Subgid-scale modeling of compressible MHD turbulence

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GTP WORKSHOP, Boulder CO, 20 May, 2013

Plan of presentation

- LES for Perfect gas
- LES for heat-conductive gas
- LES application to space plasma turbulence
- LES for forced MHD turbulence
- Conclusions

Filtered MHD equations

$$\begin{cases} \dfrac{\partial \bar{\rho}}{\partial t} + \dfrac{\partial \bar{\rho}\tilde{u}_j}{\partial x_j} = 0 \\ \dfrac{\partial \bar{\rho}\tilde{u}_i}{\partial t} + \dfrac{\partial}{\partial x_j}\left(\bar{\rho}\tilde{u}_i\tilde{u}_j + \bar{p}\delta_{ij} - \dfrac{1}{\mathrm{Re}}\tilde{\sigma}_{ij} + \dfrac{\bar{B}^2}{2M_A^2} - \dfrac{1}{M_A^2}\bar{B}_i\bar{B}_j\right) = -\dfrac{\partial \tau_{ji}^u}{\partial x_j} \\ \dfrac{\partial \bar{B}_i}{\partial t} + \dfrac{\partial}{\partial x_j}\left(\tilde{u}_j\bar{B}_i - \bar{B}_j\tilde{u}_i\right) - \dfrac{1}{\mathrm{Re}_m}\dfrac{\partial^2 \bar{B}_i}{\partial x_j^2} = -\dfrac{\partial \tau_{ji}^b}{\partial x_j} \end{cases}$$

Dimensionless form of the equations

On the right-hand sides of equations the terms designate influence of subgrid terms on the filtered part. To determine these terms, special turbulent parametrizations based on large-scale values describing turbulent MHD flow must be used.

$$\left.\begin{aligned} \tau_{ij}^u &= \bar{\rho}\left((u_j u_i)^{\sim} - \tilde{u}_j\tilde{u}_i\right) - \frac{1}{M_A^2}\left(\overline{B_i B_j} - \bar{B}_j\bar{B}_i\right) \\ \tau_{ij}^b &= \left(\overline{u_i B_j} - \bar{B}_j\tilde{u}_i\right) - \left(\overline{B_i u_j} - \tilde{u}_j\bar{B}_i\right) \end{aligned}\right\}$$

Subgrid scale (SGS) or Subfilter-scale (SFS) terms

Smagorinsky model for MHD

Approximating the subgrid energy dissipation with the aid of the local resolved dissipation rate, $\varepsilon^K \sim \overline{l^2}(2\overline{\mathbf{S}}^v : \overline{\mathbf{S}}^v)^{3/2}$ and $\varepsilon^M \sim \overline{l^2}|\bar{\mathbf{j}}|^3$ leads to the classical Smagorinsky model and its straightforward MHD extension:

$$\tau_{ij}^u - \frac{1}{3}\tau_{kk}^u \delta_{ij} = -2\nu_t (\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{kk}\delta_{ij}) \qquad \tau_{kk}^u = 2Y_1 \bar{\rho}\overline{\Delta}^2 |\tilde{S}^u|^2 \qquad |\tilde{S}^u| = (2S_{ij}S_{ij})^{1/2}$$

Turbulent viscosity: $\nu_t = C_1 \bar{\rho}\overline{\Delta}^2 |\tilde{S}^u|$

$\tilde{S}_{ij} = \frac{1}{2}(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i})$ - large-scale strain rate tensor

$\bar{J}_{ij} = \frac{1}{2}(\frac{\partial \bar{B}_i}{\partial x_j} - \frac{\partial \bar{B}_j}{\partial x_i})$ - large-scale magnetic rotation tensor

$$\tau_{ij}^b - \frac{1}{3}\tau_{kk}^b \delta_{ij} = -2\eta_t \bar{J}_{ij}$$

Turbulent diffusivity: $\eta_t = D_1 \overline{\Delta}^2 |j|$

Kolmogorov model for MHD case

If the grid filter cutoff lies within the inertial spectral range of the homogeneously turbulent system and the nonlinear exchange between resolved kinetic and magnetic energy is much smaller than the respective dissipation, kinetic subgrid-energy dissipation and magnetic subgrid-energy dissipation can be assumed to depend only on time. Thus except a unit factor carrying the necessary dimensions and an explicit filter scale dependence, both functions γ and ϕ can be absorbed by the nondimensional parameters yielding the Kolmogorov scaling model

$$\nu_t = C_2 \overline{\rho} \overline{\Delta}^{4/3} \quad \text{- } \textit{turbulent viscosity}$$

$$\tau_{kk}^{u} = 2 Y_2 \overline{\rho} \overline{\Delta}^{4/3} | \widetilde{S}^{u} | \quad \text{- } \textit{isotropic term}$$

$$\eta_t = D_2 \overline{\Delta}^{4/3} \quad \text{- } \textit{turbulent magnetic diffusivity}$$

Cross-helicity model

The cross helicity is $H^c = \int_V (u \cdot B) dV$

With regard to the mixing length framework outlined above the functions ϕ and γ are estimated as the product of subgrid dissipation and an associated length scale. However, instead of the local resolved kinetic and magnetic energy dissipation terms, the corresponding local cross-helicity dissipation expressions

$$\overline{\varepsilon}^{Cv} \sim \overline{\mathbf{S}}^{v} : \overline{\mathbf{S}}^{b} \qquad \overline{\varepsilon}^{Cb} \sim \overline{\mathbf{j}} \cdot \overline{\boldsymbol{\omega}}$$

the resolved vorticity $\overline{\boldsymbol{\omega}} = \boldsymbol{\nabla} \times \overline{\mathbf{v}}$

the electric current density $\mathbf{j} = \boldsymbol{\nabla} \times \mathbf{b}$

Cross-helicity model

The cross-helicity is related to the transfer between kinetic and magnetic energies caused by the Lorentz force. Therefore, the cross helicity allows one to estimate the energy exchange between large and small scales in the LES method:

$$\nu_t = C_3 \overline{\rho} \overline{\Delta}^2 \, | \widetilde{S}^u_{ij} \widetilde{S}^b_{ij} |^{1/2}$$ - *turbulent viscosity*

$$\tau^u_{kk} = 2 Y_3 \overline{\rho} \overline{\Delta}^2 \, | \widetilde{f} \, || \widetilde{S}^u |$$ - *isotropic term* $$\bar{f} = | \widetilde{S}^u_{ij} \overline{S}^b_{ij} |^{1/2}$$

$$\eta_t = D_3 \overline{\Delta}^2 \, \mathrm{sgn}(\bar{j} \widetilde{\omega}) \, | \bar{j} \widetilde{\omega} |^{1/2}$$ - *turbulent magnetic diffusivity*

Since the energetically most favorable configuration of the local velocity and magnetic field is **V||B**, any decrease of alignment of these two vectors increases locally the magnetic energy. The process works inversely when the local alignment increases whereby the direction of change is given by the sign of the local cross-helicity dissipation. The justification is based on the existence of the inverse magnetic helicity cascade.

Scale-similarity model for MHD case

The scale-similarity model is not of the eddy-viscosity-type. It is based on the assumption that the component of the SGS most active in the energy transfer from large to small scales can be estimated with sufficient accuracy from the smallest resolved scale, which can be obtained by filtering the subgrid-scale quantities

$$\tau_{ij}^{u} = \overline{\rho}\left((\tilde{u}_j \tilde{u}_i)^{\sim} - \tilde{\tilde{u}}_j \tilde{\tilde{u}}_i\right) - \frac{1}{M_A^2}\left(\overline{\overline{B}_i \overline{B}_j} - \overline{\overline{B}}_j \overline{\overline{B}}_i\right)$$

$$\tau_{ij}^{b} = \left(\overline{\tilde{u}_i \overline{B}_j} - \overline{\overline{B}}_j \tilde{\tilde{u}}_i\right) - \left(\overline{\overline{B}_i \tilde{u}_j} - \tilde{\tilde{u}}_j \overline{\overline{B}}_i\right)$$

Mixed model for compressible MHD turbulence

The mixed model is a combination of two subgrid-scale closures: the scale similarity model and the Smagorinsky model fro MHD case.

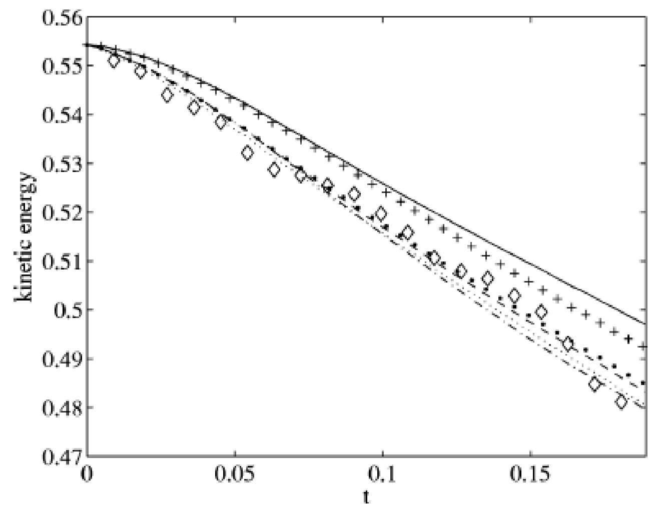
$$\tau_{ij}^{u} - \frac{1}{3}\tau_{kk}^{u}\delta_{ij} = -2C_5\bar{\rho}\overline{\Delta}^2 |\tilde{S}^{u}| (\tilde{S}_{ij} - \frac{1}{3}\tilde{S}_{kk}\delta_{ij}) + \bar{\rho}((\tilde{u}_j\tilde{u}_i)^{\sim} - \tilde{\tilde{u}}_j\tilde{\tilde{u}}_i) - \frac{1}{M_A^2}(\overline{\bar{B}_i\bar{B}_j} - \bar{\bar{B}}_j\bar{\bar{B}}_i)$$

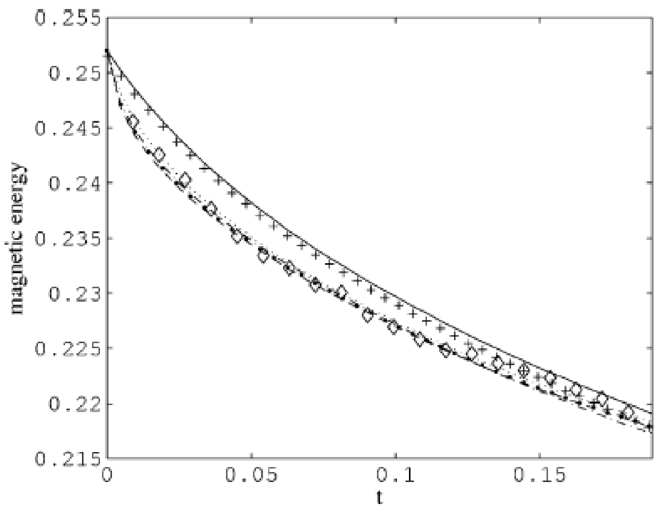
$$\tau_{kk}^{u} = 2Y_5\bar{\rho}\overline{\Delta}^2 |\tilde{S}^{u}|^2$$

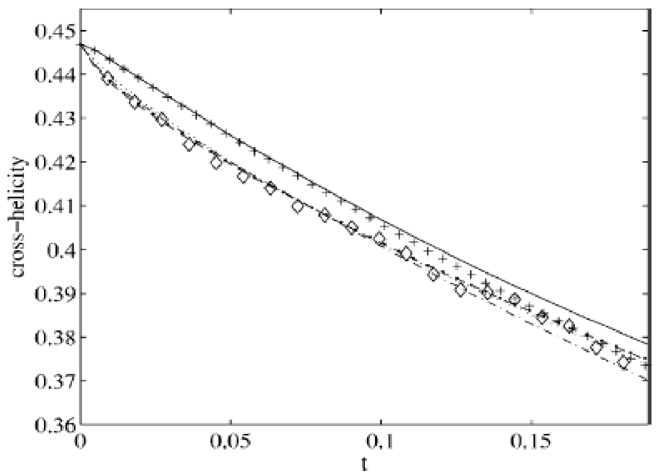
$$\tau_{ij}^{b} = -2D_5\overline{\Delta}^2 |\bar{j}| \bar{J}_{ij} + (\overline{\tilde{u}_i\bar{B}_j} - \bar{\bar{B}}_j\tilde{\tilde{u}}_i) - (\overline{\bar{B}_i\tilde{u}_j} - \tilde{\tilde{u}}_j\bar{\bar{B}}_i)$$

Computation examples

SGS model	Curve
No model	Solid
Smagorinsky model	Dashed
Kolmogorov model	Dotted
Cross-helicity model	Black point
Scale-similarity model	Marker +
Mixed model	Dashed-dotted

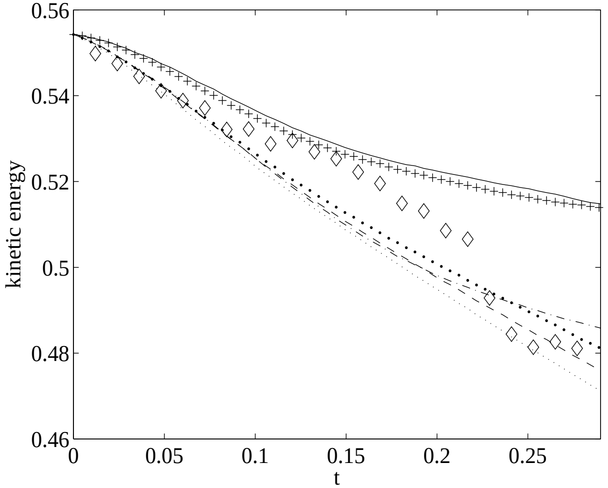




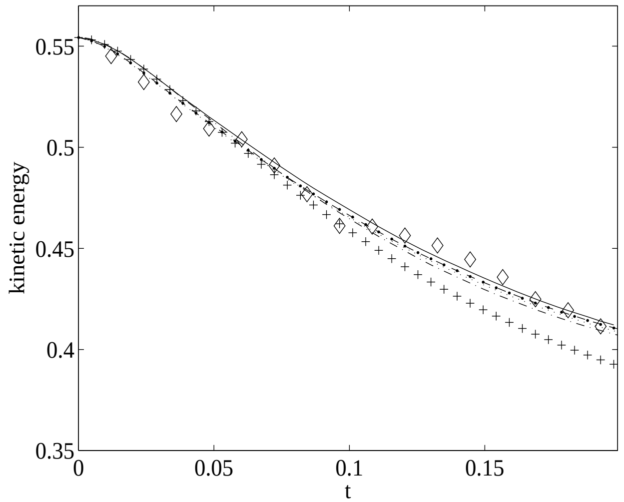


Kinetic energy

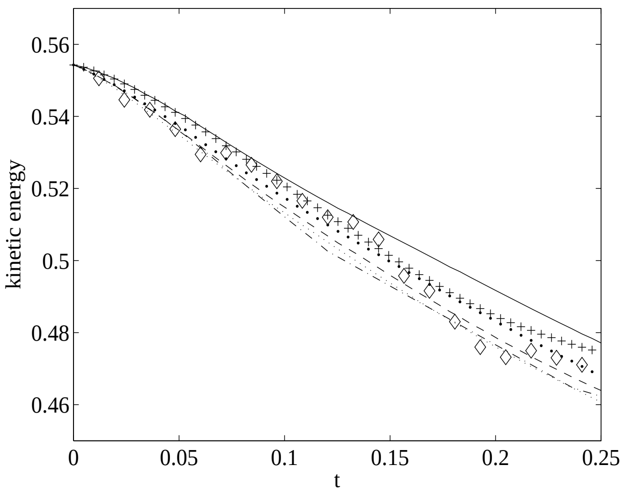
Ms=1



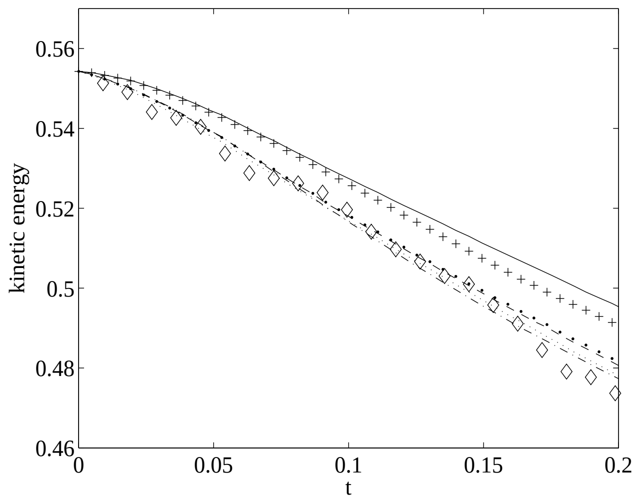
Ms=0.2



Re_m=2



Re_m=20



Kinetic energy

•For kinetic energy, larger divergence of LES results was observed with a decrease in magnetic Reynolds number using various SGS closures. The scale-similarity model shows the worst results, however, the other SGS closures increase calculation accuracy.

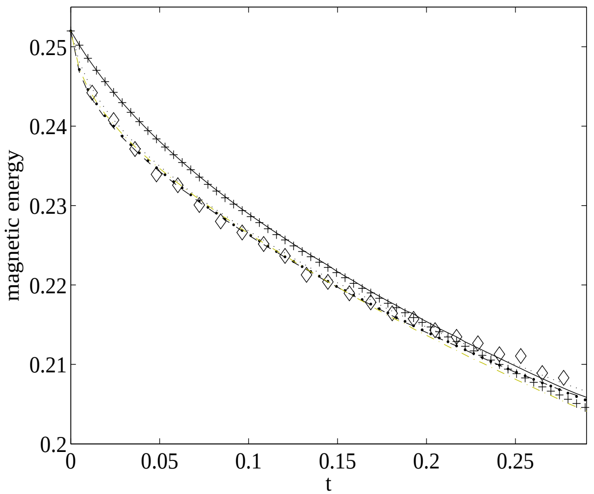
•The changing of Reynolds number produces qualitatively similar results, as the initial conditions of velocity and magnetic fields are the same, and therefore Taylor Reynolds number does not have a significant impact on the choice of subgrid parameterizations in our computations.

•Mach number *Ms* exerts essential influence on results of modeling. The divergence between DNS and LES results for kinetic energy increases with *Ms*.

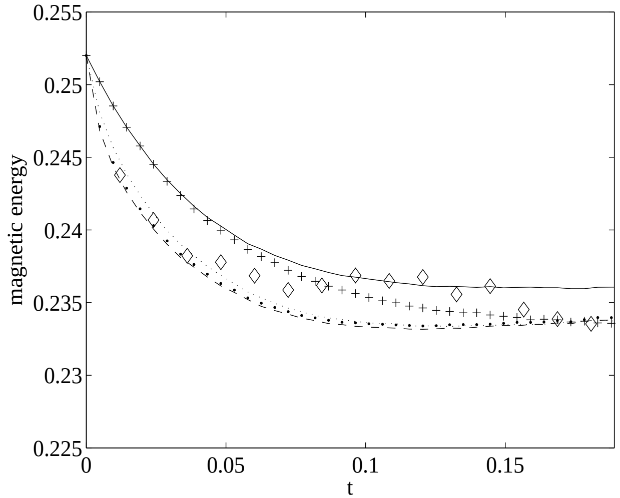
•Generally, the Smagorinsky model and the cross-helicity model yield the best accordance with DNS under various Mach number.

Magnetic energy

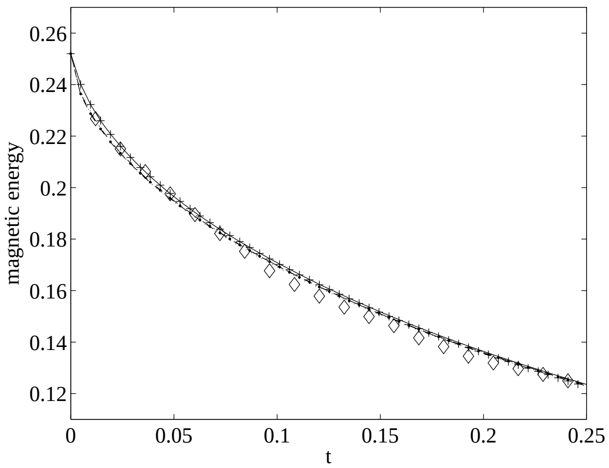
Ms=1



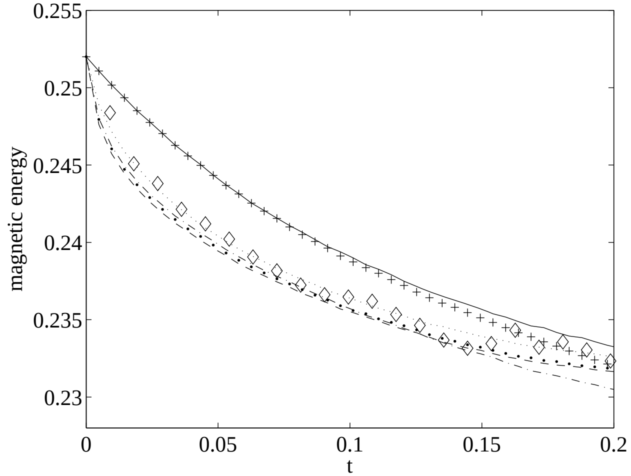
Ms=0.2



Re_m=2



Re_m=20



Magnetic energy

•The differences between SGS models for magnetic energy are shown to decrease with reducing magnetic Reynolds number and all models above demonstrate good agreement with DNS results at small value of number Re*m*.

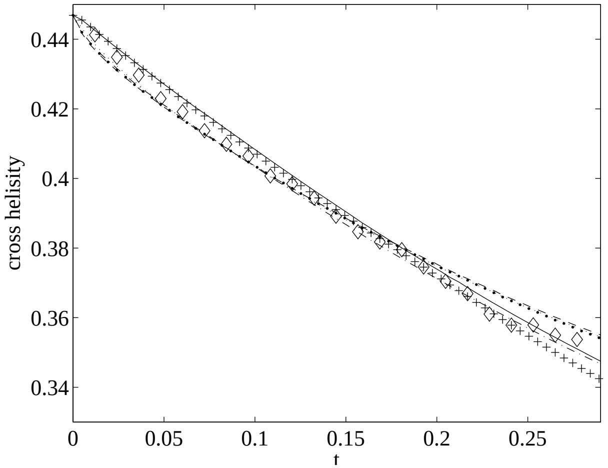
•The effect of subgrid-scale closures increases with magnetic Reynolds number for modeling of compressible MHD turbulence, but the rate of dissipation of the magnetic energy decreases with increasing Re_m.

•Generally, the best results are shown for the Smagorinsky, the Kolmogorov, and the cross-helicity models for evolution of the magnetic energy.

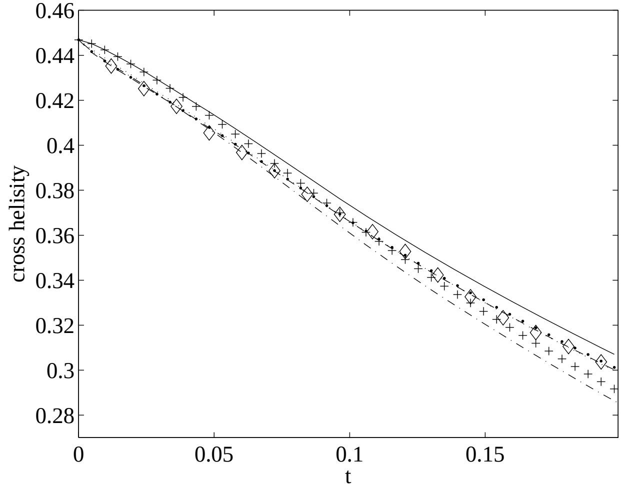
•The deviations in results for magnetic energy decrease with increasing *Ms*. It is necessary to notice, that magnetic energy reaches a stationary level more rapidly with reducing Mach number.

Cross-helicity

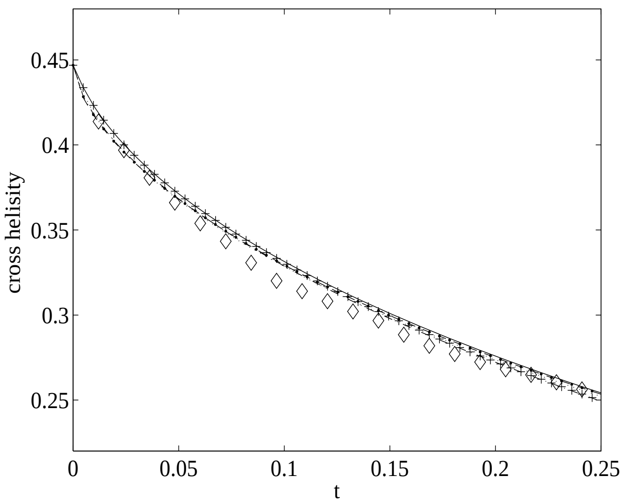
Ms=1



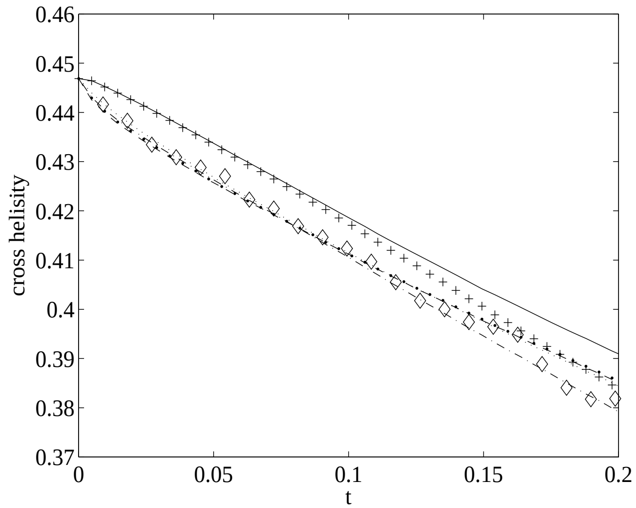
Ms=0.2



Re_m=2



Re_m=20



Cross-helicity

•For the cross-helicity, the influence of subgrid-scale parametrizations increases with magnetic Reynolds number.

•The scale-similarity model demonstrates the worst results. In the presence of adequate SGS parametrization improves calculation accuracy.

•The Smagorinsky model shows the best results for the cross-helicity both for high and for low Mach numbers.

Skewness and flatness

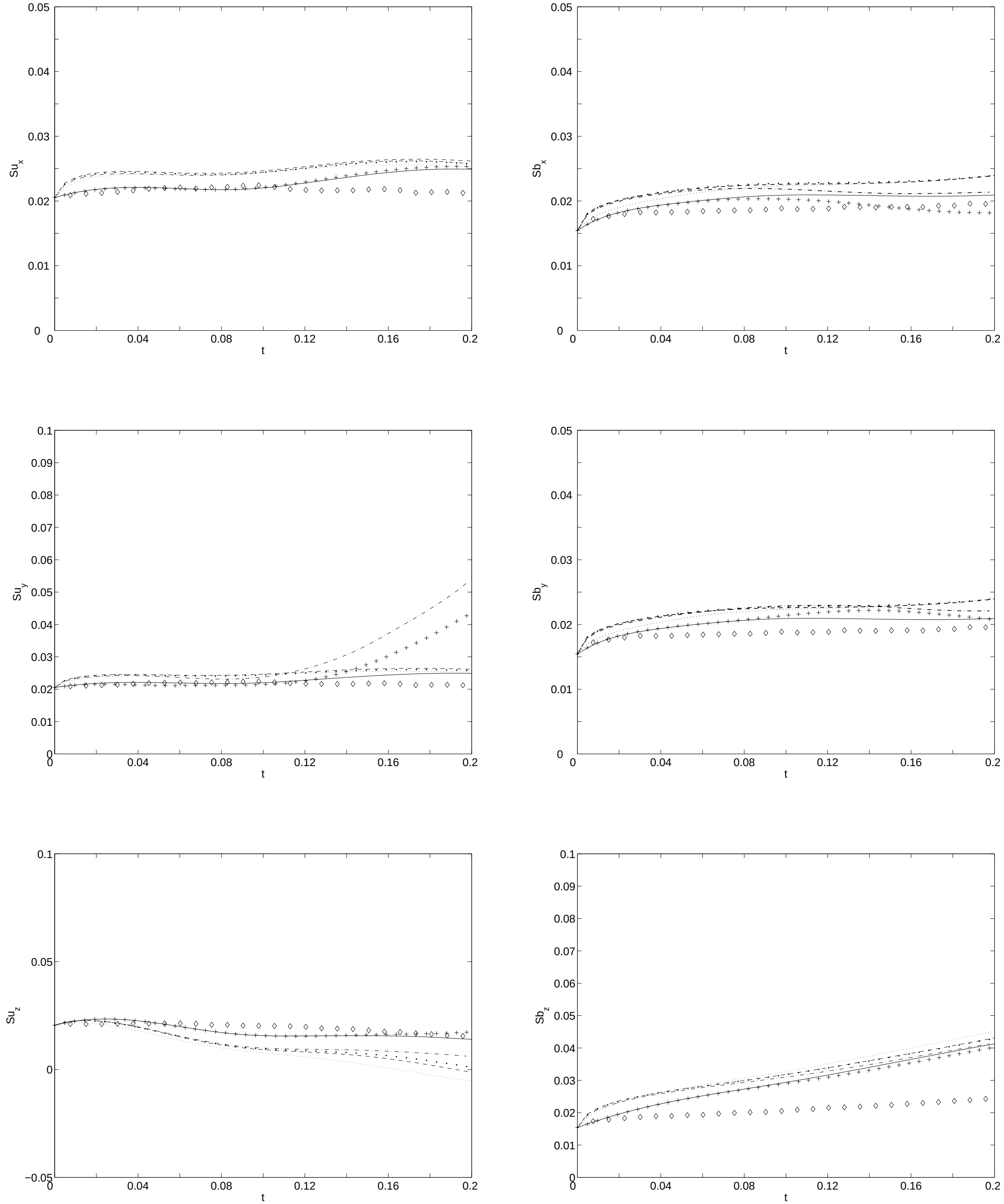
The departure from Gaussianity for fluid turbulence in the laboratory or in numerical simulations is measured in terms of the skewness and flatness factors.

The flatness factor (sometimes also called kurtosis) in turbulent flows is a measure of intermittency. The flatness is an indication of the occurrence of fluctuations far from the mean: it is an indicator of the relative frequency of rare events. Hence the flatness increases with increasing sparseness of the fluctuations:

$$Ku_j = \frac{< u_j^4 >}{(< u_j^2 >)^2} \qquad Kb_j = \frac{< B_j^4 >}{(< B_j^2 >)^2}$$

The skewness is related to the asymmetry of the probability density function of the velocity or magnetic filed fluctuations. It is a sensitive indicator of changes in the large scale structure.

$$Su_j = \frac{< u_j^3 >}{(< u_j^2 >)^{3/2}} \qquad Sb_j = \frac{< B_j^3 >}{(< B_j^2 >)^{3/2}}$$



Results	Curve
DNS	Diamond
No model	Solid
Smagorinsky model	Dashed
Kolmogorov model	Dotted
Cross-helicity model	Black point
Scale-similarity model	Marker+
Mixed model	Dashed-dot

Time evolution of skewness and flatness of velocity and magnetic field components for the case Re = 100, Re_l = 25, Re_m = 10.0, M_s = 0.6.

outcome

- applicability of LES method for studying of non-Gaussian properties of probability density function for turbulent compressible magnetic fluid flow

- potential feasibilities of various subgrid-scale parameterizations by means of comparison with DNS results are explored

- efficiency is demonstrated by various subgrid-scale models depends on similarity numbers of turbulent MHD flow. Lack of dissipation in LES model without any SGS parametrization for kinetic and magnetic energies does not have an effect on determination of the skewness and the flatness, the case without any subgrid modeling sometimes lies even closer to the DNS results. This indicates that the energy pile-up at the small scales, that is characteristic for the case without any SGS closure, does not significantly influence determination of PDF

- among the subgrid models, the best results for studying of the flatness and the skewness of the velocity and the magnetic field components are demonstrated by the Smagorinsky model for MHD turbulence and the model based on cross-helicity for MHD case.

Filtered MHD equations for heat-conducting plasma

$$
\begin{cases}
\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial \overline{\rho}\tilde{u}_j}{\partial x_j} = 0 \\
\frac{\partial \overline{\rho}\tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{\rho}\tilde{u}_i\tilde{u}_j + \overline{p}\delta_{ij} - \frac{1}{\mathrm{Re}}\tilde{\sigma}_{ij} + \frac{\overline{B}^2}{2M_A^2} - \frac{1}{M_A^2}\overline{B}_i\overline{B}_j\right) = -\frac{\partial \tau_{ji}^u}{\partial x_j} \\
\frac{\partial \overline{B}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\tilde{u}_j\overline{B}_i - \overline{B}_j\tilde{u}_i\right) - \frac{1}{\mathrm{Re}_m}\frac{\partial^2 \overline{B}_i}{\partial x_j^2} = -\frac{\partial \tau_{ji}^b}{\partial x_j} \\
\frac{\partial \overline{\rho}\tilde{E}}{\partial t} + \frac{\partial}{\partial x_j}\left[(\tilde{E} + \overline{P})\tilde{u}_j - \frac{1}{M_A^2}\overline{B}_i\overline{B}_j\tilde{u}_i\right] + \frac{\partial}{\partial x_j}\left[\frac{\tilde{q}_j}{\mathrm{Pr}\ \mathrm{Re}\ M_s^2(\gamma-1)} - \frac{1}{\mathrm{Re}}\tilde{\sigma}_{ij}\tilde{u}_i\right] - \\
\frac{\partial}{\partial x_j}\left[\frac{\eta}{\mathrm{Re}_m M_a^2}\overline{B}_i\left(\frac{\partial \overline{B}_i}{\partial x_j} - \frac{\partial \overline{B}_j}{\partial x_i}\right)\right] = -\frac{\partial}{\partial x_j}\left(\frac{1}{\gamma M_s^2}Q_j + \frac{1}{2}J_j + \frac{1}{2M_a^2}V_j - \frac{1}{M_s^2}G_j\right)
\end{cases}
$$

Equation of state: $\overline{p} = \frac{\tilde{T}\overline{\rho}}{\gamma M_s^2}$

$$E = \rho e + \frac{1}{2}\rho u_i u_i + \frac{1}{2M_a^2}B_iB_i$$

total energy

$$e = \frac{T\rho}{\gamma(\gamma-1)M_s^2}$$

internal energy

Subgrid-scale terms of filtered MHD equations for heat-conducting plasma

$$\tau_{ij}^{u} = \bar{\rho}((u_j u_i)^{\sim} - \tilde{u}_j \tilde{u}_i) - \frac{1}{M_A^2}(\overline{B_i B_j} - \bar{B}_j \bar{B}_i)$$ - SGS stresses

$$\tau_{ij}^{b} = (\overline{u_i B_j} - \bar{B}_j \tilde{u}_i) - (\overline{B_i u_j} - \tilde{u}_j \bar{B}_i)$$ - magnetic SGS stresses

$$Q_j = \bar{\rho}((u_j T)^{\sim} - \tilde{u}_j \tilde{T})$$ - SGS heat flux

$$J_j = \bar{\rho}((u_j u_k u_k)^{\sim} - \tilde{u}_j (u_k u_k)^{\sim})$$ - SGS turbulent diffusion

$$V_j = (\overline{B_k B_k u_j} - \overline{B_j B_i} \tilde{u}_j)$$ - SGS magnetic energy flux

$$G_j = (\overline{B_j B_k u_k} - \tilde{u}_k \bar{B}_k \bar{B}_k)$$ - SGS energy of the interaction between the magnetic tension and the velocity

Models for SGS terms

For SGS stresses we use the Smagorinsky model for the MHD case:

$$\tau_{ij}^{u} = -2C\,\overline{\rho}\overline{\Delta}^{2}\,|\widetilde{S}^{u}|\,(\widetilde{S}_{ij} - \frac{1}{3}\widetilde{S}_{kk}\delta_{ij}) + \frac{2}{3}Y\,\overline{\rho}\overline{\Delta}^{2}\,|\widetilde{S}^{u}|^{2}\,\delta_{ij}$$

$$\tau_{ij}^{b} = -2D\overline{\Delta}^{2}\,|\,\bar{j}\,|\,\overline{J}_{ij}$$

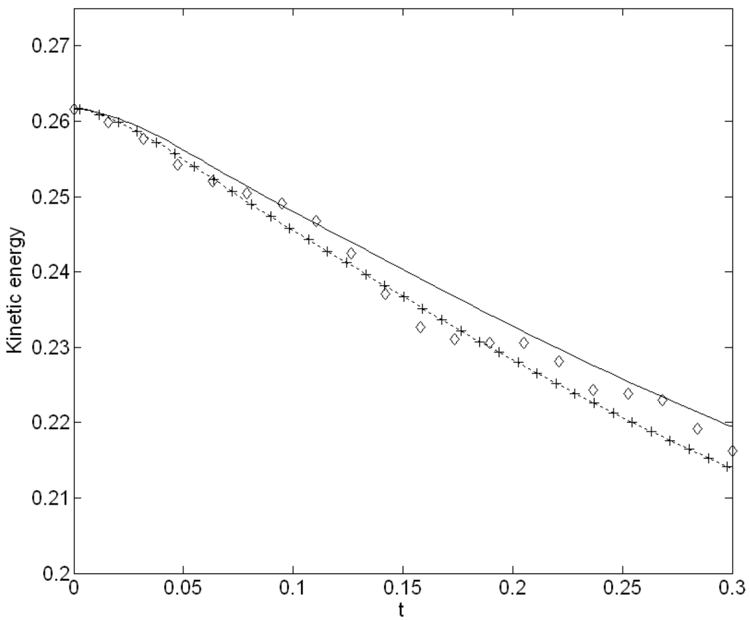
The eddy diffusivity model is used for the closure of the subgrid-scale heat flux. This eddy diffusivity model is similar to the molecular heat flux term, but the molecular viscosity and Prandtl number have been replaced by the dynamic eddy viscosity and the turbulent Prandtl number:

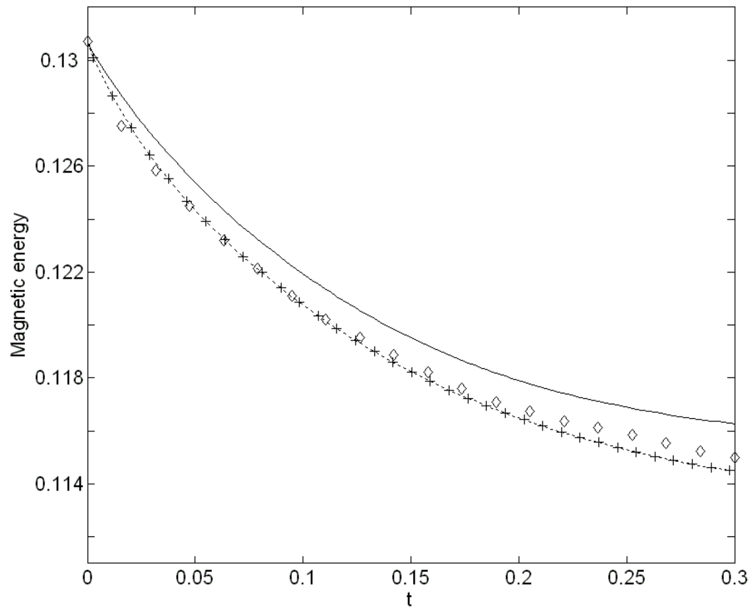
$$Q_{j} = -C_{s}\,\frac{\overline{\Delta}^{2}\overline{\rho}\,|\widetilde{S}^{u}|}{\mathrm{Pr}_{T}}\frac{\partial\widetilde{T}}{\partial x_{j}}$$

The model for Jj is based on an analogy to Reynolds-averaged Navier-Stokes equations and on the assumption that $\widetilde{u}_{i} \cong \widetilde{\widetilde{u}}_{i}$

$$J_{j} = \widetilde{u}_{k}\tau_{jk}^{u}$$

M=0.38

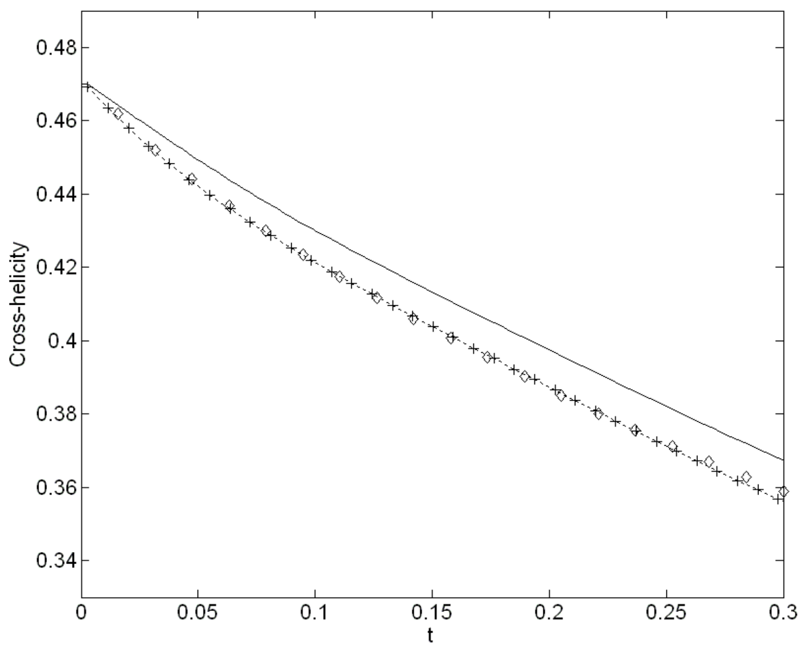


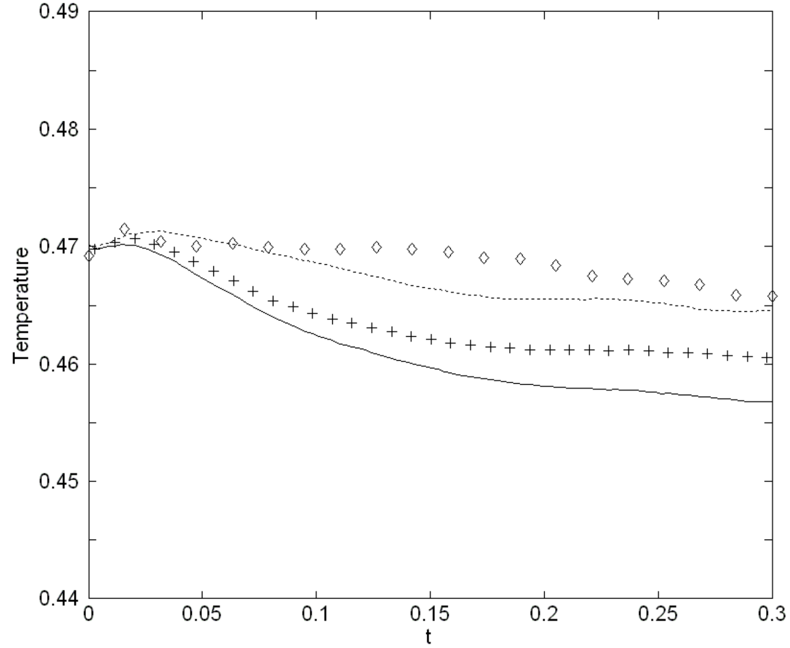


Time dynamics of kinetic and magnetic energy

Case	Curve
DNS	Diamond line
LES without any SGS models	Solid line
LES	Dotted line
LES without energy SGS terms	Marker +

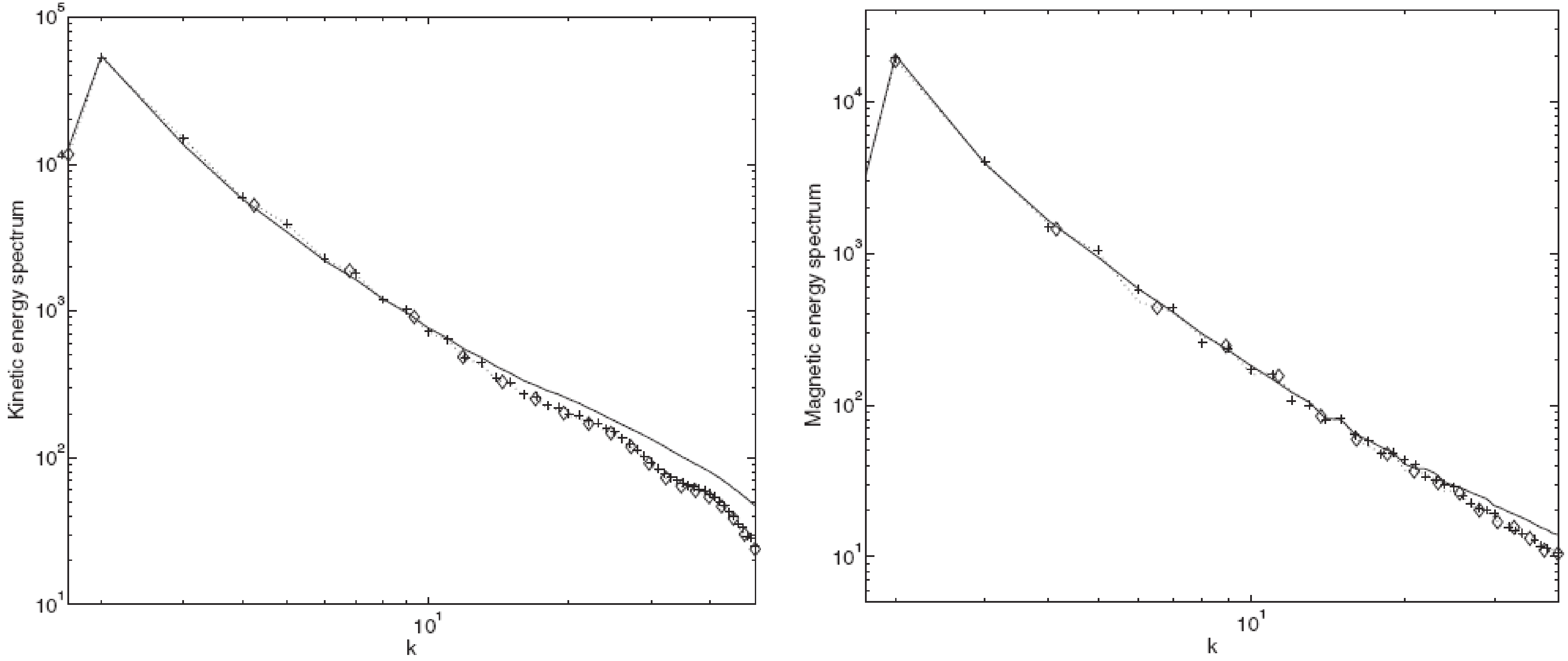
M=0.38





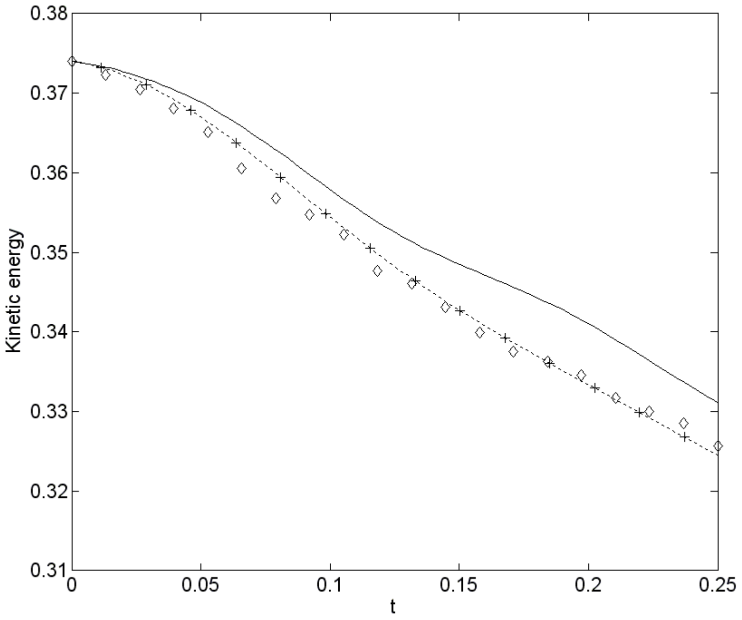
Time evolution of cross-helicity and temperature

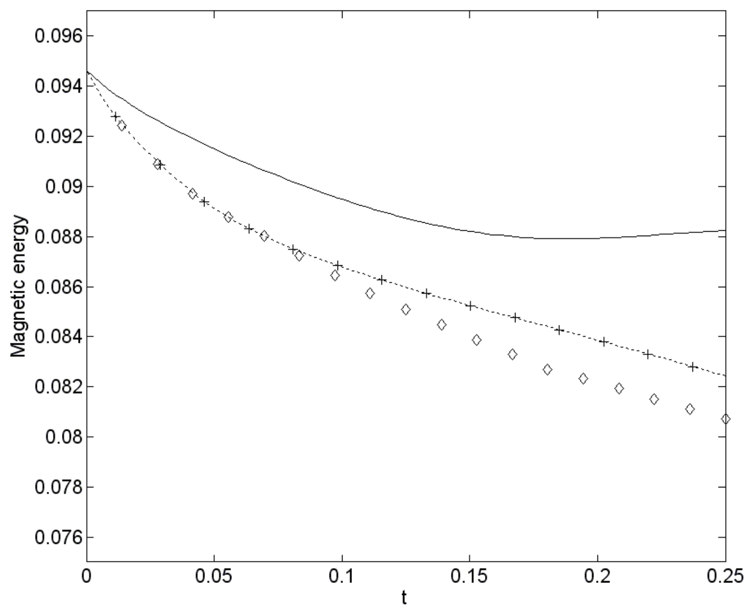
M=0.38



Kinetic and magnetic energy spectra.

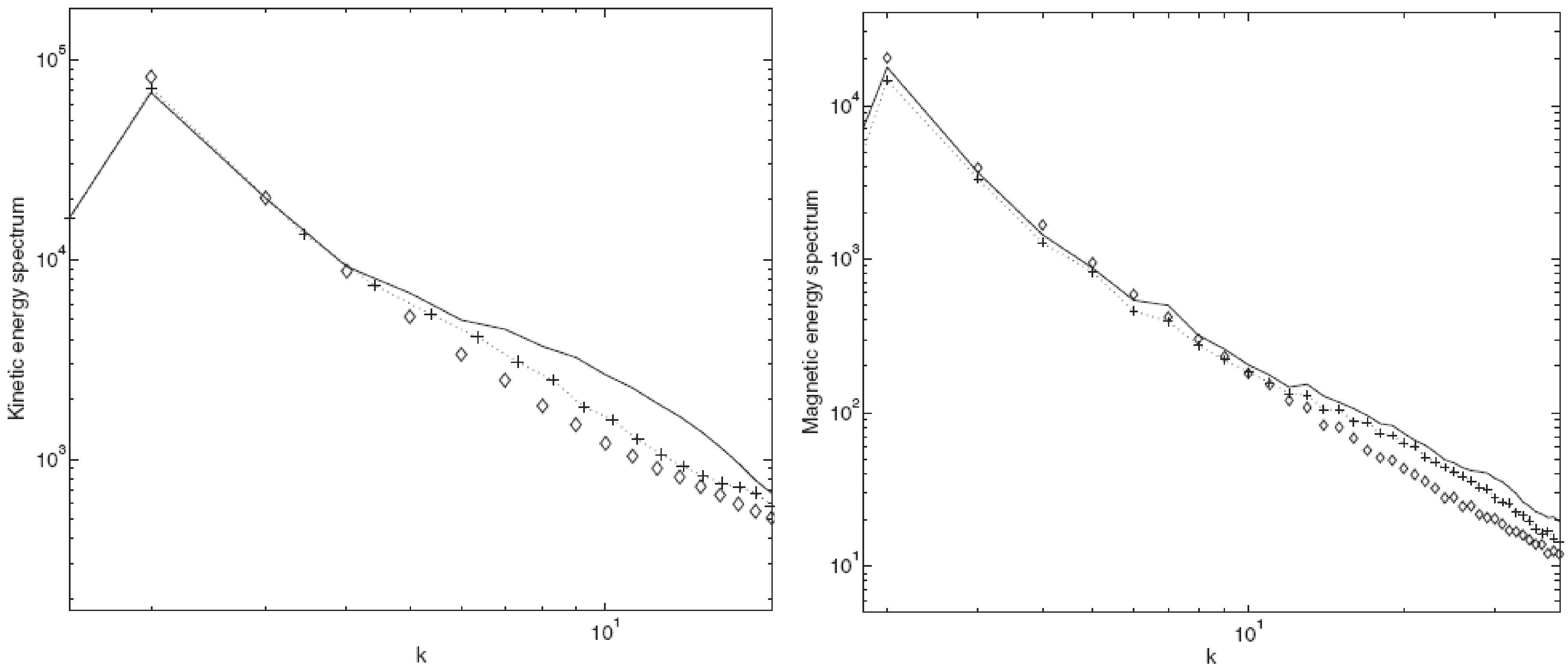
M=0.70





Time dynamics of kinetic and magnetic energy

M=0.70



Kinetic and magnetic energy spectra.

outcome

• The system of the filtered MHD equations with the total energy equation using the mass-weighted filtering procedure has been obtained. Novel subgrid-scale terms arise in total energy equation due to the presence of energy equation.

• New subgrid-scale models for the SGS terms, appearing after filtering procedure in the total energy equation in the presence of magnetic field, are suggested.

• Consideration of the SGS terms in the energy equation scarcely affects the kinetic and the magnetic energy even at high Mach numbers, while for the temperature (same as for the internal energy) the presence of SGS models in the energy equation is an important condition for improvement of calculation accuracy of thermodynamic quantities.

• Generally, LES method using explicit mass-weighted filtering demonstrates good results for modeling of electrically and heat conducting fluid in MHD turbulence when the medium is weakly or moderately compressible.

Linear forcing

Idea essentially consists in adding a force proportional to the fluctuating velocity. Linear forcing resembles a turbulence when forced with a mean velocity gradient, that is, a shear. This force appears as a term in the equation for fluctuating velocity that corresponds to a production term in the equation of turbulent kinetic energy.

The equation for the fluctuating part of the velocity in a compressible MHD turbulent flow are written as

$$\rho[\frac{\partial \acute{u}_i}{\partial t} + U_j \frac{\partial \acute{u}_i}{\partial x_j}] = -\frac{\partial \acute{p}}{\partial x_j} + \frac{\partial \acute{\sigma}_{ij}}{\partial x_j} - \boxed{\rho \acute{u}_j \frac{\partial U_i}{\partial x_j}} - [\rho \acute{u}_j \frac{\partial \acute{u}_i}{\partial x_j} - \rho \langle \acute{u}_j \frac{\partial \acute{u}_i}{\partial x_j} \rangle] - \frac{\partial}{\partial x_j} \frac{\acute{B}^2}{8\pi} + \frac{1}{4\pi}[ß_j \frac{\partial \acute{B}_i}{\partial x_j} + \acute{B}_j \frac{\partial ß_i}{\partial x_j}] - \frac{1}{4\pi}[\acute{B}_j \frac{\partial \acute{B}_i}{\partial x_j} - \langle \acute{B}_j \frac{\partial \acute{B}_i}{\partial x_j} \rangle]$$

Here following decomposition referred to as the Reynolds decomposition is used:

$$u_i = U_i + \acute{u}_i,\ B_i = ß_i + \acute{B}_i,\ B_i = ß_i + \acute{B}_i,\ p = P + \acute{p},\ \sigma_{ij} = \Sigma_{ij} + \acute{\sigma}_{ij}$$

Linear forcing

In symbolic terms, derivation of turbulent kinetic energy equation can be written as $\langle u \cdot NS\ eq. \rangle - U \langle NS\ eq. \rangle$ which yields:

$$\frac{\partial}{\partial t}\langle \frac{1}{2}\rho \acute{u}^2 \rangle + \frac{\partial}{\partial x_j}(\langle \frac{1}{2}\rho \acute{u}^2 \rangle U_j + \langle \frac{1}{2}\rho \acute{u}^2 \acute{u}_j \rangle - \langle \beta_{ij} \acute{u}_i \rangle) = -\langle \acute{u}_i \frac{\partial \acute{p}}{\partial x_i} \rangle + \langle \acute{u}_i \frac{\partial \acute{\sigma}_{ij}}{\partial x_j} \rangle - \langle \rho \acute{u}_i \acute{u}_j \frac{\partial U_i}{\partial x_j} \rangle - \langle \beta_{ij} \frac{\partial \acute{u}_i}{\partial x_j} \rangle$$

where $\beta_{ij} = \frac{\acute{B}_i \acute{B}_j}{4\pi} - \frac{\acute{B}^2}{8\pi}\delta_{ij}$ - turbulent magnetic tensor

production of turbulent energy per unit volume per unit time resulting from the interaction between the Reynolds stress and the mean shear.

Linear forcing

$F_i^u = \Theta \rho u_i$ - driving term proportional to the velocity

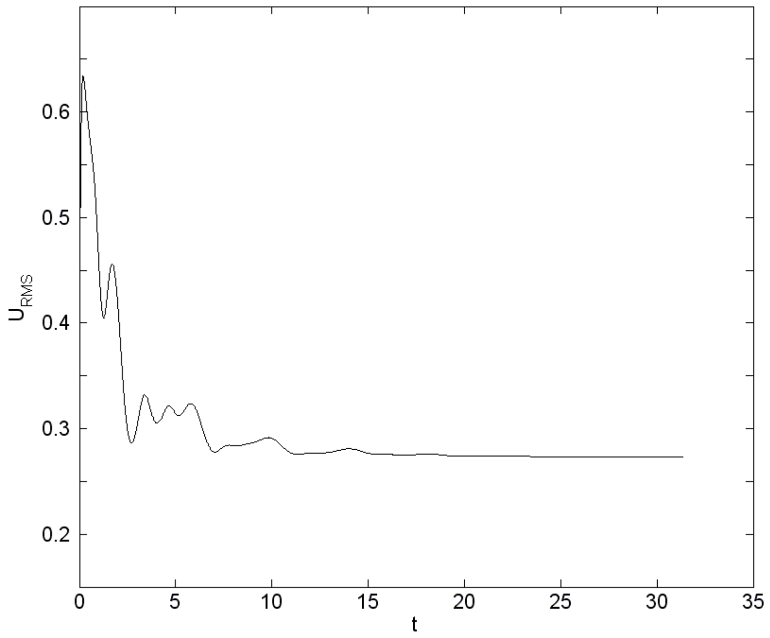
Θ ↓ coefficient which is determined from a balance of kinetic energy for a statistically stationary state:

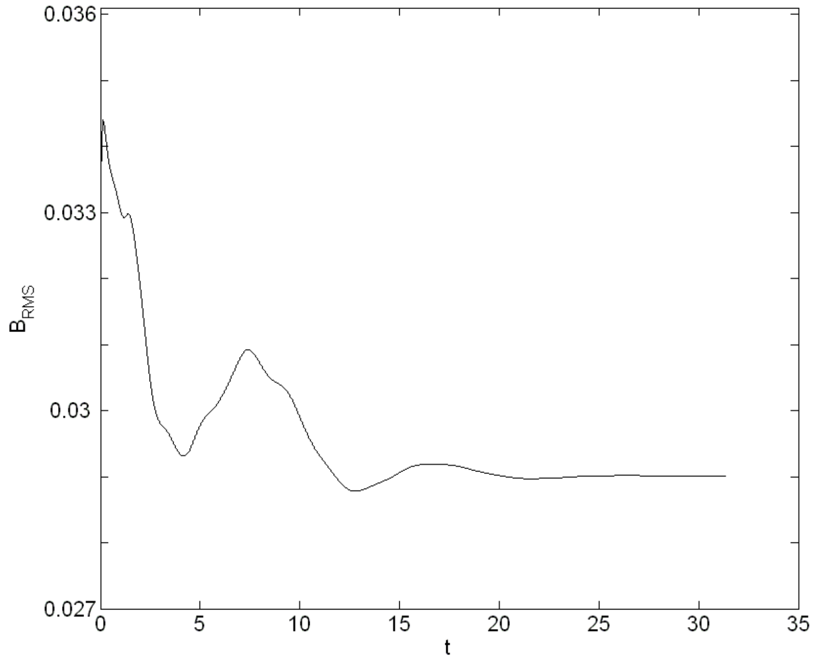
$$\Theta = \frac{1}{3\langle\rho\rangle u_{rms}^2}[\langle u_j \frac{\partial}{\partial x_j} p\delta_{ij}\rangle + \varepsilon + \frac{1}{8\pi}\langle u_j \frac{\partial}{\partial x_j} B^2 \delta_{ij}\rangle]$$

$\varepsilon = -\langle u_j \; \partial\sigma/\partial x_j\rangle$ - mean dissipation rate of turbulent energy into heat

$$1/(\langle\rho u^2\rangle) = 1/(3\langle\rho\rangle u_{rms}^2)$$

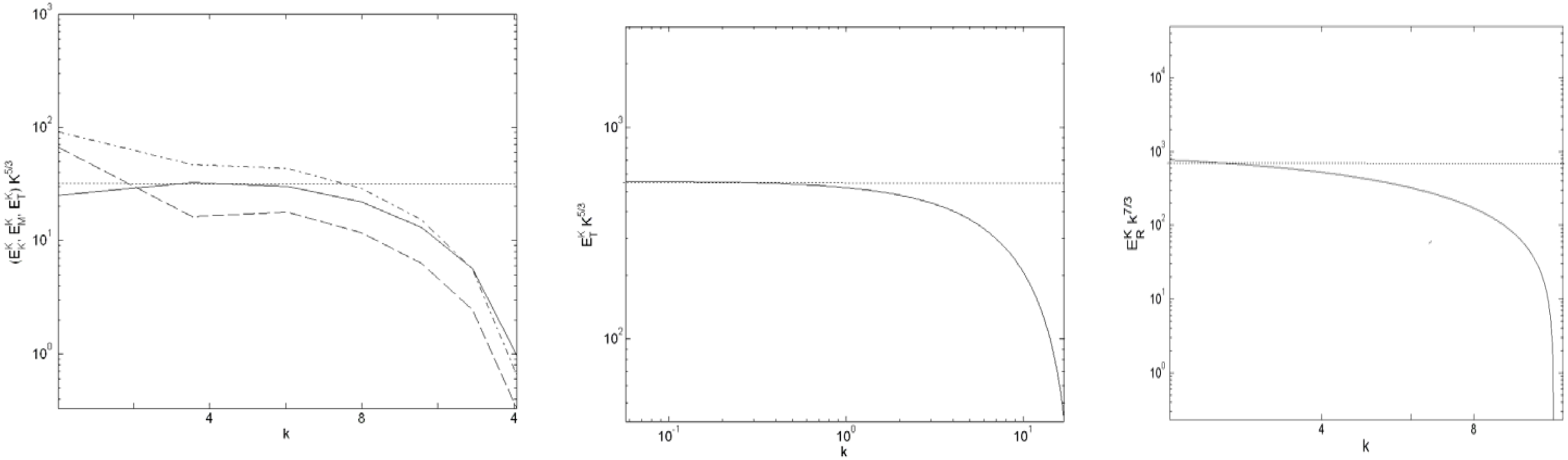
Polytropic plasma -1





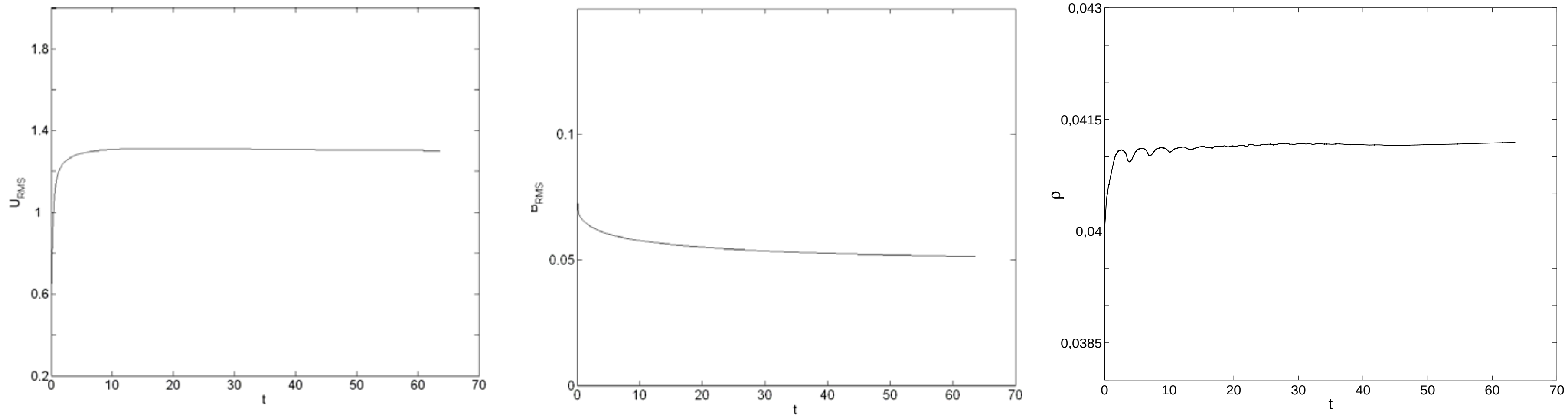
Time evolution of U_rms and B_rms

Polytropic plasma

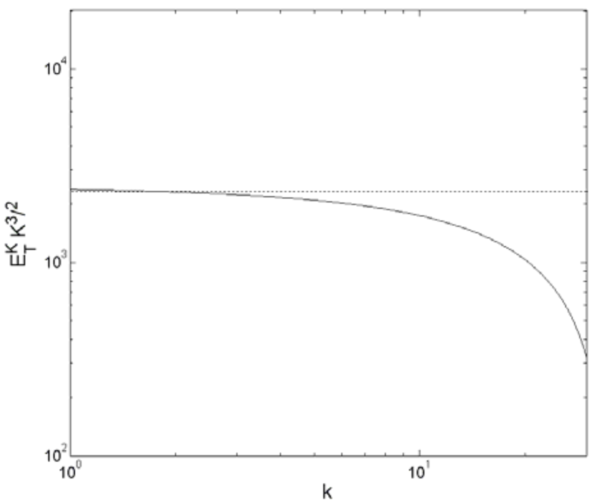


Spectra of MHD turbulence

Polytropic plasma



Time dynamics of rms velocity, rms magnetic field and mean density.



Spectrum of total energy

outcome

The theory of linear forcing is developed for study of compressible MHD turbulence in coordinate space. The expressions of external force which provide obtaining a statistically stationary regime of turbulence are derived. The formulae used for the formulation of large eddy simulation approach are obtained. The potential possibilities of LES method to reproduce physics of flow under investigation in a stationary regime both for polytropyc and for heat-conducting plasmas are studied.

Spectra of MHD turbulence is obtained and studied. Type of obtained spectra is determined. Kolmogorov and Iroshnikov-Kraichnan spectra of total energy are obtained and conditions of their occurrence are showed.

Efficiency of LES method for studying of scale-invariant properties of compressible MHD turbulence is demonstrated.