Helical flows in GFD turbulence, *and how to model them*

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* Where does the presence of helicity matter?
  Examples of rotating flows and slow decay

• Where does it come from?
  Rotation + stratification

• Does it need to be modeled specifically?
  Two examples

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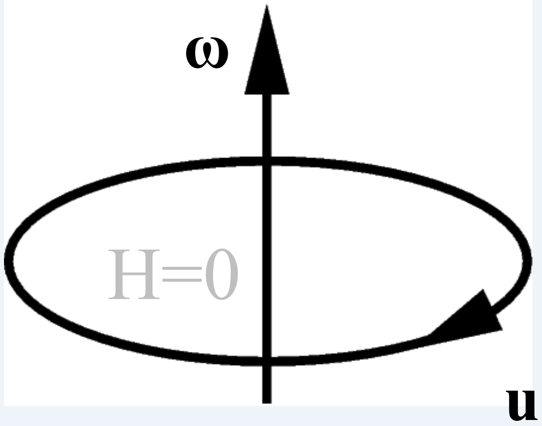
Kinetic helicity

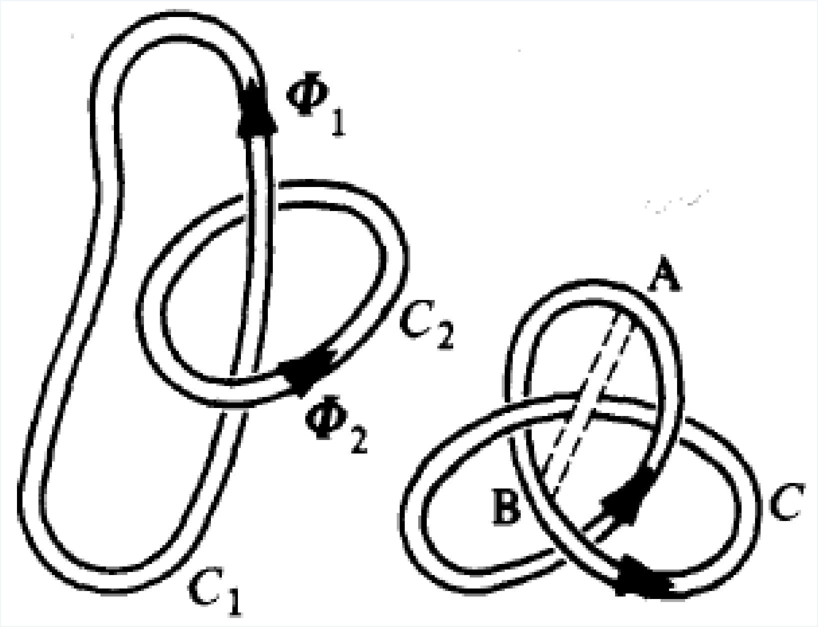
H is a pseudo (axial) scalar



$$H = \int \boldsymbol{\omega} \cdot \mathbf{u} dV$$

$$\langle u_i(\mathbf{k}) u_j^*(-\mathbf{k}) \rangle = U_E(|k|)\, P_{ij}(|k|)$$

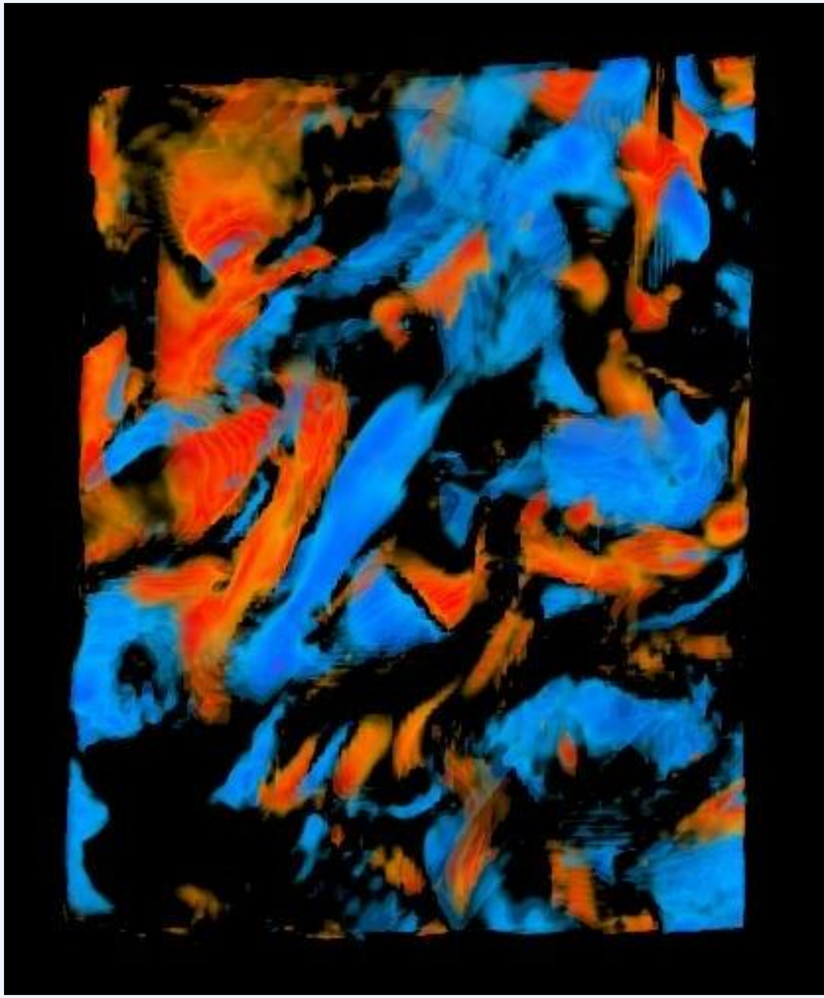
$$+ \quad \varepsilon_{ijl} k_l\, U_H(|k|)$$



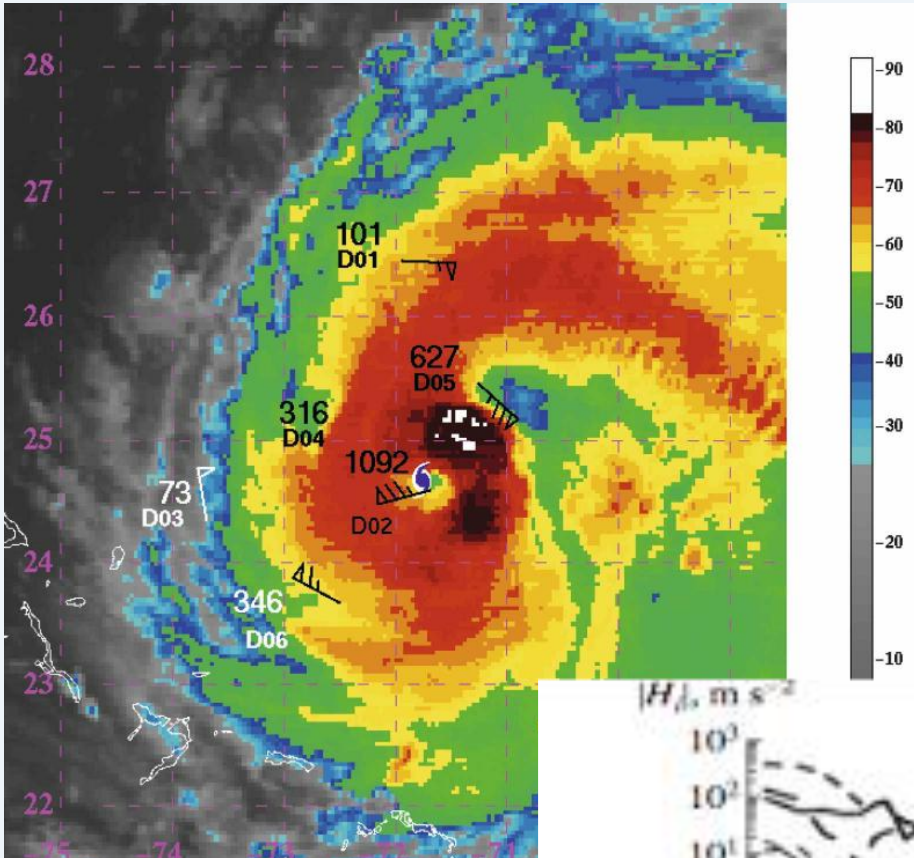
Vorticity $\omega = \nabla \times v$ & Relative helicity $h = \cos(\mathbf{v}, \omega)$

Local v-ω alignment (Beltramization) (*Tsinober & Levich, 1983; Moffatt, 1985).*

→ no mirror symmetry, together with weak nonlinearities in the small scales



Blue, h> 0.95 ; Red, h<-0.95



Molinari & Vollaro, 2008

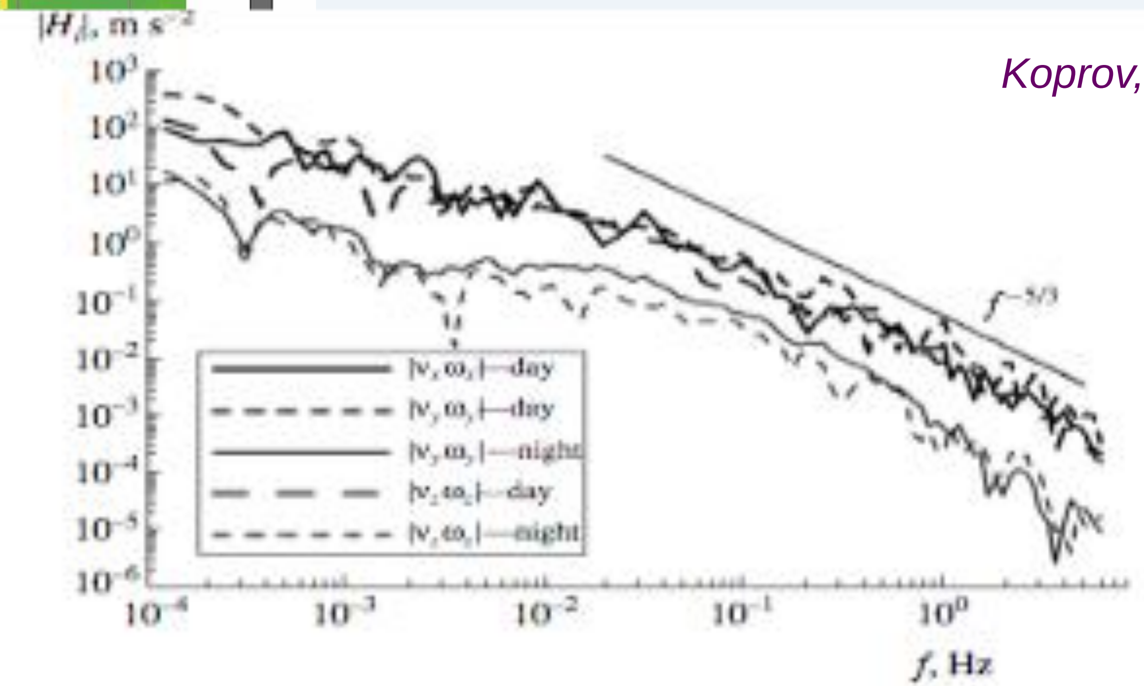


Fig. 4. Spectra of helicity components.

Koprov, 2005

Kinetic helicity in other geophysical flows

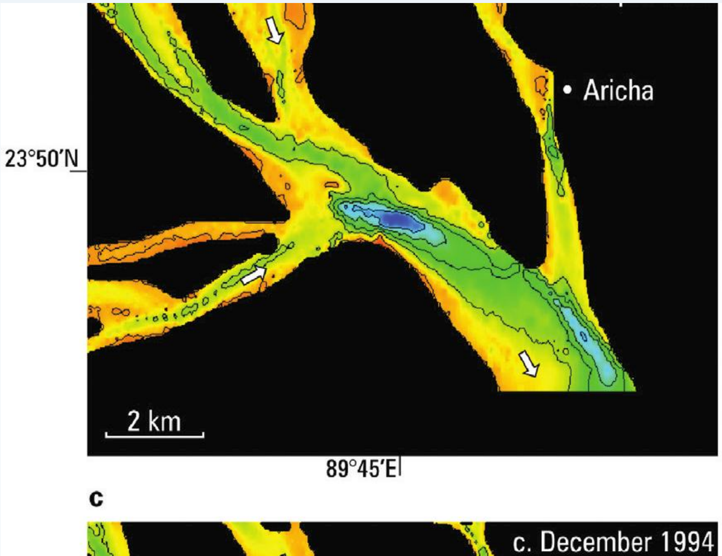
→ Secondary currents in river bends, effect on salt distribution

→ Mixing in estuaries, interactions with tidal flows, water quality

→ Isopycnals are helical surfaces when eq. of state is nonlinear

→ Helicity and large-scale instabilities, as in hurricanes

→ Production of large-scale
helical magnetic fields *(& shear)*



Kinetic helicity: old and new results

- Craya-Herring-Waleffe decomposition into ± circularly polarized waves: triad interactions (s,s',s'') where s,s',s''= ±
- Restrict to one-sign interactions → inverse cascade of energy in 3D NS *(Biferale et al., 2013),* and regularity of ideal flow *(Biferale & Titi 2013)*
- But Kraichnan (1973) showed that one-signed triad interactions are subdominant: overall direct cascade

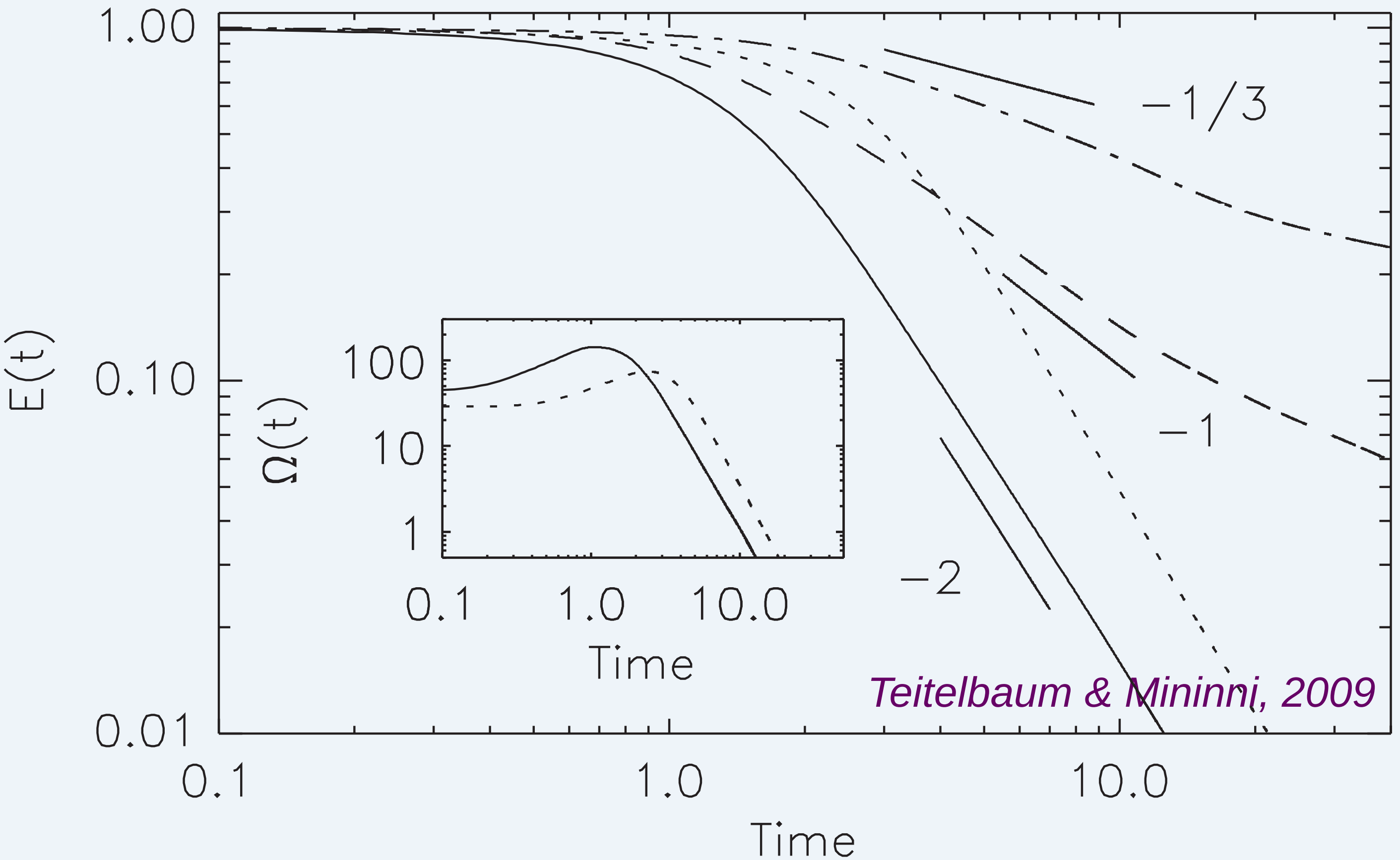
- Production of point-wise helicity *(Matthaeus et al. 2008)*

- Relative helicity decreases as 1/k, but there are strong helical vortex filaments in the dissipation range

In the presence of rotation:

* If no rotation, same decay rate (but delay when $H_V \neq 0$)
* *In the presence of waves: slower decay*

- *MORE SO when waves and helicity are both present*
- *Similar results for stratification* *(Rorai et al., 2013)*



Teitelbaum & Mininni, 2009

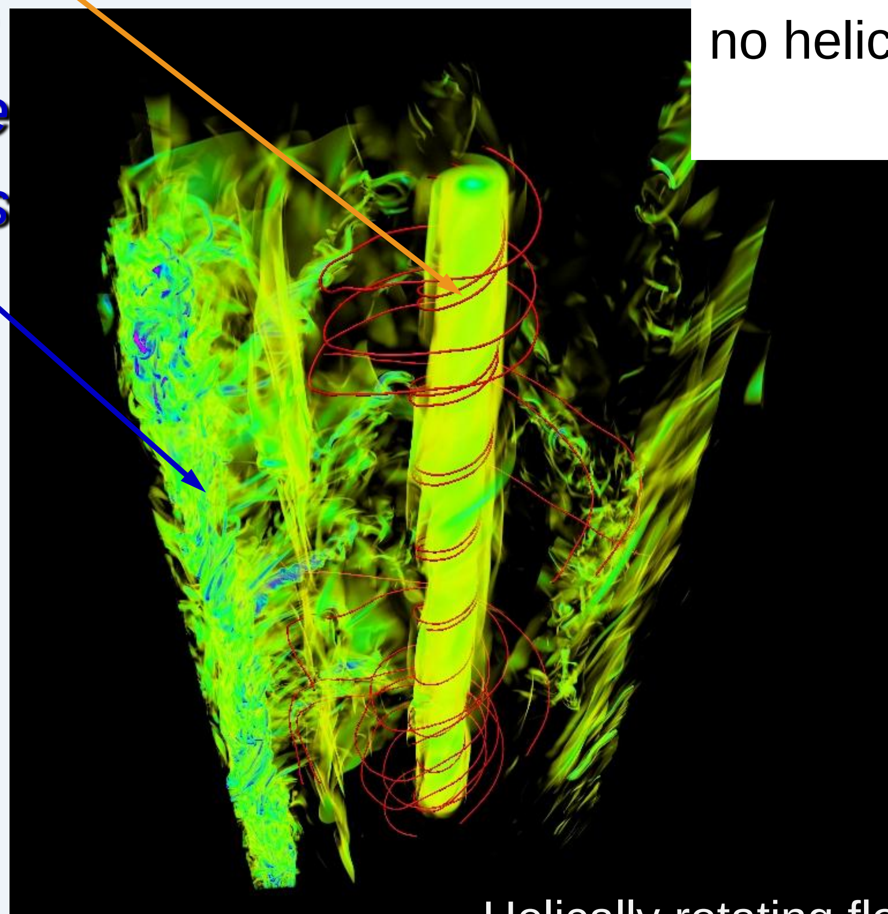
Mininni & AP (2010, 2012)

Role of helicity in rotating flows

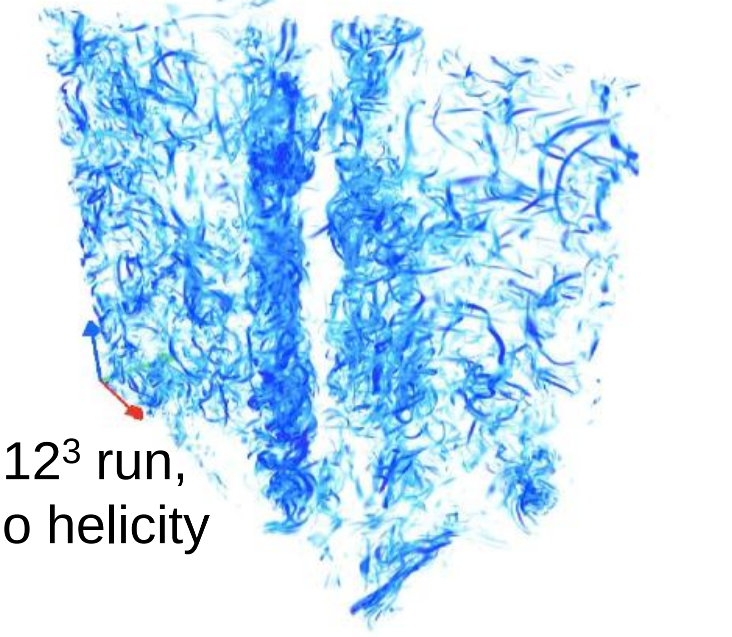
Zoom on a Beltrami core vortex

amidst a tangle of smaller-scale vortex filaments

Together with particle trajectories

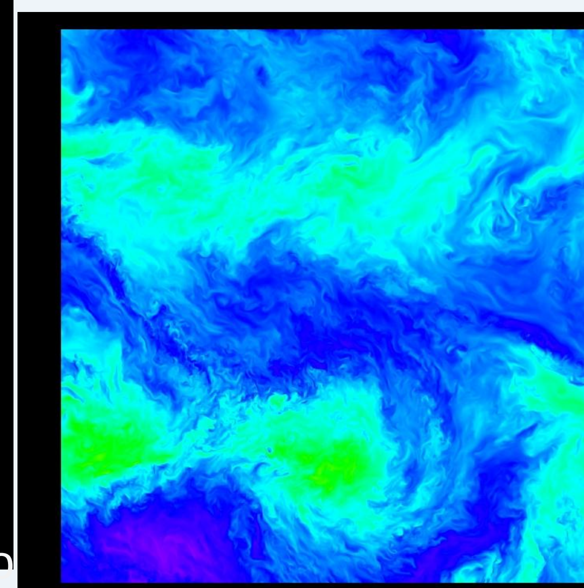


1536^3 grid, $k_F=7$,
Re=5100,
Ro=0.06,
fixed time

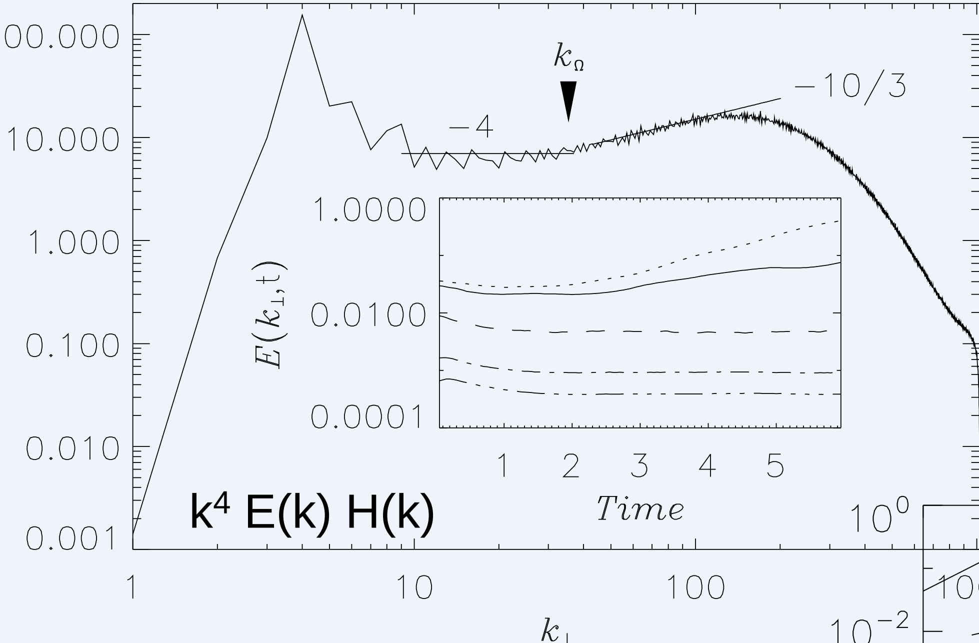


512^3 run, no helicity

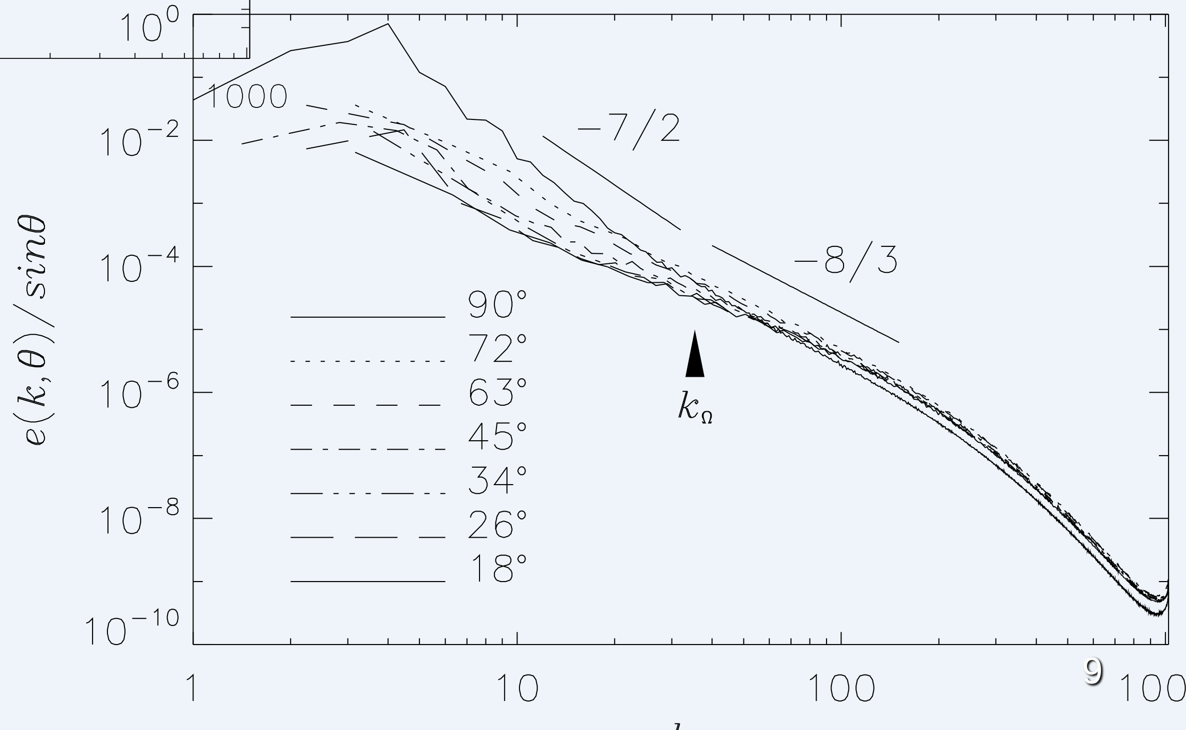
3072^3, isotropy & K41 recovered at small scale



Rotating flows: two direct cascades



Anisotropy at large scale
Isotropy at small scale



3072^3 points
Re~ 27000
Ro ~ 0.07
K_F ~ 4

Creation of helicity
Boussinesq equations

GB

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} - \nu\Delta\mathbf{u} = -\nabla P - Nb e_z - 2\Omega e_z \times \mathbf{u} + \mathrm{F}$$

$= 0$

$$\partial_t b + \mathbf{u}\cdot\nabla b - \kappa\Delta b = Nw\ ,$$

$$\nabla\cdot\mathbf{u} = 0\ .$$

Take the curl of GB → thermal winds

Then, dot with Coriolis force →

$$\langle H_\perp\rangle_\perp \equiv \langle u_\perp \cdot \nabla \times u_\perp\rangle_\perp = \frac{N}{f}\langle b\ \frac{\partial w}{\partial z}\rangle_\perp$$

f=2Ω

Parameter: N/f

Hide, 1976

GHOST code

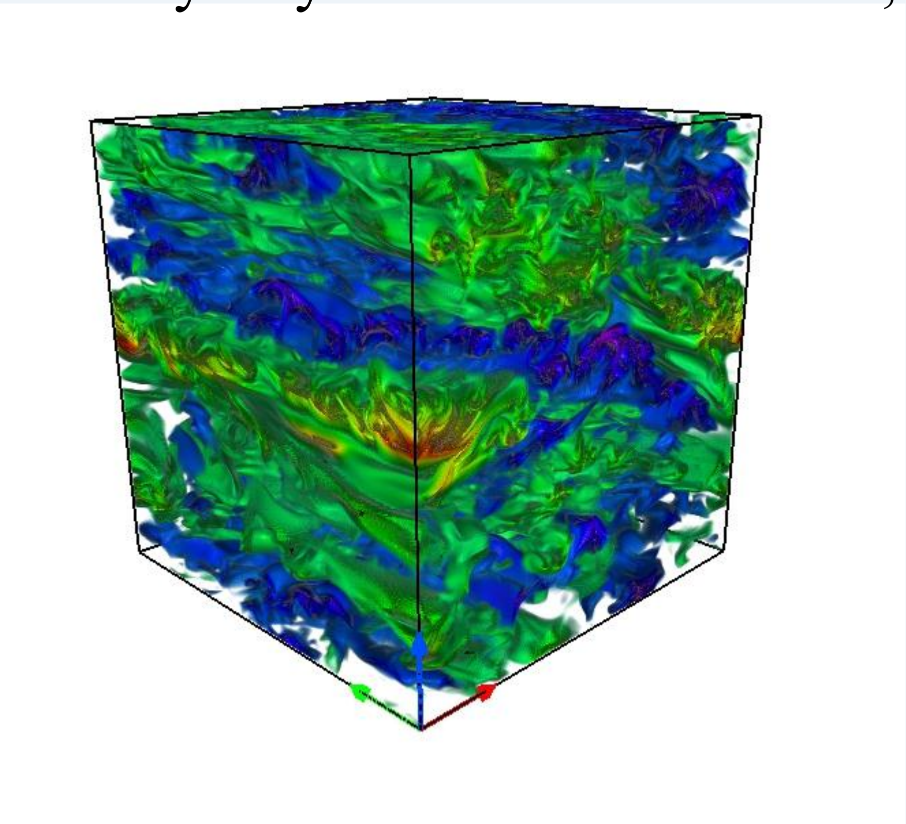
- Geophysical High Order Suite for Turbulence *(Gomez & Mininni)*
- Pseudo-spectral, 2D & 3D, tri-periodic BC, Runge-Kutta.
- Incompressible Navier-Stokes, with rotation, passive scalar, and magnetic fields (MHD, + Hall current). Boussinesq & SQG.
- LES: alpha model & simpler variants; helical spectral model.
- *``Soon:'' Lagrangian tracers and tetrads (with A. Pumir)*

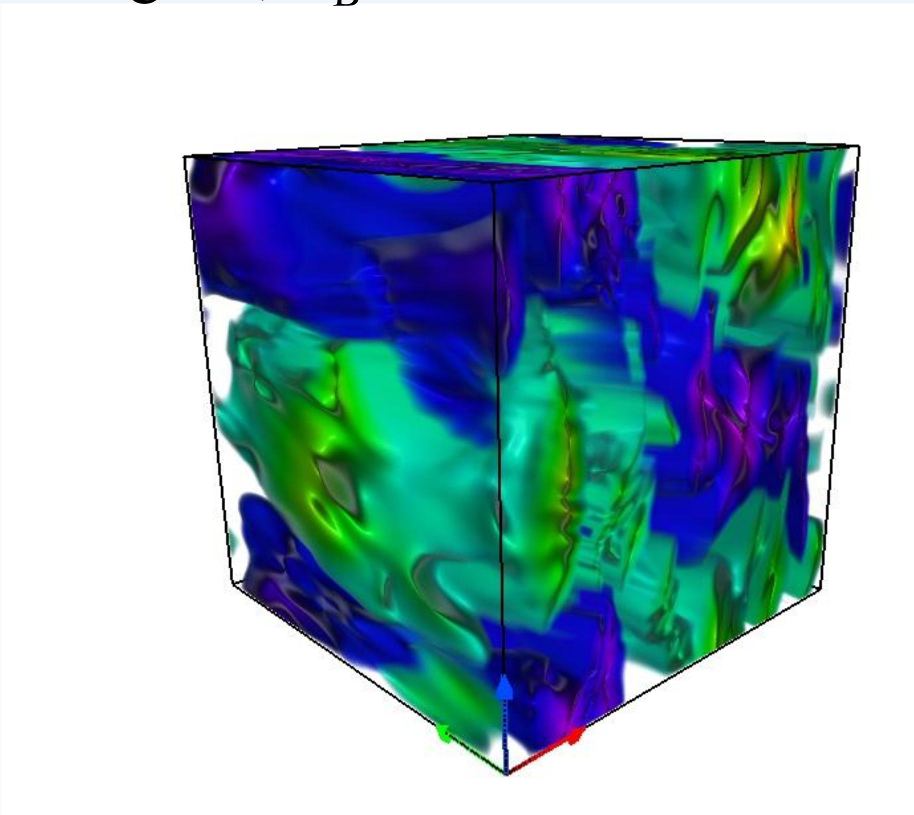
- The code parallelizes ~ linearly up to 98,000 processors (grid of 6144^3), using hybrid Open-MP / MPI *Mininni et al. 2011, Parallel Comp. 37*

- Available Data: *2048^3 forced Navier-Stokes turbulence with and without helicity and/or stratification; 1536^3 and 3072^3 helically forced rotating turbulence; 1536^3 decaying turbulence with a magnetic field, 6144^3 ideal and 2048^3 decaying MHD with imposed symmetries.*

- 3D visualization with *VAPOR* (NCAR) freeware.

Buoyancy $Re \sim 8000$, 512^3 grids, $R_B = ReFr^2$

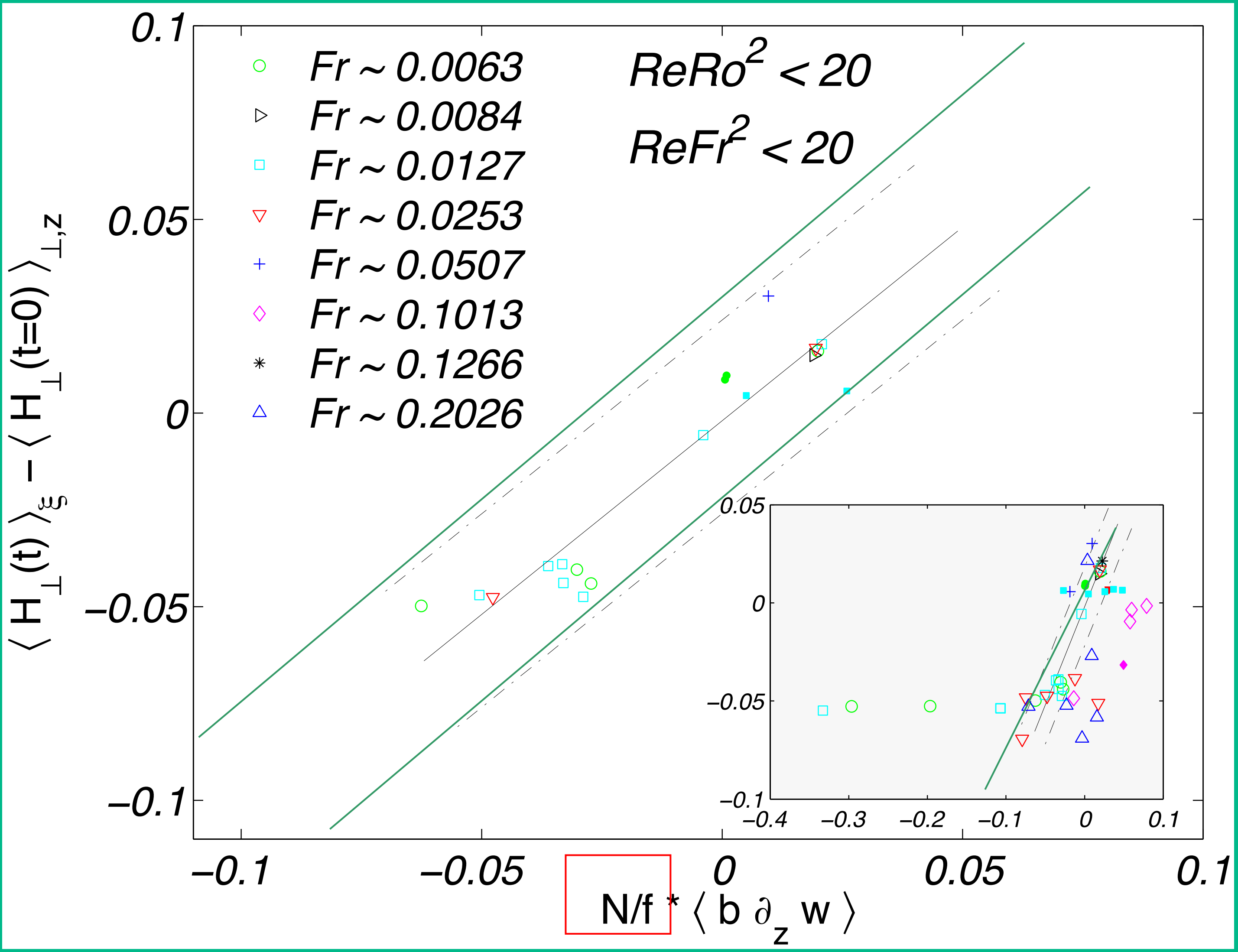




$Fr \sim 0.11$, $Ro \sim 0.4$,
$R_B \sim 100$, $N/f \sim 3.6$

$Fr \sim 0.025$, $Ro \sim 0.05$,
$R_B \sim 5$, $N/f = 2$

Marino et al., 2013



Selection of data from 45 runs, 9 on 512 grids *(filled symbols)*

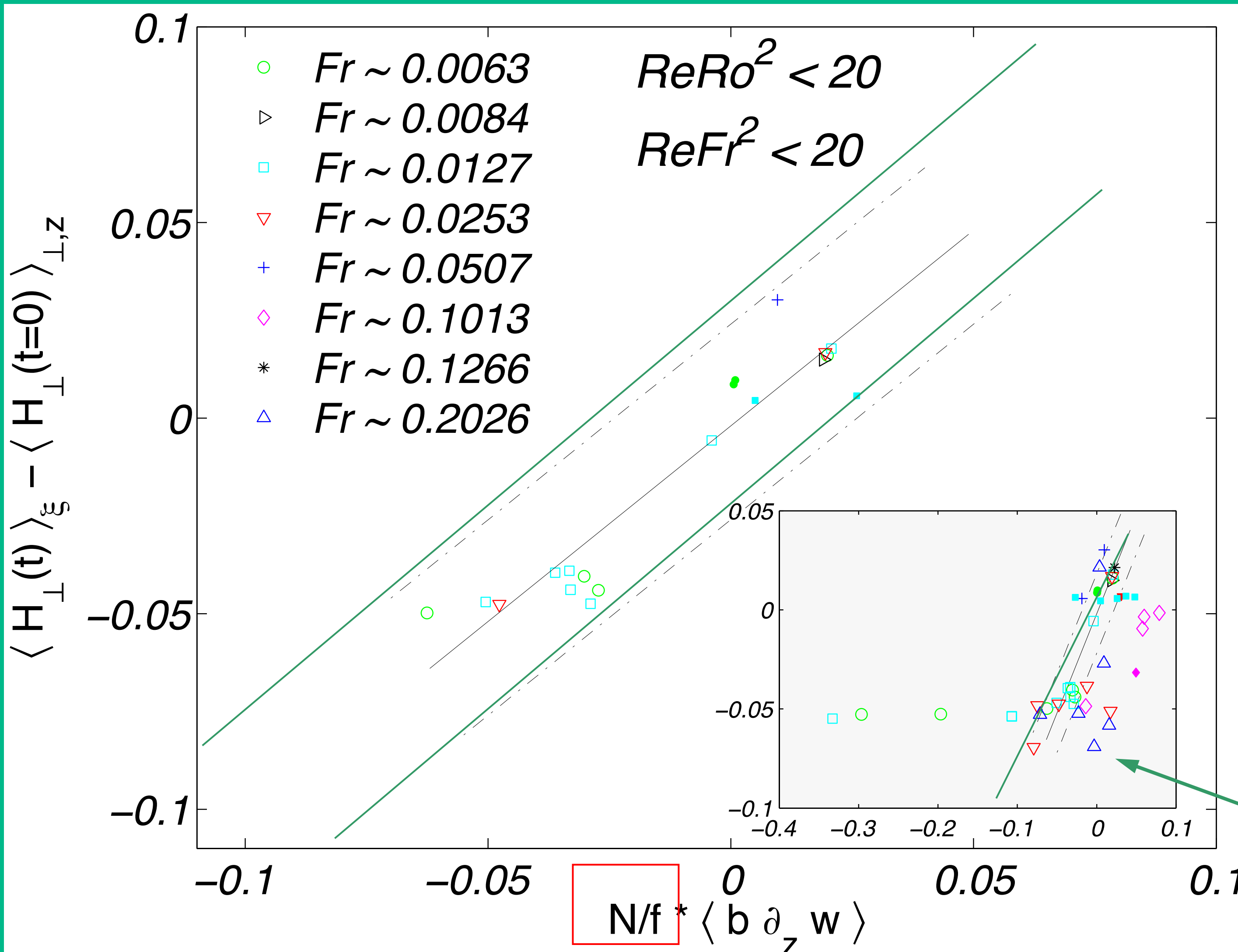
Criterion:

$ReFr^2 < 20$, and $ReRo^2 < 20$

(similar results with N/f<3)

$$\langle H_\perp \rangle_\perp \equiv \langle u_\perp \cdot \nabla \times u_\perp \rangle_\perp = \frac{N}{f} \langle b \, \frac{\partial w}{\partial z} \rangle_\perp$$

Marino et al., 2013



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Criterion:

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Shaded box: all runs

$$\langle H_\perp \rangle_\perp \equiv \langle u_\perp \cdot \nabla \times u_\perp \rangle_\perp = \frac{N}{f} \langle b \, \frac{\partial w}{\partial z} \rangle_\perp$$

Marino et al., 2013

Modeling of helical flows

SGS Reynolds stress

Yokoi, 2010

$$\mathcal{R}^{\alpha\beta} \equiv \overline{u'^{\alpha} u'^{\beta}} = \frac{2}{3} K_{\rm S} \delta^{\alpha\beta} - \nu_{\rm S} \mathcal{S}^{\alpha\beta} + \eta_{\rm S} \left[\overline{\omega}^{\alpha} \frac{\partial H_{\rm S}}{\partial x^{\beta}} + \overline{\omega}^{\beta} \frac{\partial H_{\rm S}}{\partial x^{\alpha}} - \frac{2}{3} \delta^{\alpha\beta} \left(\overline{\boldsymbol{\omega}} \cdot \nabla \right) H_{\rm S} \right]$$

SGS viscosity $\quad \nu_{\rm S} = C_{\rm S} \Delta K_{\rm S}^{1/2}$

Helicity-related coefficient $\quad \eta_{\rm S} = C_{\eta{\rm S}} \Delta^{3} K_{\rm S}^{-1/2}$

Modeling of helical flows

$$\tilde{\nu}^{>}(k|k_c,t) = \int\int_{\Delta^{>}} \frac{\theta_{kpq} S_{E_4}(k,p,q,t)}{2k^2 H(k,t)} dp\,dq$$

$$= \int\int_{\Delta^{>}} \theta_{kpq} \frac{1}{2k^2 q} z(1-y^2) H(q,t)\, dp\,dq.$$

→ $\nu_{turb} k^2 v_k + \nu^{H}_{turb} k^2 \omega_k$

à la Chollet-Lesieur (1981), *EDQNM-based closure*, Baerenzung et al. 2008

Modeling of helical flows

$$\tilde{\nu}^{>}(k|k_c,t) = \int\int_{\Delta^{>}} \frac{\theta_{kpq} S_{E_4}(k,p,q,t)}{2k^2 H(k,t)} dpdq$$

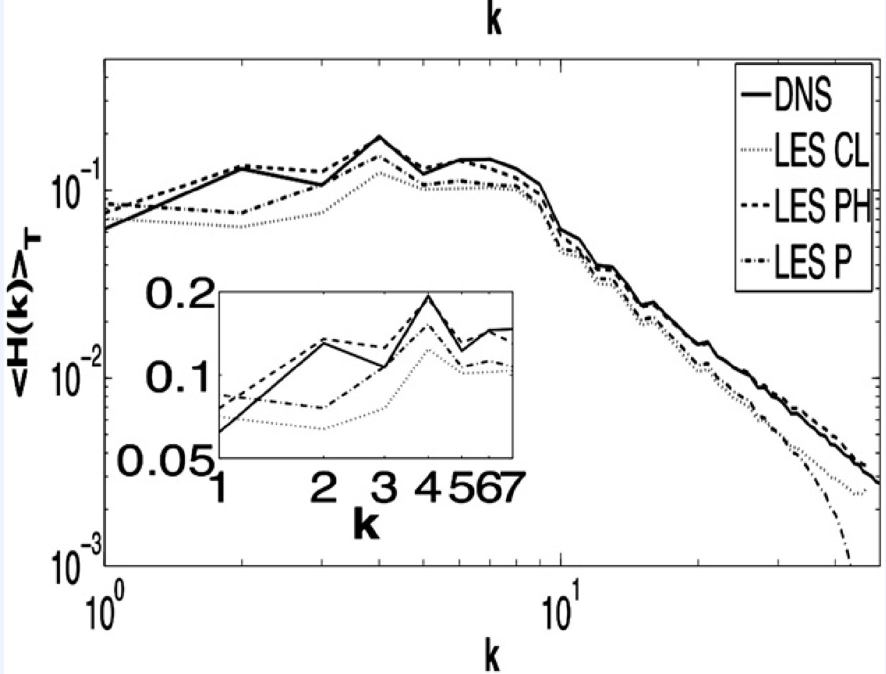
$$= \int\int_{\Delta^{>}} \theta_{kpq} \frac{1}{2k^2 q} z(1-y^2) H(q,t) dpdq .$$

$\rightarrow$ $\nu_{turb} k^2 v_k + \nu^{H}_{turb} k^2 \omega_k$

+ Eddy noise (or back-scatter: *Rose 1977, Mason & Thomson 1992, Sura 2011, Palmer 2012*),

with again a helical contribution

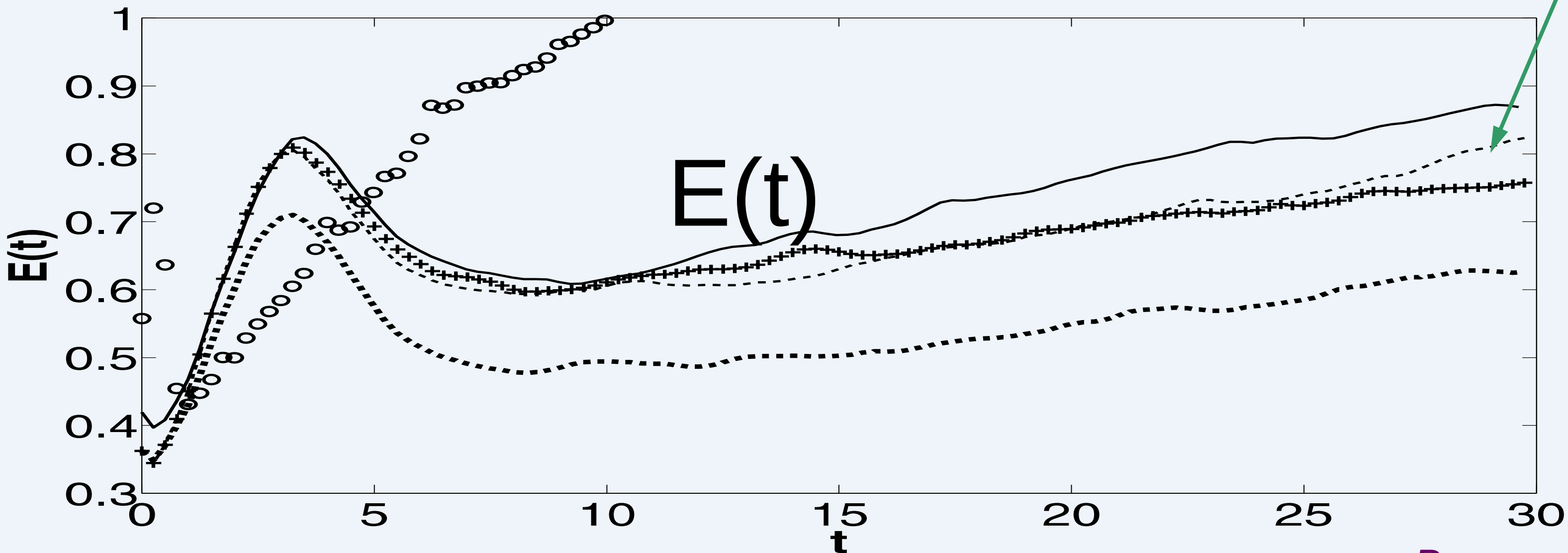
EDQNM-based closure, Baerenzung et al. 2008



Rotating turbulence

Helical model is better for H(k)

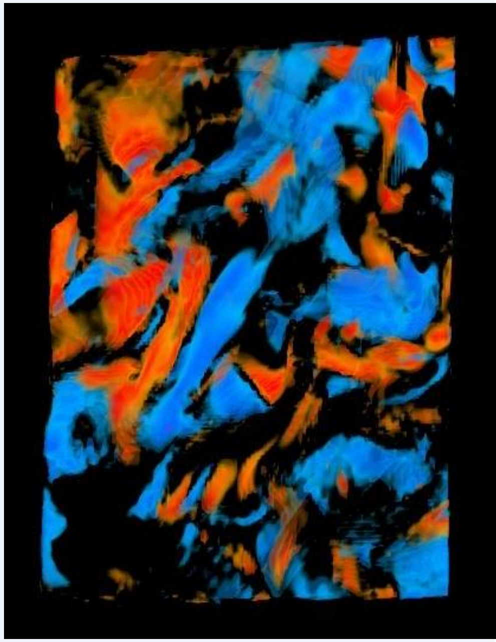
Helical model is closer to DNS



Baerenzung et al. 2011

Conclusion as to the role of helicity

- *Large-scale: helicity produced by geostrophic balance*
- *All scales: helicity produced by local shear alignment*
- Helicity is maximal when vorticity is strong; it kills nonlinear interactions, making structures to be long-lived



- Helicity is cascaded to small scales
- It has a measurable effect on rotating flows
- It is created in rotating stratified flows

- Large scale helicity can be strengthened through an instability due to anisotropic small-scale helicity *(cf. the dynamo alpha effect)*

- How does one model helical flows? How does one take into account in models the anisotropy induced by rotation/stratification?