Summary.

1. Kinetic MHD models of weakly collisional astrophysical plasmas require a SGS model that captures the effect of microscopic instabilities (firehose and mirror) on pressure anisotropy, heat fluxes, and the Hall term at high β.

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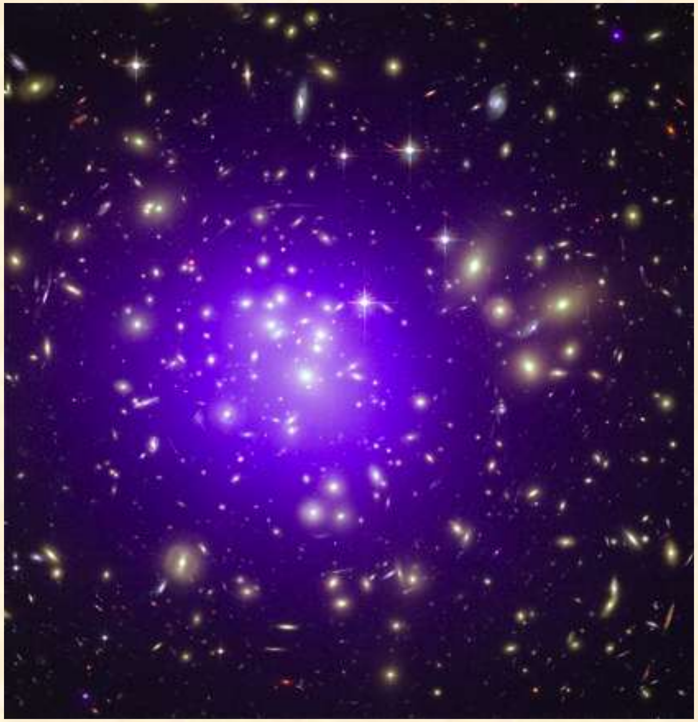
Kinetic Effects in Astrophysical MHD

Example: X-ray emitting plasma in clusters of galaxies is in the *kinetic MHD* regime: $\lambda << L$, $\lambda >> \rho$ (gyro-radius)

For example: Abell 1689



Purple=Chandra X-ray image
Yellow=Hubble optical image

$T \sim 4.5$ keV, $n \sim 10^{-3}$-10^{-4} cm^{-3}, B~ 1μG implies $\lambda_{mfp} \sim 10^{22}$ cm $\sim 0.1R_V$

$\rho \sim 10^8$ cm

Global PIC simulations impossible

Simplest model of low-collisionality regime, use continuum (fluid) equations plus anisotropic transport coefficients (Braginskii 1965):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot [\rho \mathbf{v}] = 0$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot [\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} + \mathsf{P}^*] = -\nabla \cdot \mathbf{\Pi}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + P^*)\mathbf{v} - \mathbf{B}(\mathbf{B} \cdot \mathbf{v})] = -\nabla \cdot \mathbf{Q} - \nabla \cdot [\Pi \cdot \mathbf{v}]$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0$$

MHD equations

$$E = \frac{P}{\gamma - 1} + \frac{1}{2}\rho v^2 + \frac{B^2}{2},$$

Definition of total energy

$$\boxed{\mathbf{Q} = -\chi \hat{\mathbf{b}}\hat{\mathbf{b}} \cdot \nabla T}$$

Anisotropic heat flux (χ = conductivity)

$$\boxed{\mathbf{\Pi} = -3\eta \left(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{1}{3}\mathsf{I}\right)\left(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{1}{3}\mathsf{I}\right) : \nabla \mathbf{v}}$$

Anisotropic viscous stress tensor

New dynamics in kinetic MHD: magneto-thermal instability

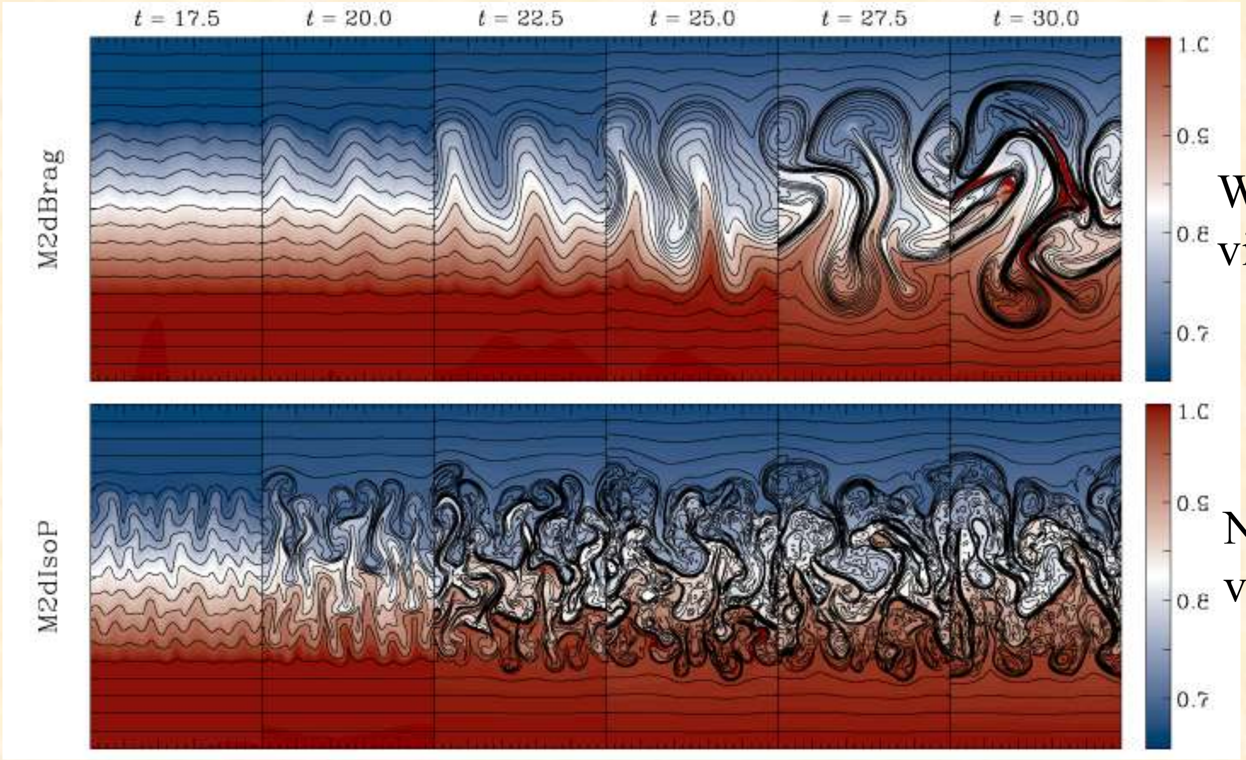
Balbus 2000; Parrish & Stone 2005; 2007

With anisotropic conduction, atmospheres with temperature decreasing upward are convectively unstable, regardless of entropy profile

cold

Increasing temperature

hot



With Braginskii viscosity

No Braginskii viscosity

Kunz et al. 2012

Colors = Temperature

Lines = B-field

Problems with Braginskii

Large pressure anisotropies in the plasma drive instabilities at microscopic (close to Larmor radius) scale.

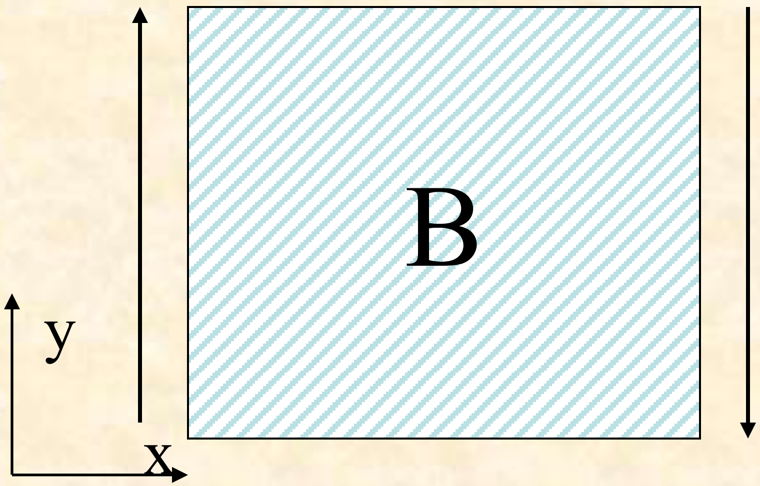
When $P_{perp} << P_{para}$: firehose instability
$P_{perp} >> P_{para}$: mirror instability

But Braginskii gets the wrong growth rates for both. Moreover, fastest growth rate is near Larmor radius, which simulations can never resolve.

- Saturation of firehose and mirror at small scales can strongly affect MHD on large scales by tangling field and limiting P anisotropy
- Need sub-grid model for firehose and mirror at large β

Developing a sub-grid model with hybrid PIC.

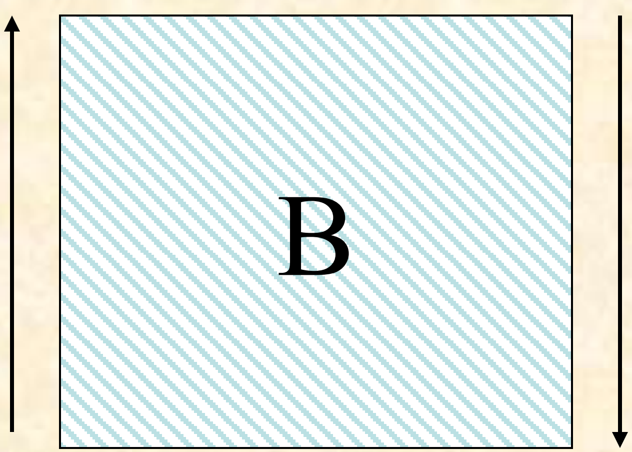
Study **firehose instability** in decreasing magnetic field.



B_x = constant
$|B_y|$ *decreasing*

From adiabatic invariance of
$\mu \propto mv^2{}_\perp/B \sim T_\perp/B$, must
have $v_\perp$ *decreasing*

mirror instability in shear amplification of magnetic field.



B_x = constant
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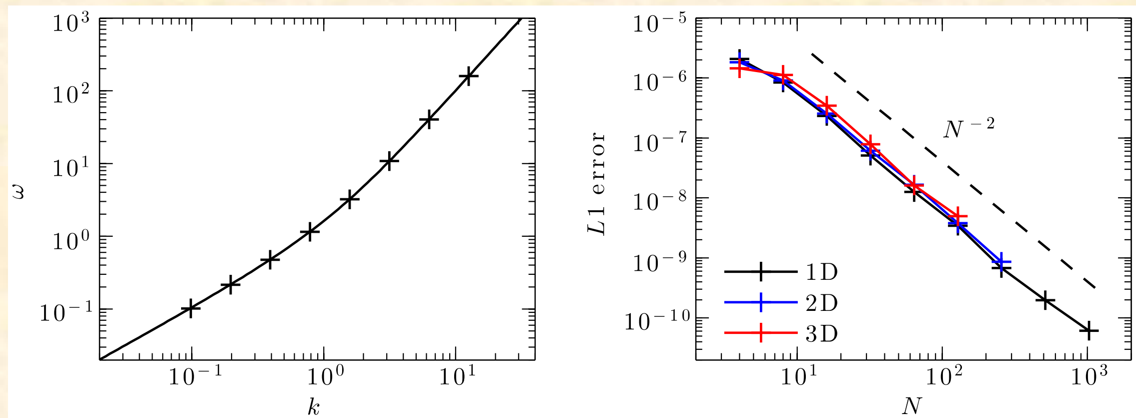
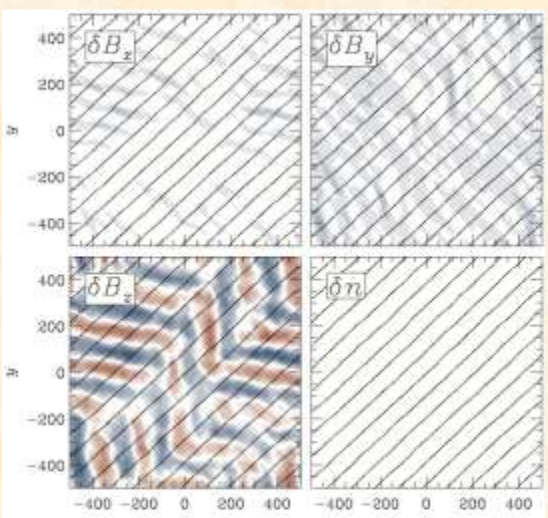
Firehose and mirror with hybrid PIC

(M. Kunz & JMS)

Fluid electrons and kinetic ions
Written entirely new code using novel algorithms for some steps
Uses δf methods to achieve extremely low noise.

Test: dispersion relation for whistler waves



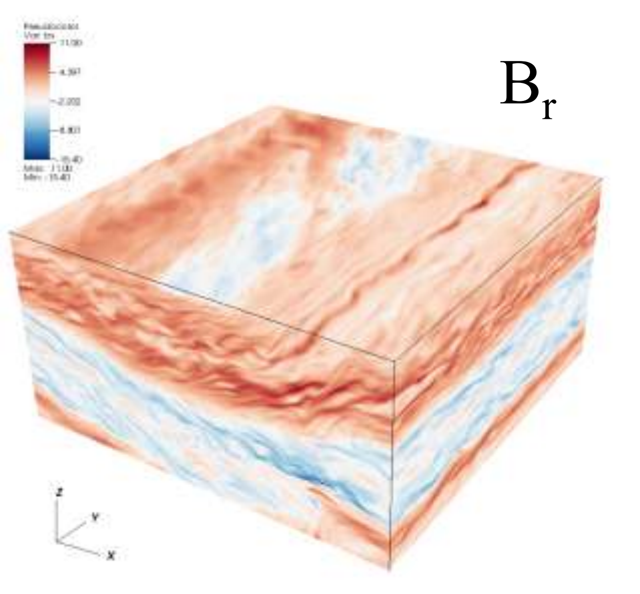


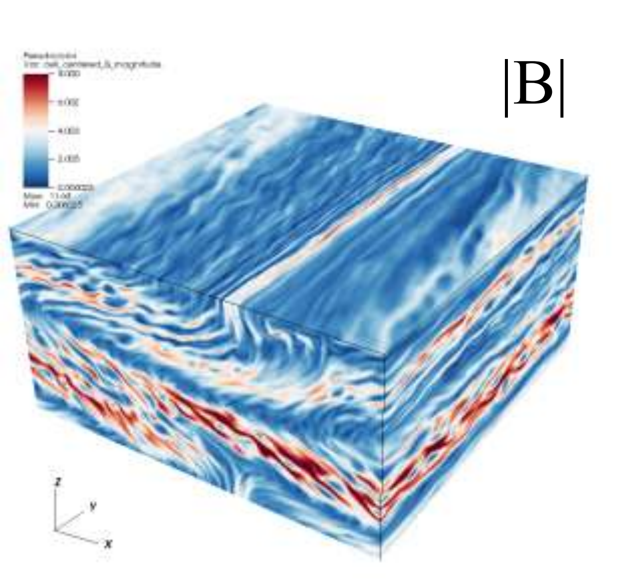
Early results: driven firehose saturates by tangling field

Kinetic MRI studied with hybrid PIC

(M. Kunz & JMS)

- Local shearing-box simulation of the MRI
- Box size 2120 x 2120 x 1060 $c/\omega_{p,I}$
- Roughly 2 billion particles, 256^3 grid





- Box is big enough to be in MHD regime on large scales, while still capturing kinetic effects on scales of Larmor radius.
- Direct comparison to kinetic MHD simulations can be used to validate SGS model.

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New hybrid-PIC code

Lagrangian ion particles solve 6D characteristics of

$$\left(\frac{\partial}{\partial t} - \sigma_0 x \frac{\partial}{\partial y}\right) f_s + \boldsymbol{v}' \cdot \boldsymbol{\nabla} f_s$$
$$+ \left[\frac{q_s}{m_s}\left(\boldsymbol{E}' + \frac{\boldsymbol{v}'}{c} \times \boldsymbol{B}\right) - 2\varpi_0 \hat{\boldsymbol{e}}_z \times \boldsymbol{v}' + \sigma_0 v'_x \hat{\boldsymbol{e}}_y + \boldsymbol{g}_{\rm eff}\right] \cdot \frac{\partial f_s}{\partial \boldsymbol{v}'} = 0$$

using a generalized Boris push for shearing box with
$\boldsymbol{v}' \equiv \boldsymbol{v} + \sigma_0 x \hat{\boldsymbol{y}}$ (orbit preserving)

PEGASUS
KUNZ, STONE & BAI 2013

Electrons are massless fluid; provide Ohm's law:

$$\boldsymbol{E} + \frac{\boldsymbol{u}_{\rm i}}{c} \times \boldsymbol{B} = -\frac{T_{\rm e} \boldsymbol{\nabla} n_{\rm i}}{e n_{\rm i}} + \frac{(\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B}}{4\pi Z e n_{\rm i}}$$

2nd-order accurate constrained transport solution to

$$\left(\frac{\partial}{\partial t} - \sigma_0 x \frac{\partial}{\partial y}\right) \boldsymbol{B} = -c \boldsymbol{\nabla} \times \boldsymbol{E}' - \sigma_0 B_x \hat{\boldsymbol{e}}_y$$

with upwinding along characteristics and optional orbital advection (for SB)