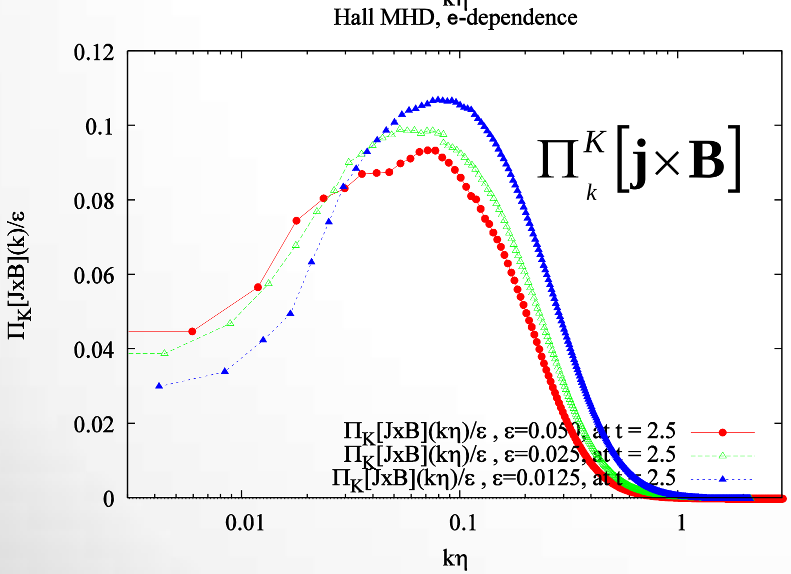


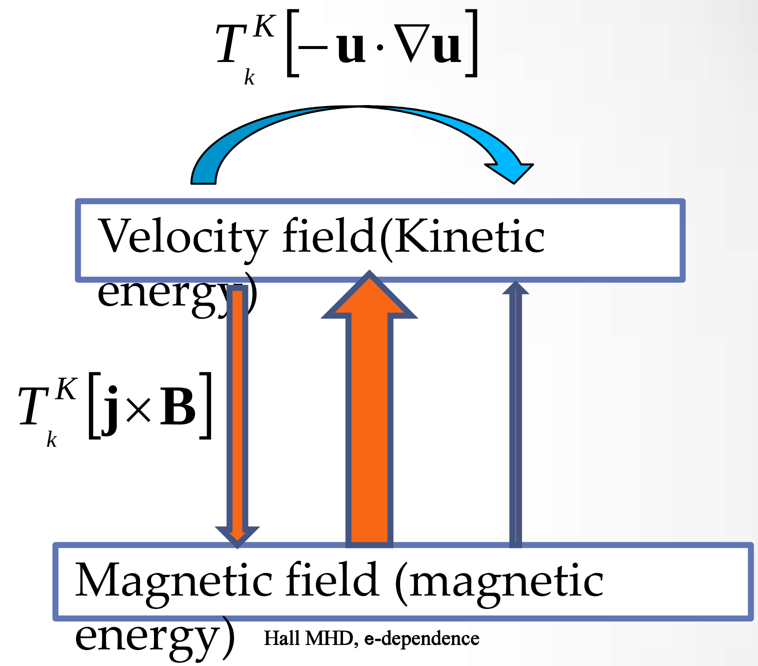
Note

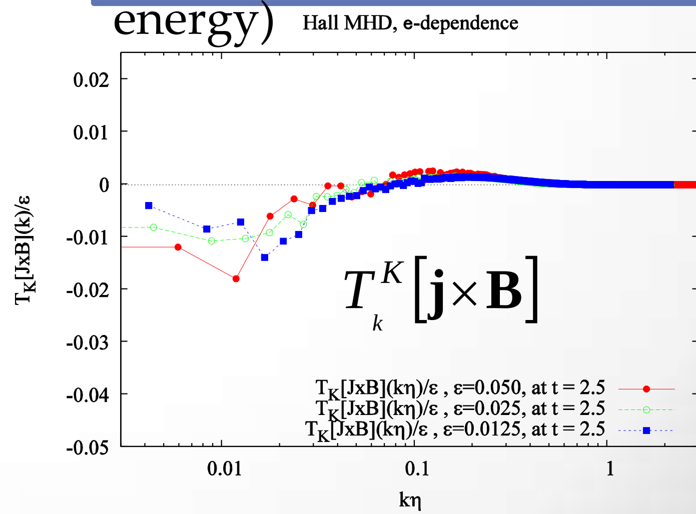
1. The concept of the energy flux is not clear because we decompose the energy into the kinetic energy and the magnetic energy.
2. We need to keep the decomposition of the energy into the two parts because we are going to consider to apply the analysis to the LES of a compressible system.

Kinetic energy budget is sustained by the JxB force

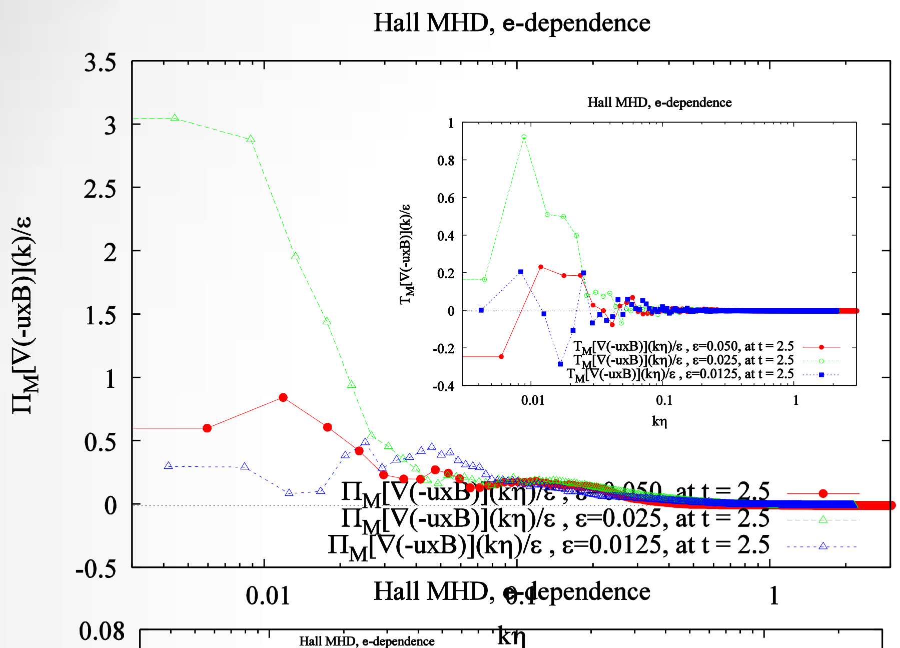


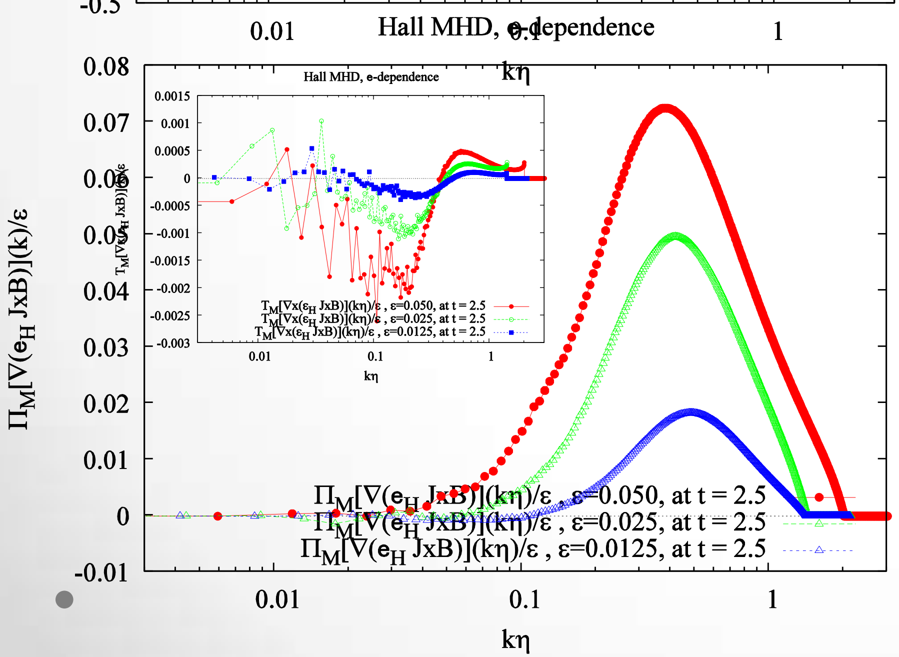




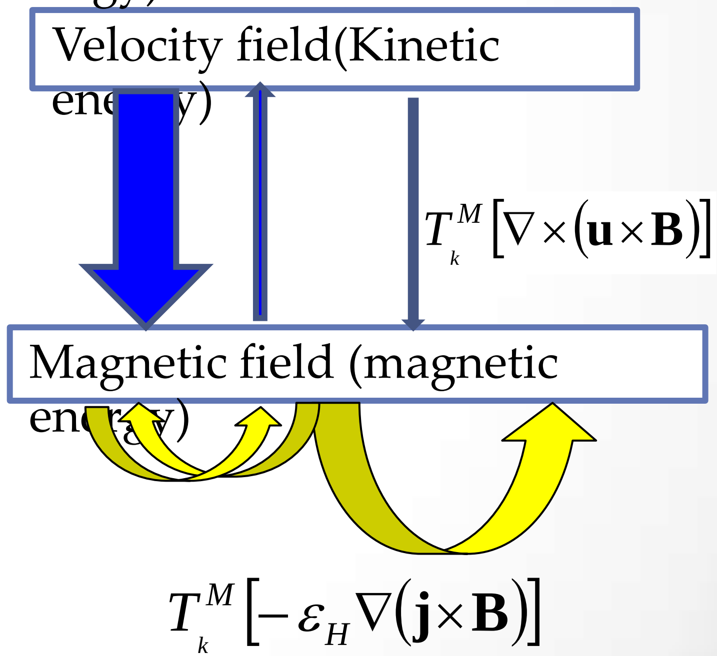


The Hall term dominates the small scale





Large scale magnetic field generation comes from the dynamo action (energy flux from the kinetic energy).



Does the Hall MHD system allows a Smagorinsky-type model ?

- Classical Smagorinsky-type model (Hamba & Tsuchiya, PoP 2010)

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} = -\nabla\cdot\left(\overline{\mathbf{u}}\,\overline{\mathbf{u}} - \overline{\mathbf{B}}\,\overline{\mathbf{B}}\right) - \nabla\left(\overline{p} + \frac{1}{2}\left|\overline{\mathbf{B}}\right|^2\right) + \nu\nabla^2\overline{\mathbf{u}} + \nabla\cdot\left[\left(\overline{\mathbf{u}}\,\overline{\mathbf{u}} - \overline{\mathbf{u}\mathbf{u}}\right) - \left(\overline{\mathbf{B}}\,\overline{\mathbf{B}} - \overline{\mathbf{B}\mathbf{B}}\right)\right],$$

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = -\nabla\times\left[\overline{\mathbf{u}}\times\overline{\mathbf{B}} - \left(\overline{\mathbf{u}}\times\overline{\mathbf{B}} - \overline{\mathbf{u}\times\mathbf{B}}\right) + \eta\,\overline{\mathbf{j}}\right],$$

$$\nabla\cdot\left[\left(\overline{\mathbf{u}}\,\overline{\mathbf{u}} - \overline{\mathbf{u}\mathbf{u}}\right) - \left(\overline{\mathbf{B}}\,\overline{\mathbf{B}} - \overline{\mathbf{B}\mathbf{B}}\right)\right] = \nabla\cdot\left[\nu_{SGS}\overline{S}_{ij}\right], \quad \overline{S}_{ij} = \frac{\partial\overline{u}_i}{\partial x_j} + \frac{\partial\overline{u}_j}{\partial x_i}, \quad \nu_{SGS} = C_\nu\Delta^2\left(\frac{1}{2}C_\nu\overline{S}_{ij}^{\,2} + C_\lambda\overline{j}_i\overline{j}_i\right)^{1/2}$$

$$\left(\overline{\mathbf{u}}\times\overline{\mathbf{B}} - \overline{\mathbf{u}\times\mathbf{B}}\right) = C_\lambda\Delta^2\left(\frac{1}{2}C_\nu\overline{S}_{ij}^{\,2} + C_\lambda\overline{j}_i\overline{j}_i\right)^{1/2}, \quad C_\lambda = \frac{5}{7}C_\nu, \quad C_\nu = 0.046.$$

How much the Hall MHD turbulent field can be approximated by the model ?

Q: Role of the anisotropic substructure for predicting power laws of isotropized spectra, as $k^{-3/2}$, $k^{5/3}$, k^{-2},

Relevance of statistical theory of homogeneous turbulence, towards very high Reynolds numbers, as generalized EDQNM, from isotropy to strong anisotropy,

Anisotropy and dynamics, with Alfv\'en, inertial and internal gravity waves combined together and with turbulence, in rotating stratified MHD,

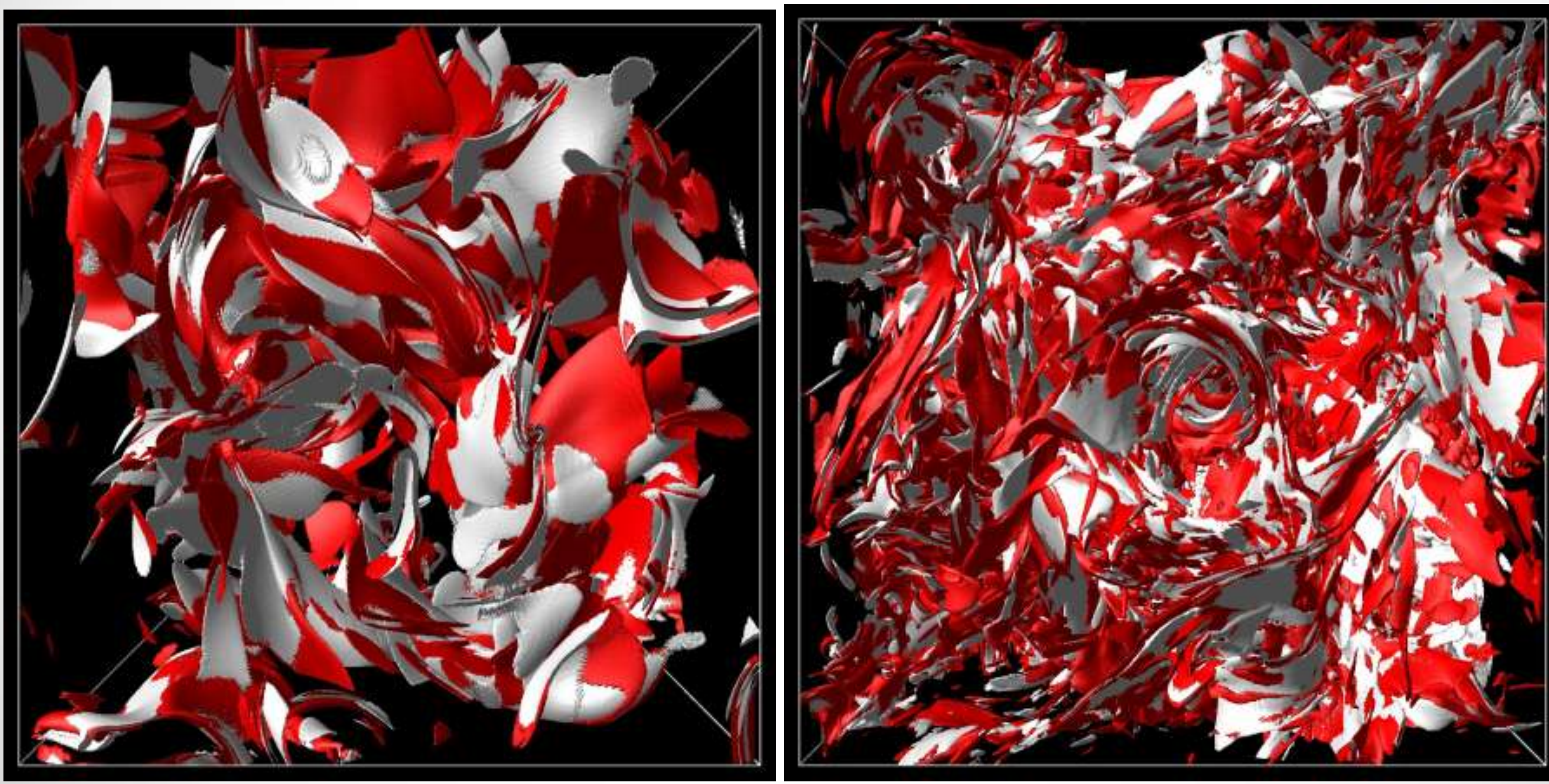
A list of questions follows: -

- Role of the anisotropic substructure for predicting power laws of isotropized spectra, as $k^{-3/2}$, $k^{5/3}$, k^{-2},
- Relevance of DNS and LES in periodic cubic boxes, from isotropy to strong anisotropy, with respect to computations in explicitly bounded, e.g. sperical, domains.
- Relevance of statistical theory of homogeneous turbulence, towards very high Reynolds numbers, as generalized EDQNM, from isotropy to strong anisotropy, -
- Anisotropy and dynamics, with Alfv\'en, inertial and internal gravity waves combined together and with turbulence, in rotating stratified MHD,
- Role of Hall effects on the local structures such as the enstrophy density and the current density,
- Hideaki Miura, ``DNS of Hall and non-Hall MHD turbulence''.

Local structures of turbulence … intermittency, and dissipative structure

- Energy, helicity, hybrid-helicity in Hall MHD
  they constrain global structures.
- MHD and Hall MHD
  The introduction of the Hall term brings about the correction of both the magnetic and the kinetic energy spectra at relatively high wave numbers.
- Though the change by the Hall term is not very large in global statistics, but very large in local structures.
- Are global statistics free from local structures ?
- How can we model the sub-grid-scales ?

Local structures, MHD vs Hall MHD in freely decaying turbulence (1) MHD



Local structures, MHD vs Hall MHD in freely decaying turbulence (2)Hall MHD