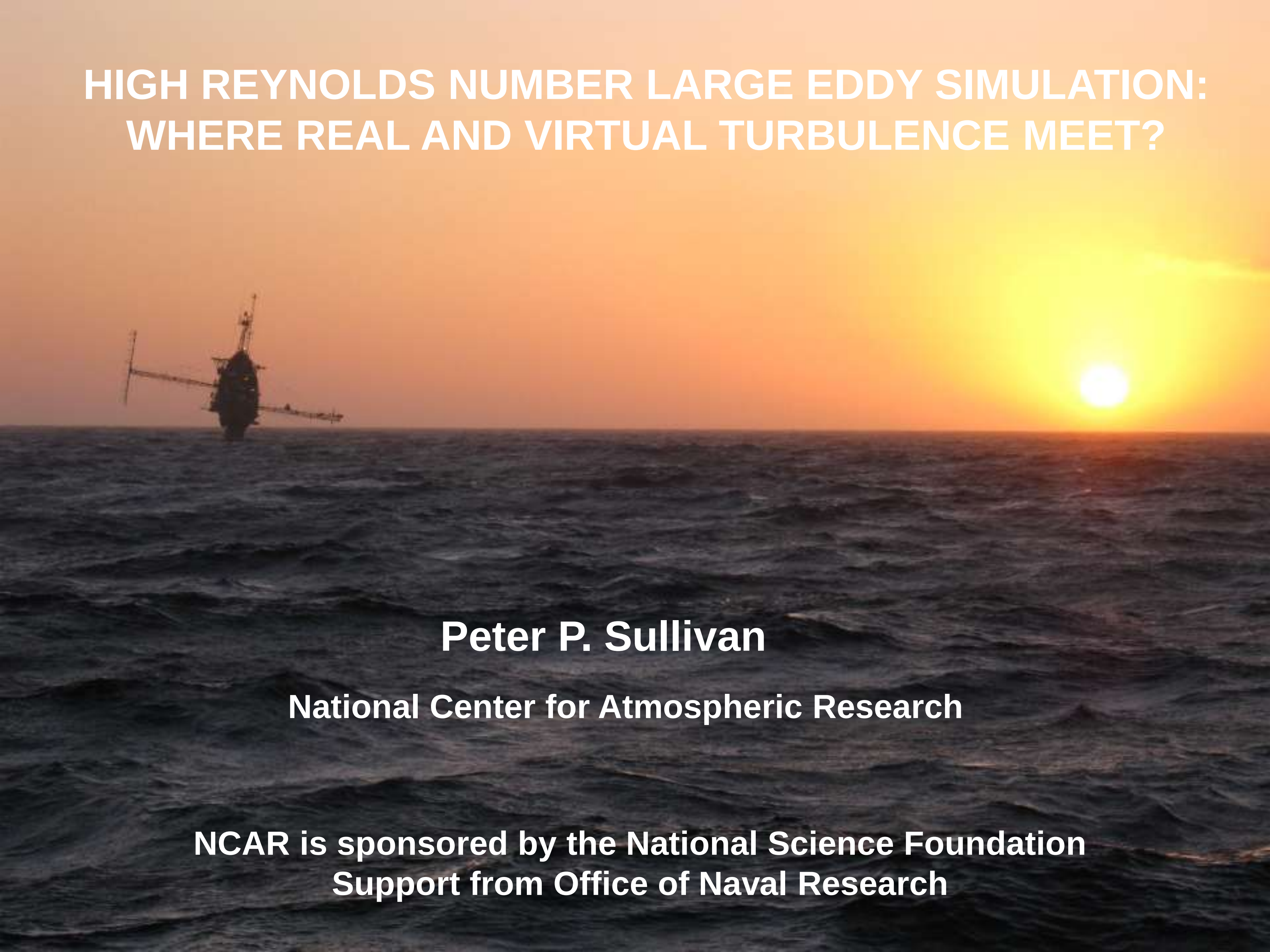


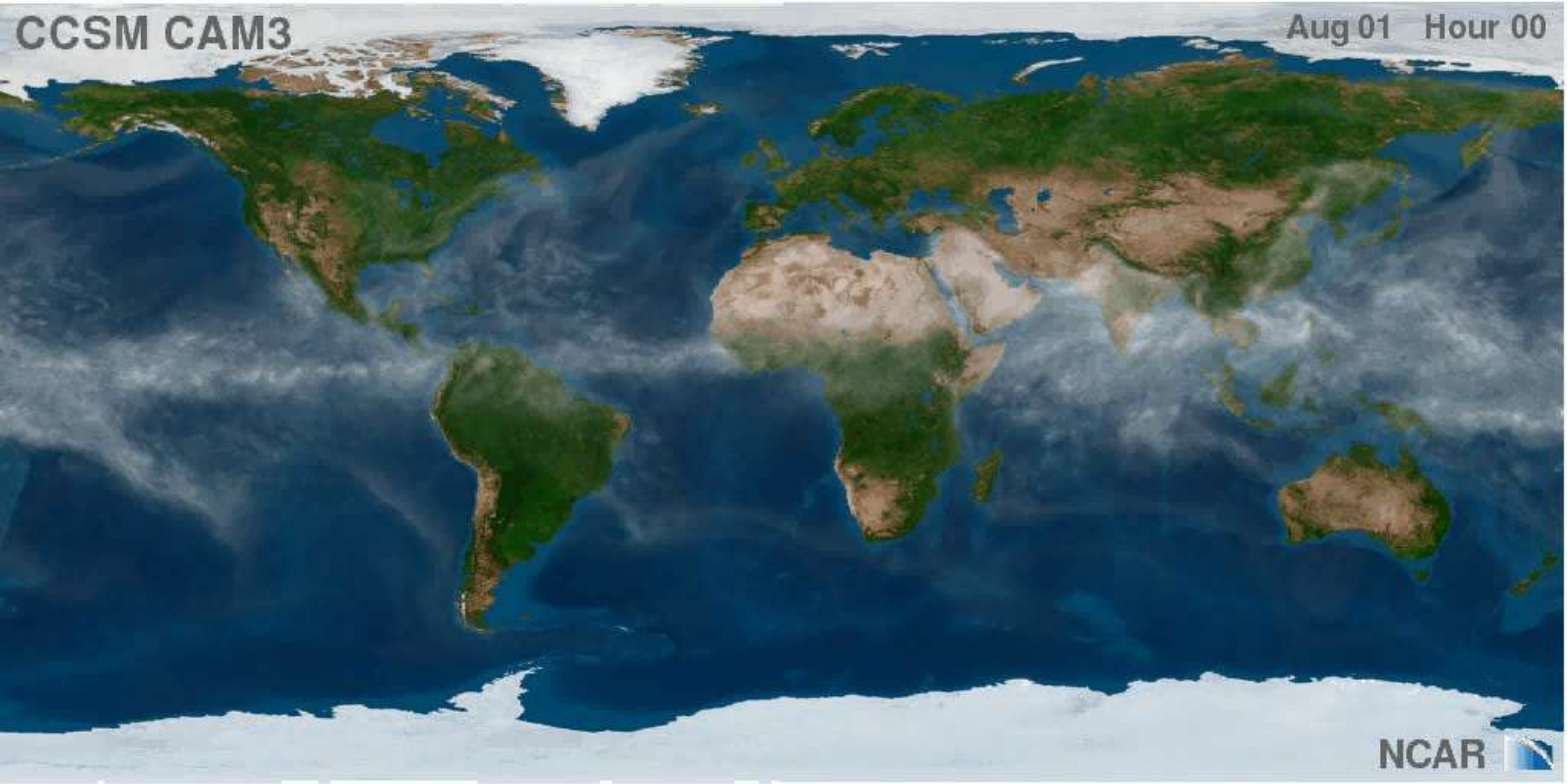
HIGH REYNOLDS NUMBER LARGE EDDY SIMULATION: WHERE REAL AND VIRTUAL TURBULENCE MEET?

Peter P. Sullivan

National Center for Atmospheric Research

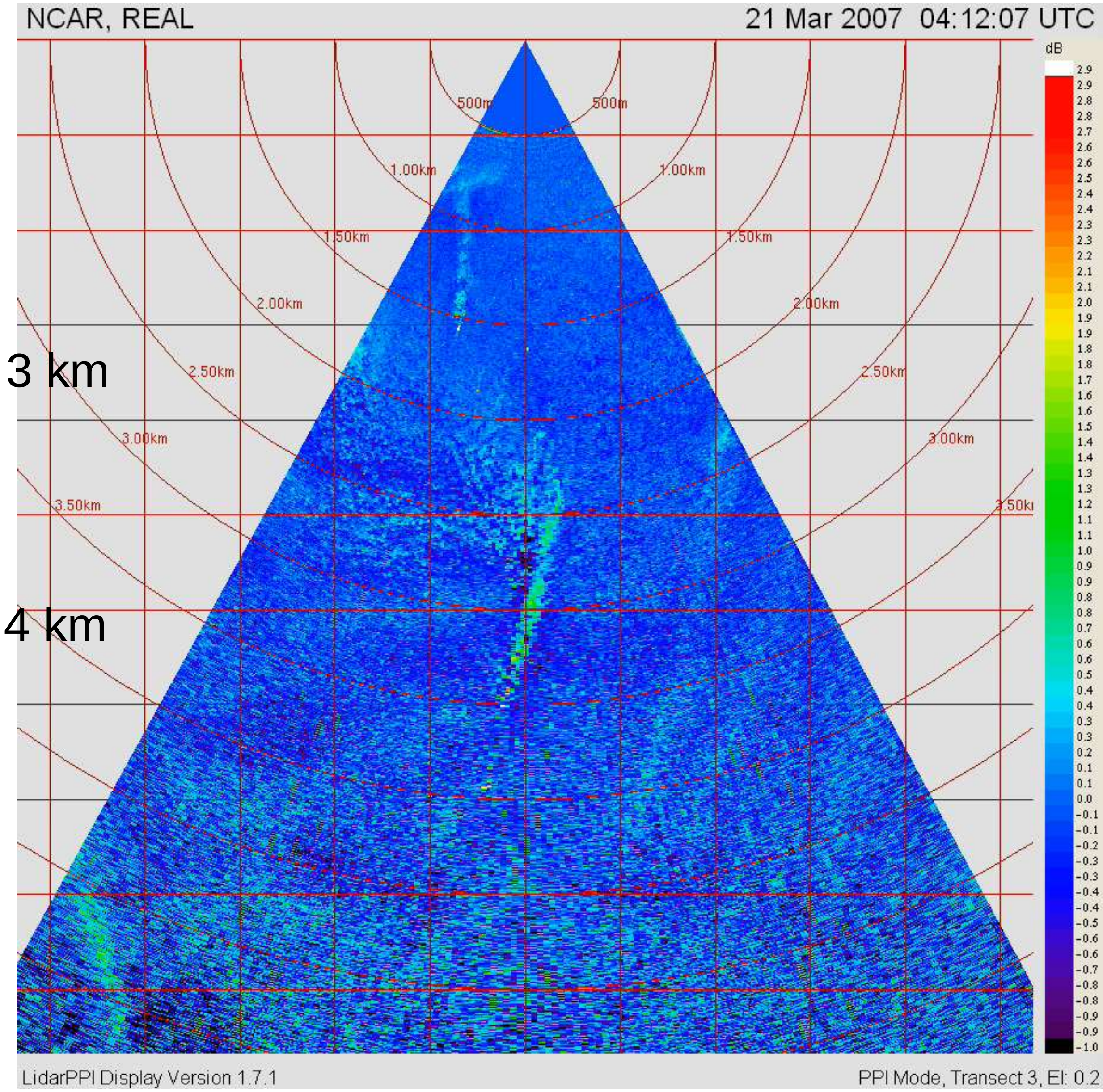
NCAR is sponsored by the National Science Foundation
Support from Office of Naval Research

HIGH RESOLUTION GLOBAL SIMULATION
Courtesy J. Hurrell and CGD/NCAR



AN ABUNDANCE OF TIME AND SPACE SCALES!

BOUNDARY-LAYER PROCESSES

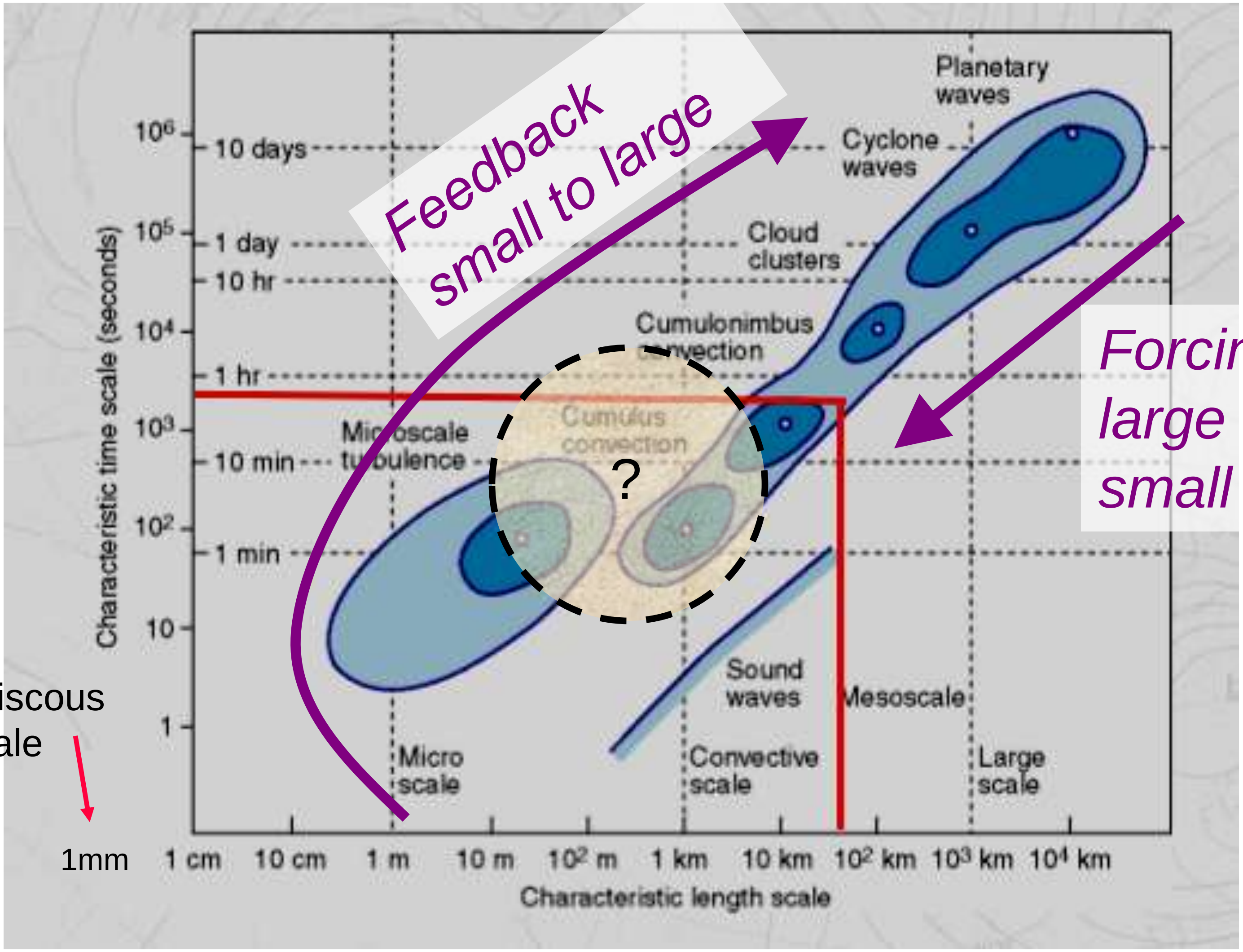


Stably stratified turbulent flow just above a walnut orchard during the Canopy Horizontal Array Turbulence Study (CHATS)

Provided by Shane Mayor & Ned Patton

SCALES OF TURBULENCE RELATIVE TO NWP SCALES

Kolmogorov viscous dissipation scale



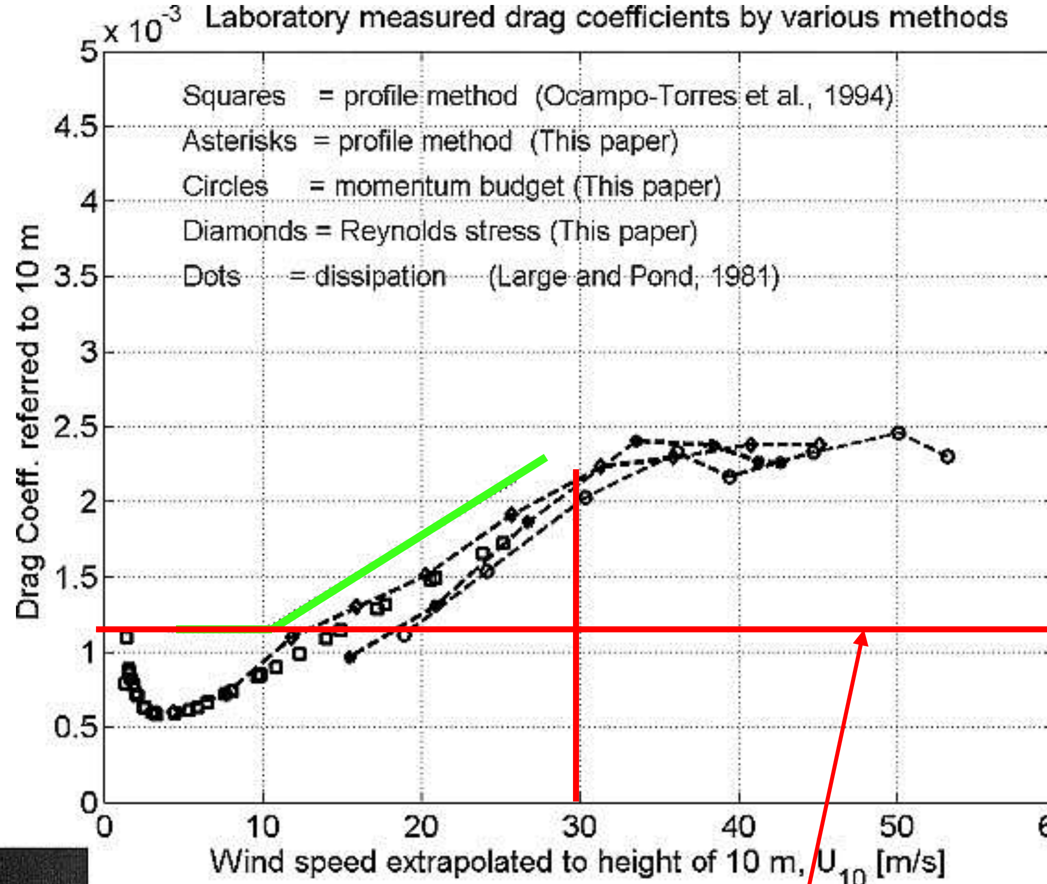
Courtesy Anton Beljaars (ECMWF)

LOW LEVEL FLIGHT IN HURRICANE ISABEL Courtesy M. Black

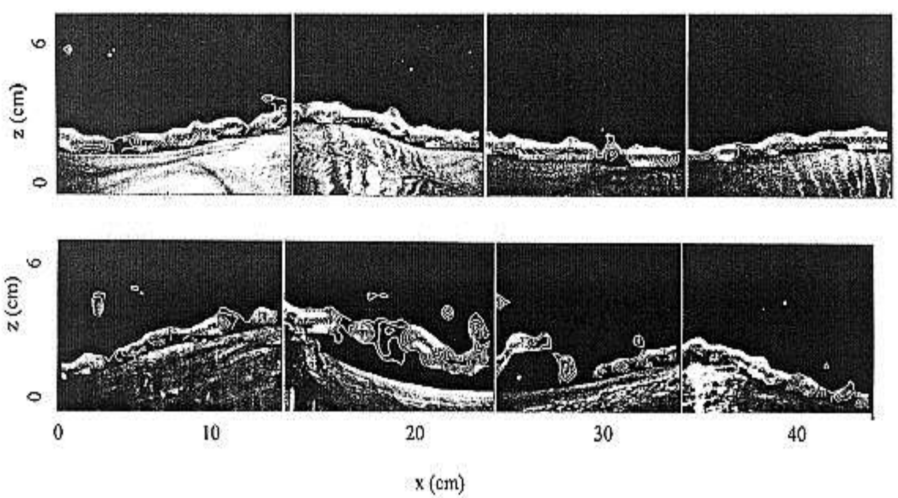
C_D Donelan etal, 2004

$$v_{max}^2 = \frac{T_s - T_o}{T_o}\frac{C_k}{C_D}(k_s^* - k)$$

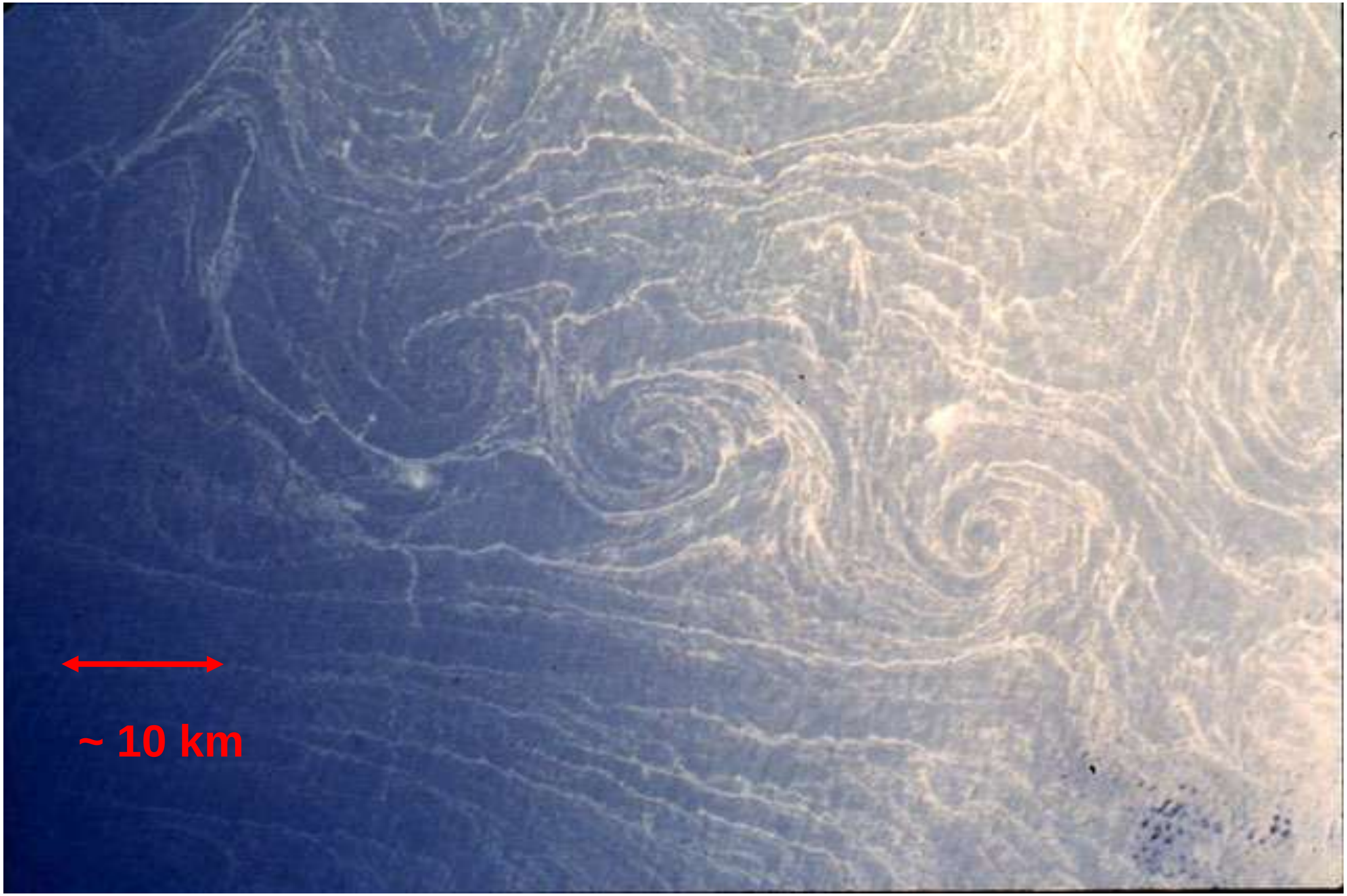
Maximum azimuthal wind speed (Emanuel, 2004)



Oceanic value?

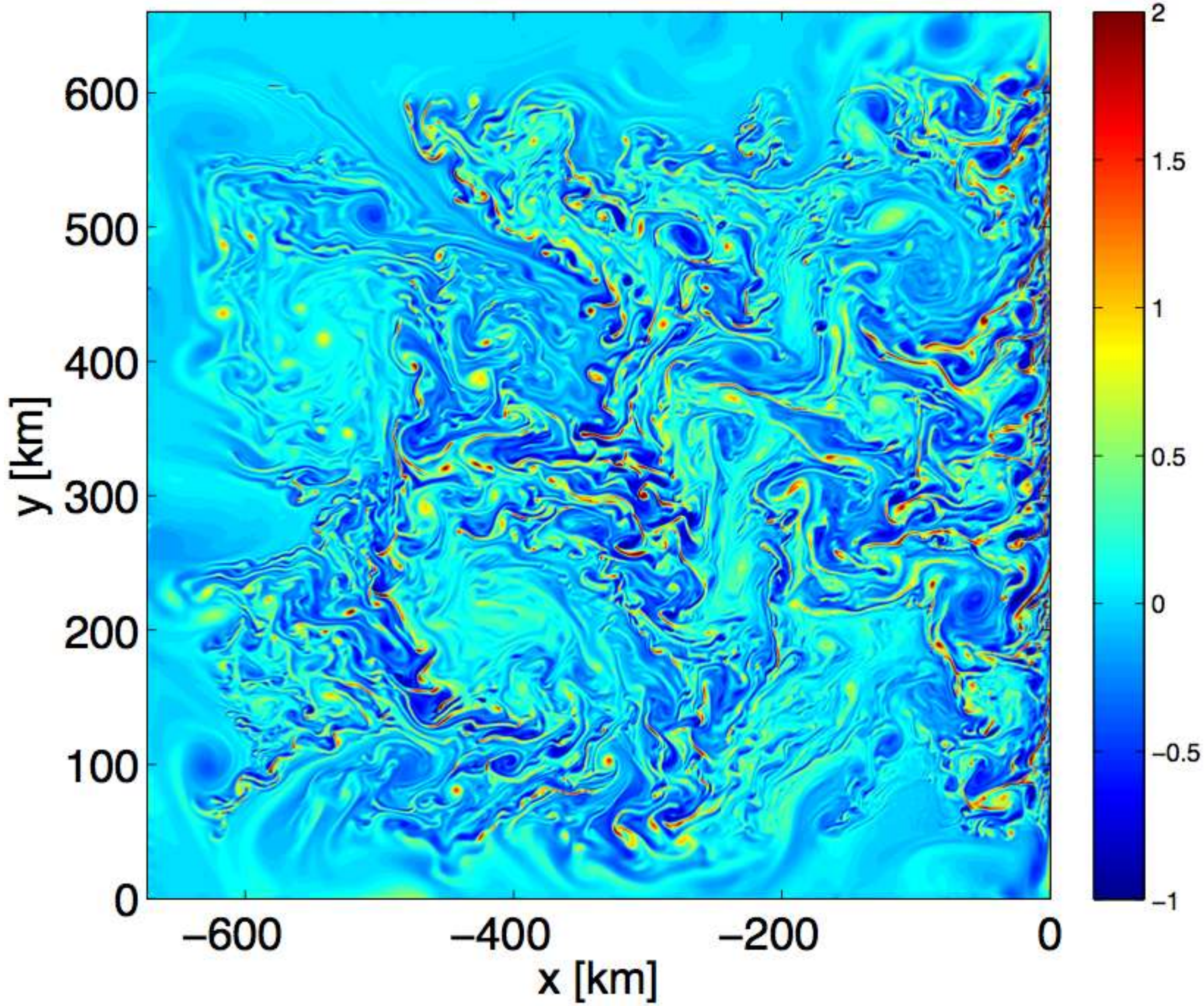


SPIRALS ON THE SEA



Photograph of a cyclonic spiral-eddy street off the coast of the Egyptian/Libyan border. Eddy radii are ≈ 5 km, and scum convergence lines are ∼ 100s m wide. The street configuration suggests a recent vortex roll-up from an unstable submesoscale front or wake. (Scully-Power, 1986), courtesy J. McWilliams.

OCEAN WEATHER SYSTEMS

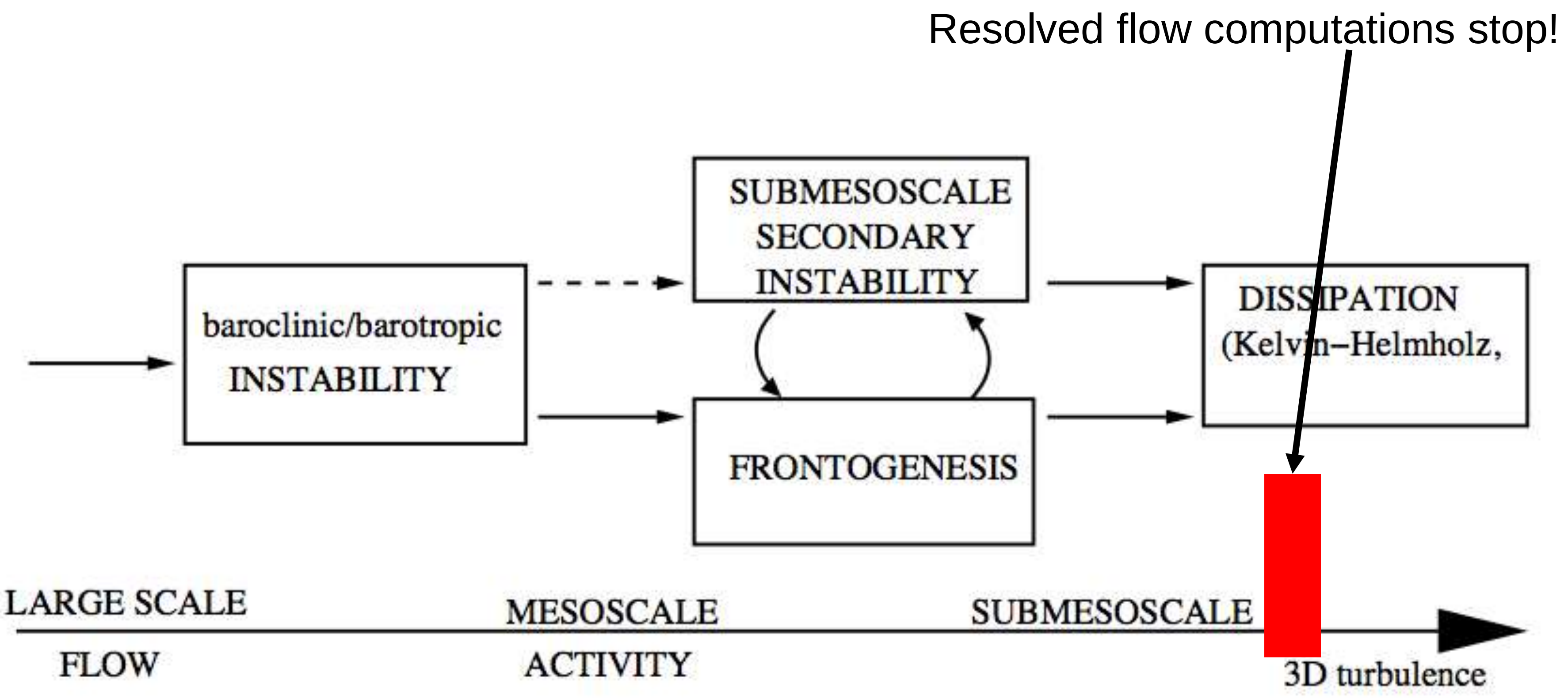


Simulated vertical vorticity normalized by f illustrating the interaction between fronts, instabilities and vortices in the ocean. Courtesy J. McWilliams (UCLA)

SUMMARY OF SUBMESOSCALE TURBULENCE

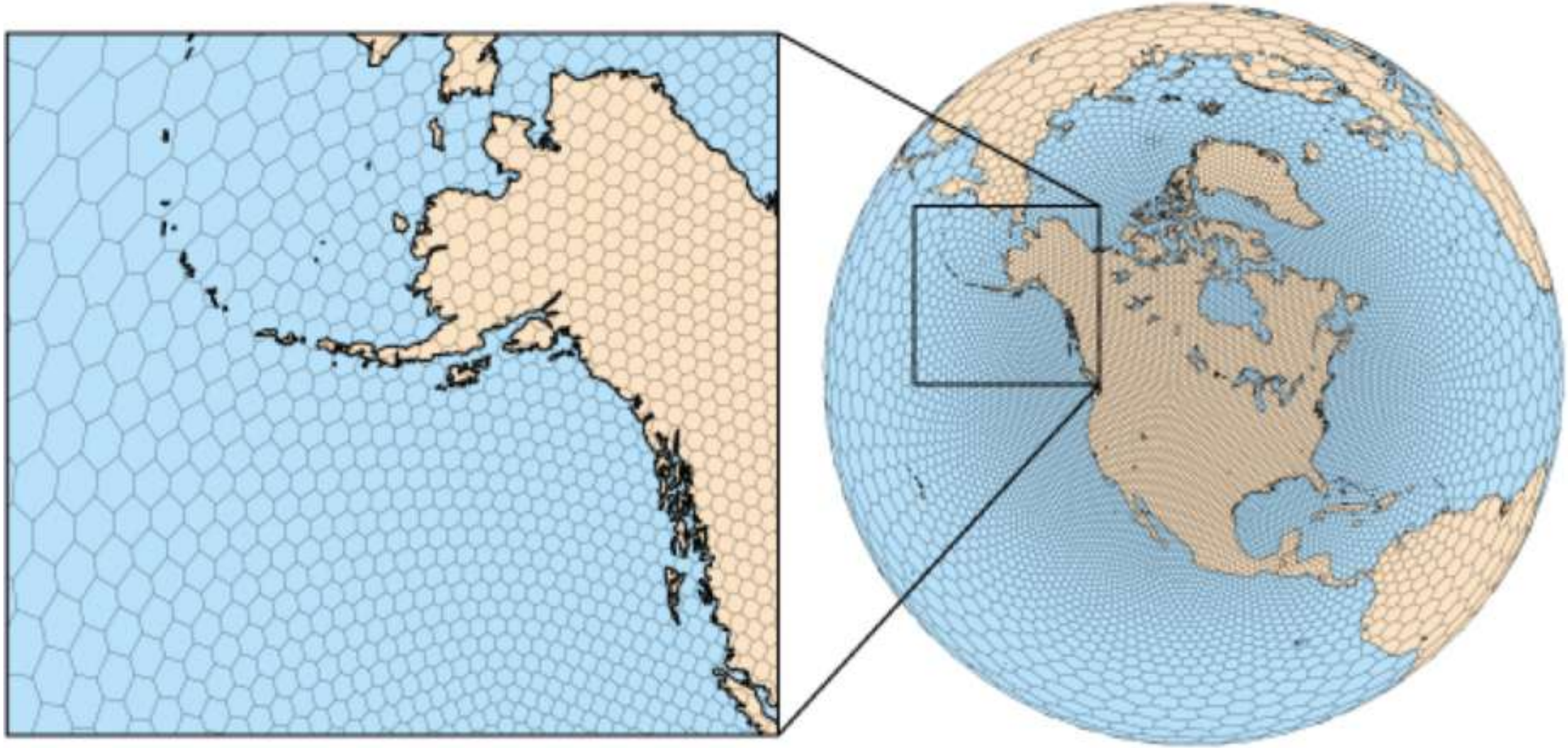
Courtesy Jim McWilliams (UCLA)

- Surface-intensified frontogenesis induced by mesoscale straining
- Frontal instabilites of several types
- Kinetic energy forward cascade towards turbulence dissipation
- ...



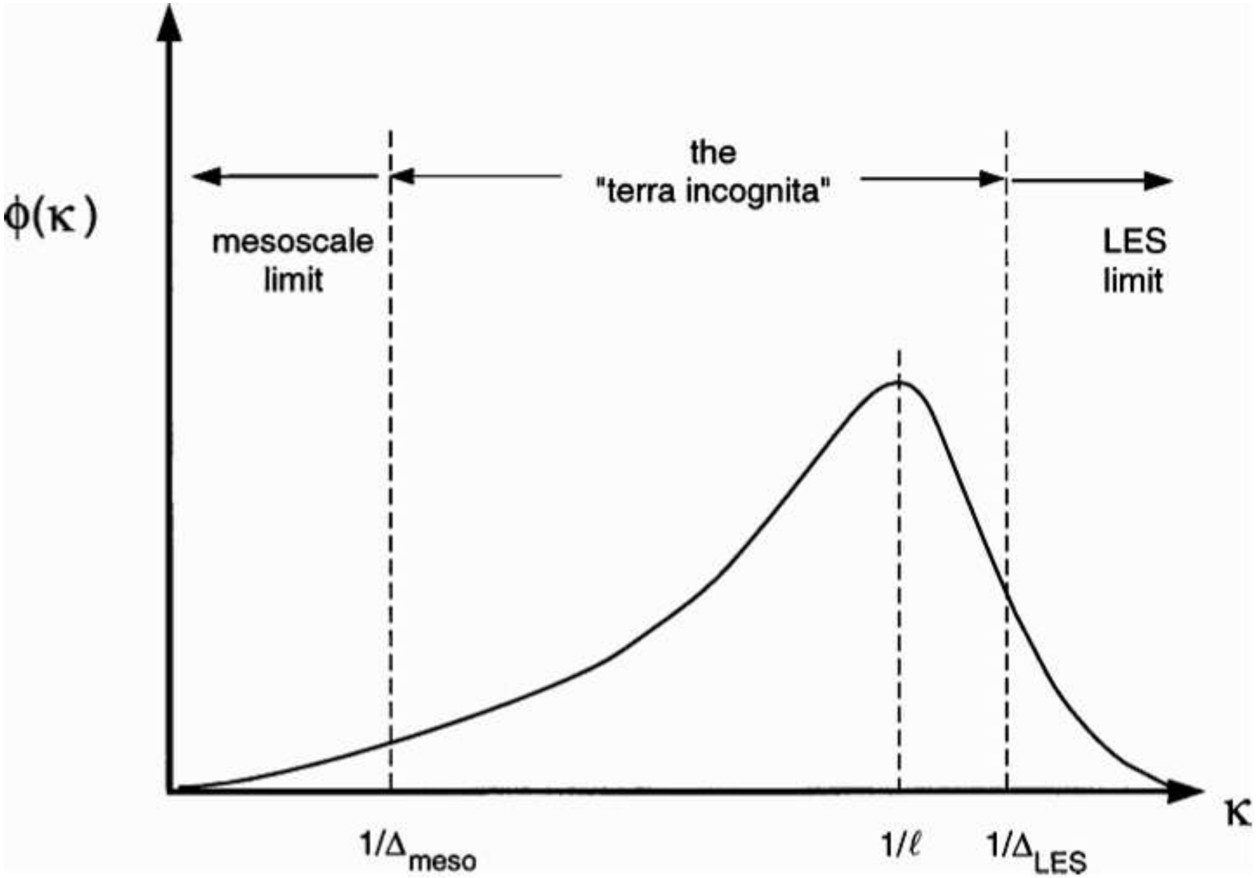
AN EXAMPLE OF AN MPAS MESH BASED ON SPHERICAL CENTROIDAL VORONOI TESSELLATIONS

http://public.lanl.gov/ringler/Voronoi.html

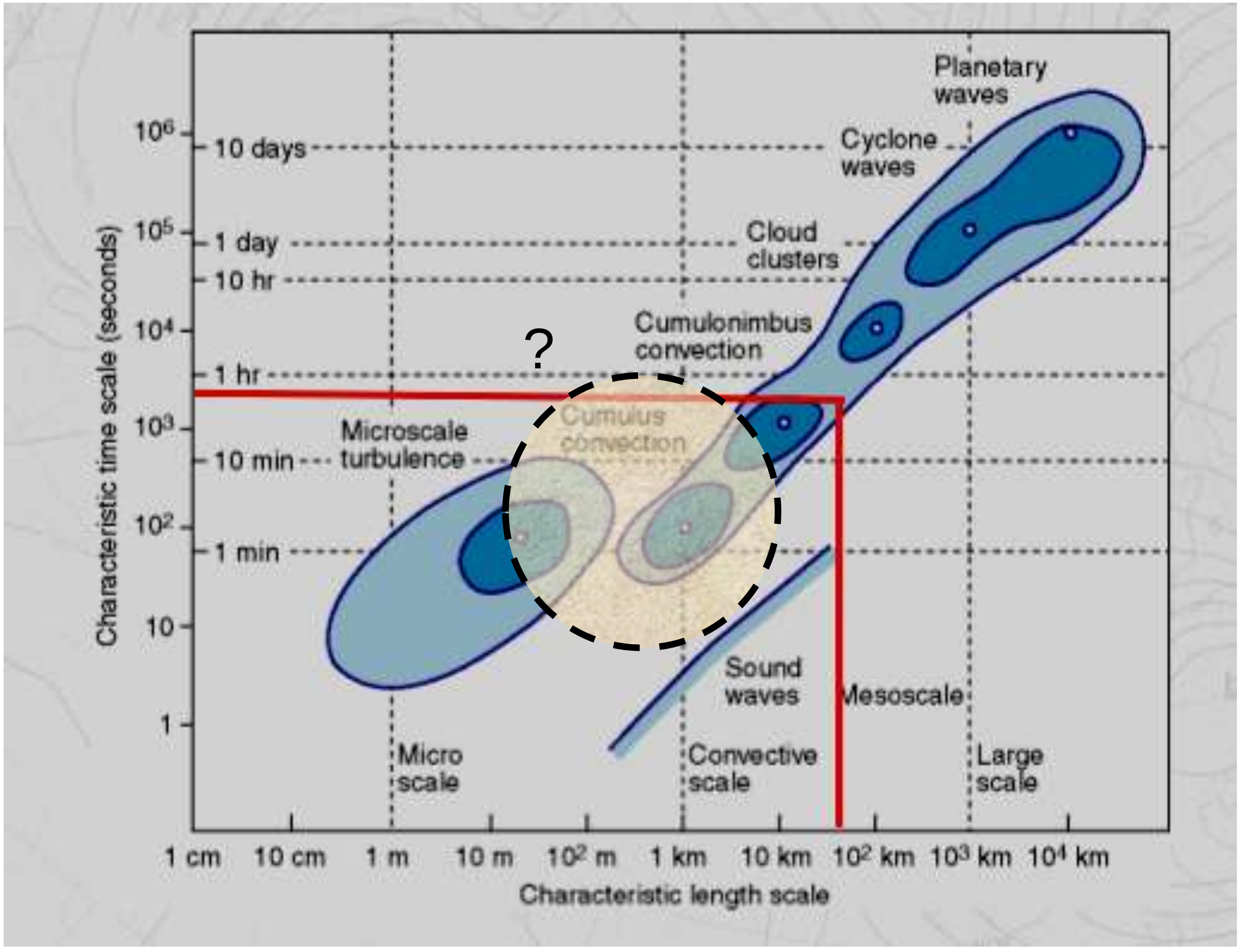


A need for scale aware parameterizations?

What are the appropriate (LES) equations for Terra-Incognita?



TOO MANY SCALES!



GOVERNING EQUATIONS

Given incompressible Navier Stokes equations:

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu\frac{\partial^2 u_i}{\partial x_j^2}$$

COMMENTS ON THE FILTERED EQUATIONS AND THE SUBGRID SCALE STRESS TENSOR

$$
\begin{aligned}
\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} &= -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} - \frac{\partial}{\partial x_j}\left(\mathcal{T}_{ij} - \cancelto{0}{\nu \frac{\partial \overline{u}_i}{\partial x_j}}\right) \\
\mathcal{T}_{ij} &= \overline{u_i u_j} - \overline{u}_i \overline{u}_j
\end{aligned}
$$

- The filtered equations are generally applicable in the LES and mesoscale limits and also in Terra-Incognita [*Lilly*(1967)]

- The filtered equations contain two parameters, molecular viscous Reynolds number and the SGS stress tensor $\mathcal{T}_{ij}$

- $\mathcal{T}_{ij}$ is unknown! It needs to be expressed in terms of known resolved fields $\overline{u}_i$

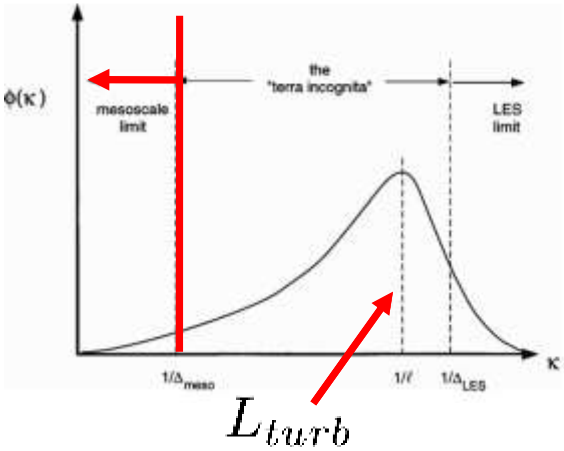
- $\mathcal{T}_{ij}$ controls how the flow transitions from mesoscale limit $\Longleftrightarrow$ LES limit

RESOLVED FLOW IN MESOSCALE LIMIT

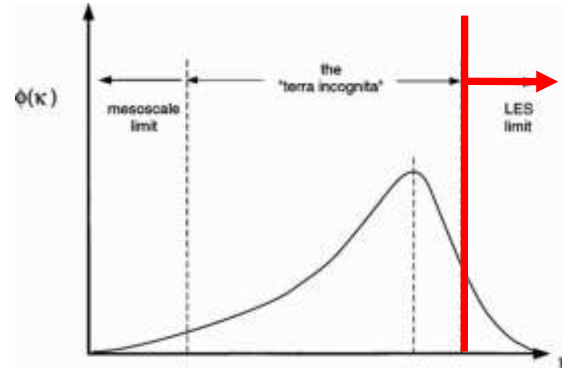
Simple eddy viscosity closure (model for deviatoric stress)

$$\tau_{ij} \sim K\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right), \quad K = e^{1/2} l$$

In mesoscale limit: $L_{turb} < \triangle_f$



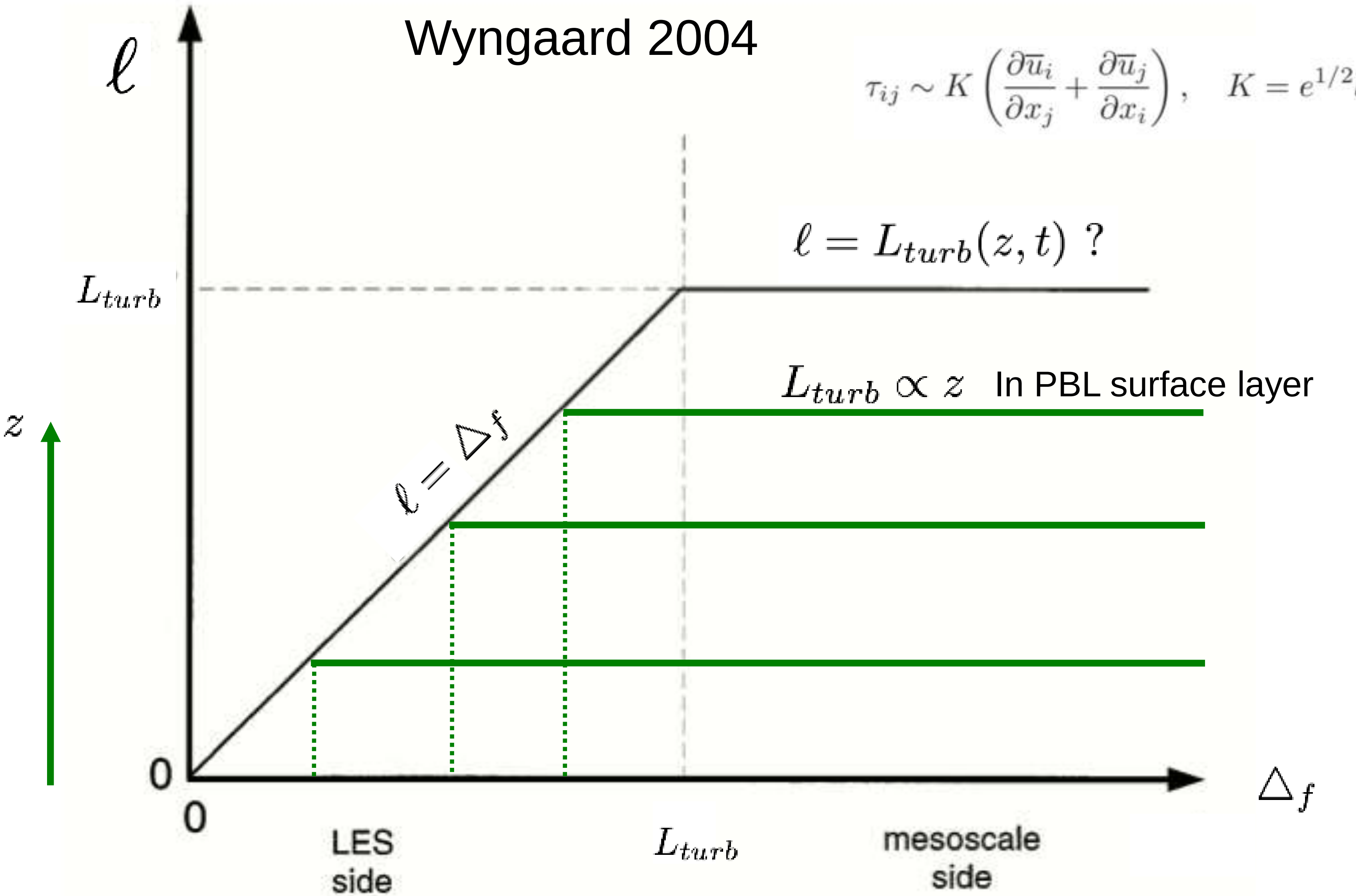
RESOLVED FLOW IN LES LIMIT



Simple eddy viscosity closure (model for deviatoric stress)

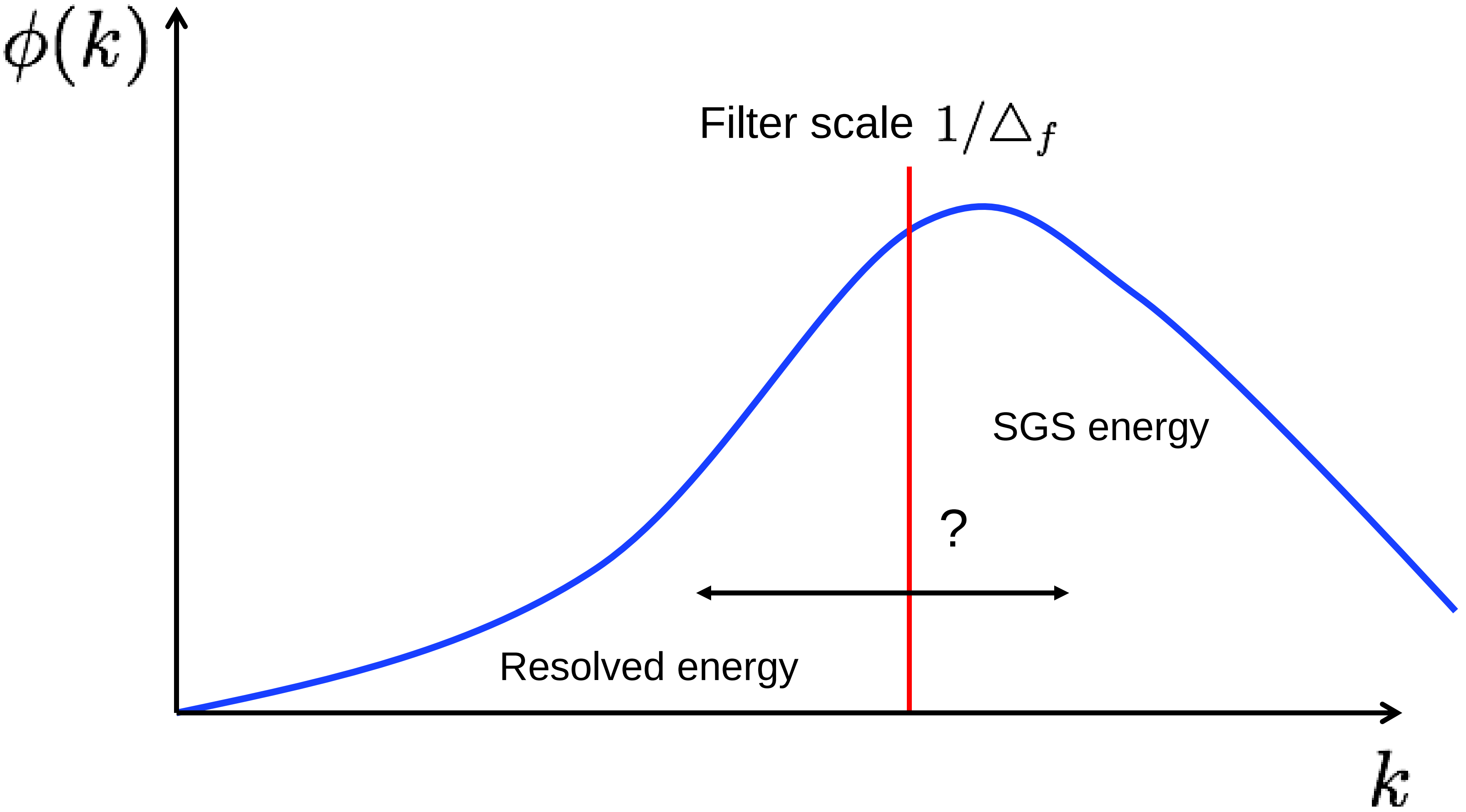
$$\tau_{ij} \sim K \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right), \quad K = e^{1/2} l$$

In LES limit: $\triangle_f < L_{turb}$



Suggested behavior of the length scale ℓ of the unresolved turbulence as function of Δ_f for a K-model

HOW DOES ENERGY FLOW BETWEEN RESOLVED AND SGS FIELDS?



HOW DOES ENERGY FLOW BETWEEN RESOLVED AND SGS FIELDS?

- Need to build energy transport equations:

Resolved energy

$$
\begin{aligned}
\overline{u}_i \frac{\partial \overline{u}_i}{\partial t} &= \ldots - \overline{u}_i \frac{\partial \tau_{ij}}{\partial x_j} \\
&= \ldots \tau_{ij} \frac{1}{2} \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) \\
&= \ldots \tau_{ij} S_{ij}
\end{aligned}
$$

Energy transfer term

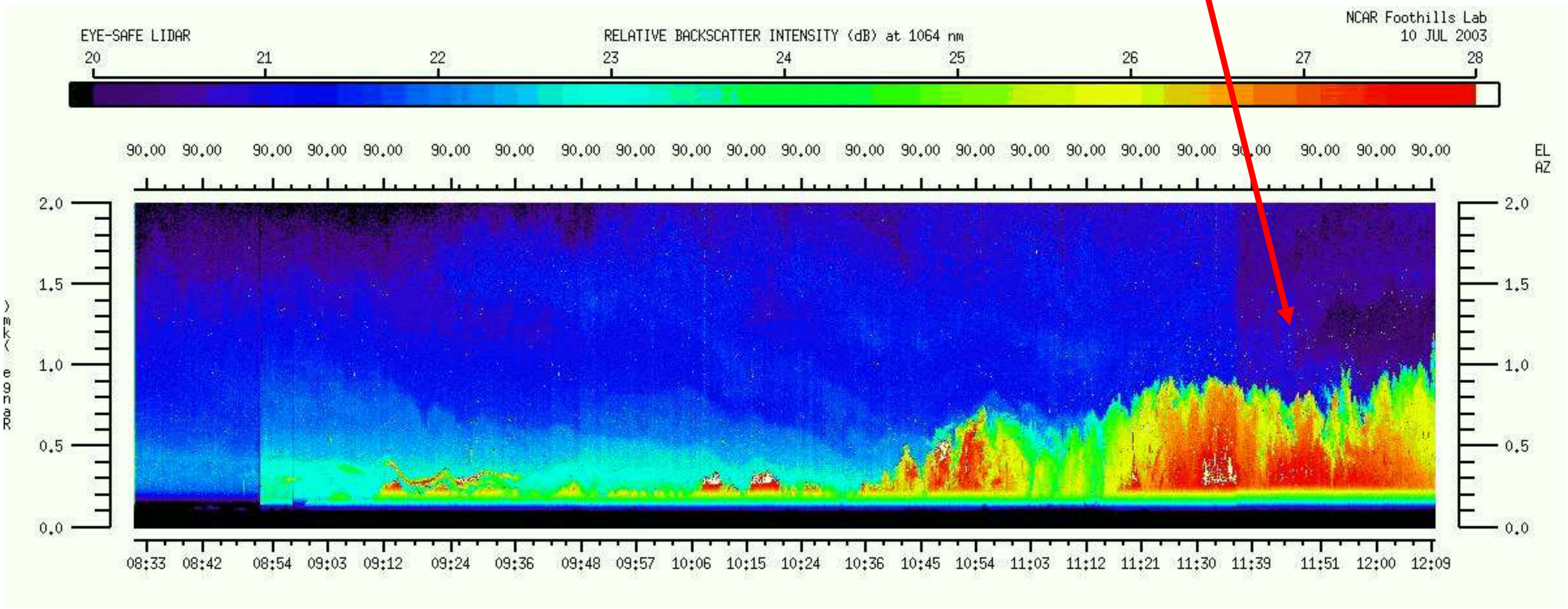
SGS energy

$$
\overline{u_i \frac{\partial u_i}{\partial t}} - \overline{u}_i \frac{\partial \overline{u}_i}{\partial t} = \ldots - \tau_{ij} S_{ij} - \epsilon
$$

Molecular dissipation

DO HIGH-Re ROUGH-WALL LES STATISTICS CONVERGE WITH MESH REFINEMENT?

Daytime Convective BL



8 am **Local time** **Noon**

Courtesy Shane Mayor

$$Re_{\mathcal{L}} = \mathcal{UL}/\nu = 10^8$$

LES EQUATIONS FOR DRY ATMOSPHERIC PBL

Momentum

$$\frac{D\overline{\mathbf{u}}}{Dt} = -\mathbf{f}\times\overline{\mathbf{u}} - \nabla\pi + \hat{\mathbf{z}}g\frac{\overline{\theta}}{\theta_*} - \nabla\cdot\mathbf{T}$$

Scalar

$$\frac{D\overline{b}}{Dt} = -\nabla\cdot\mathbf{B}$$

TKE

$$\frac{De}{Dt} = -\mathbf{T}:\mathbf{S} + \mathbf{B}\cdot\hat{\mathbf{z}} - \mathcal{E} + \nabla\cdot(2\nu_t\nabla e)$$

Subgrid-scale momentum and scalar fluxes

$$\begin{aligned}\mathbf{T} &= \overline{u_i u_j} - \overline{u_i}\,\overline{u_j}\\ \mathbf{B} &= \overline{u_i b} - \overline{u_i}\,\overline{b}\end{aligned}$$

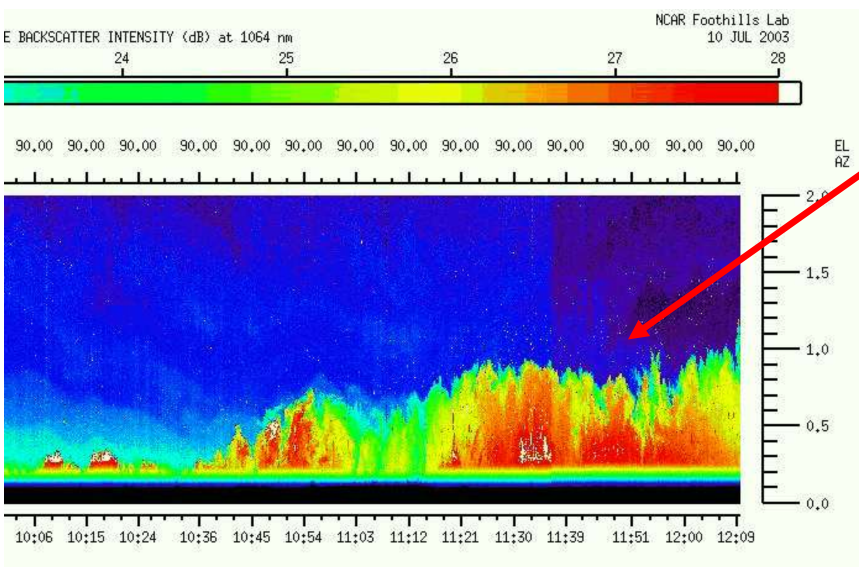
Incompressible Boussinesq flow

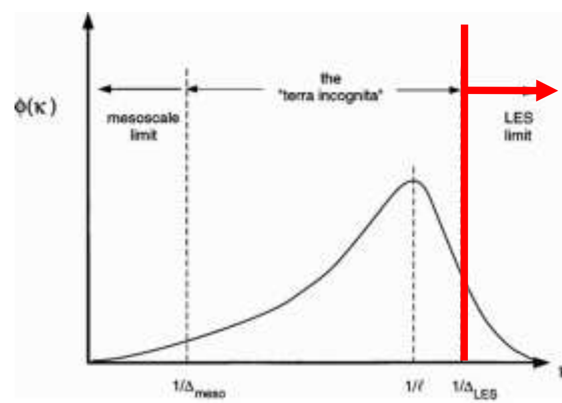
$$\nabla\cdot\overline{\mathbf{u}} = 0 \implies \nabla^2\pi = s$$

DO LES STATISTICS CONVERGE WITH MESH REFINEMENT?

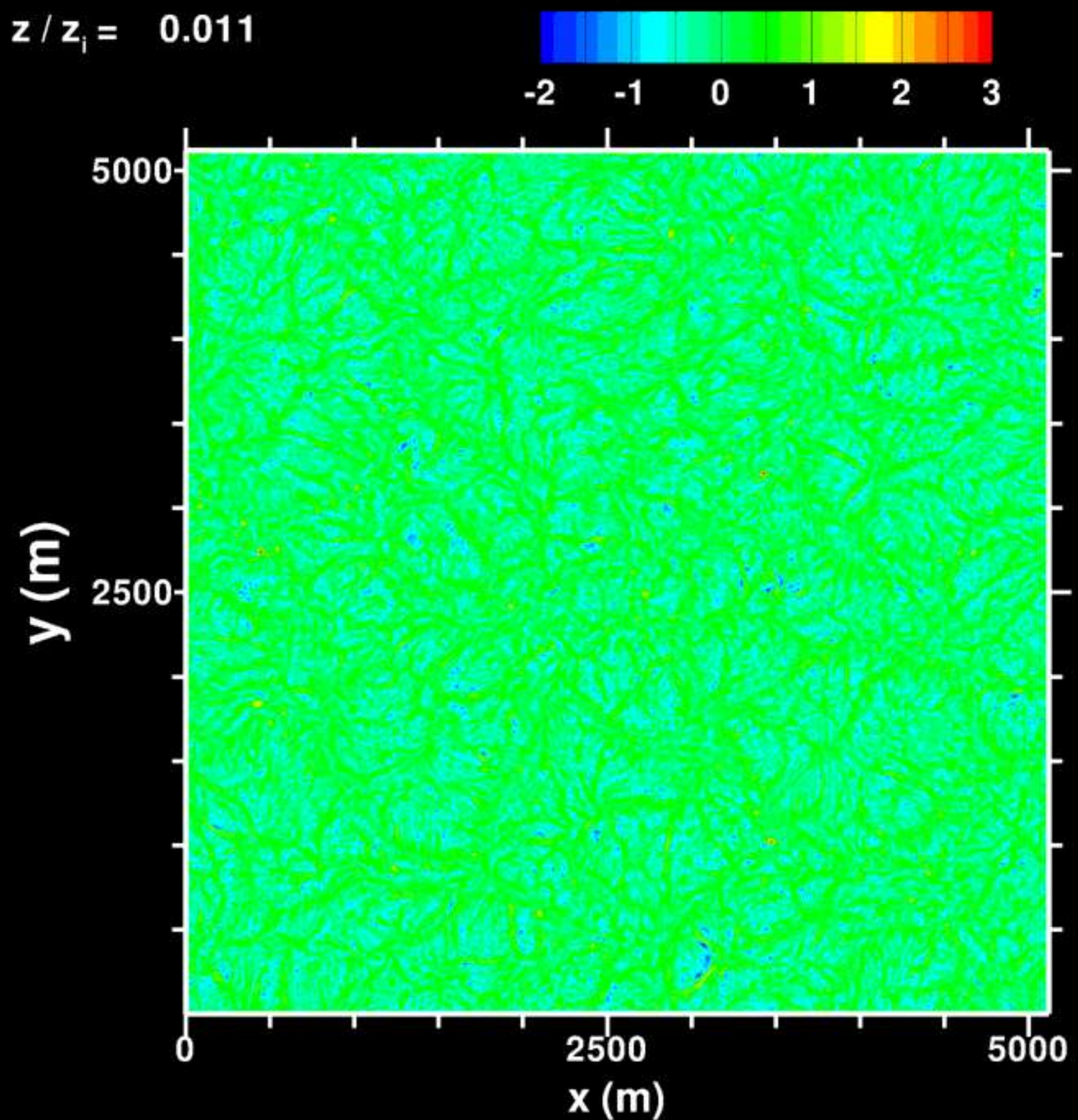
$$(X_L, Y_L, Z_L) = (5, 5, 2)\ km$$

Gridpoints	$(\triangle x, \triangle y, \triangle z)$ (m)	$z_i/\triangle_f$	$z_i/C_s\triangle_f$
32^3	(160,160,64)	7.2	40
64^3	(80,80,32)	14.5	80
128^3	(40,40,16)	28.5	158
256^3	(20,20,8)	56.6	314
512^3	(10,10,4)	113.3	630
1024^3	(5,5,2)	229.0	1272





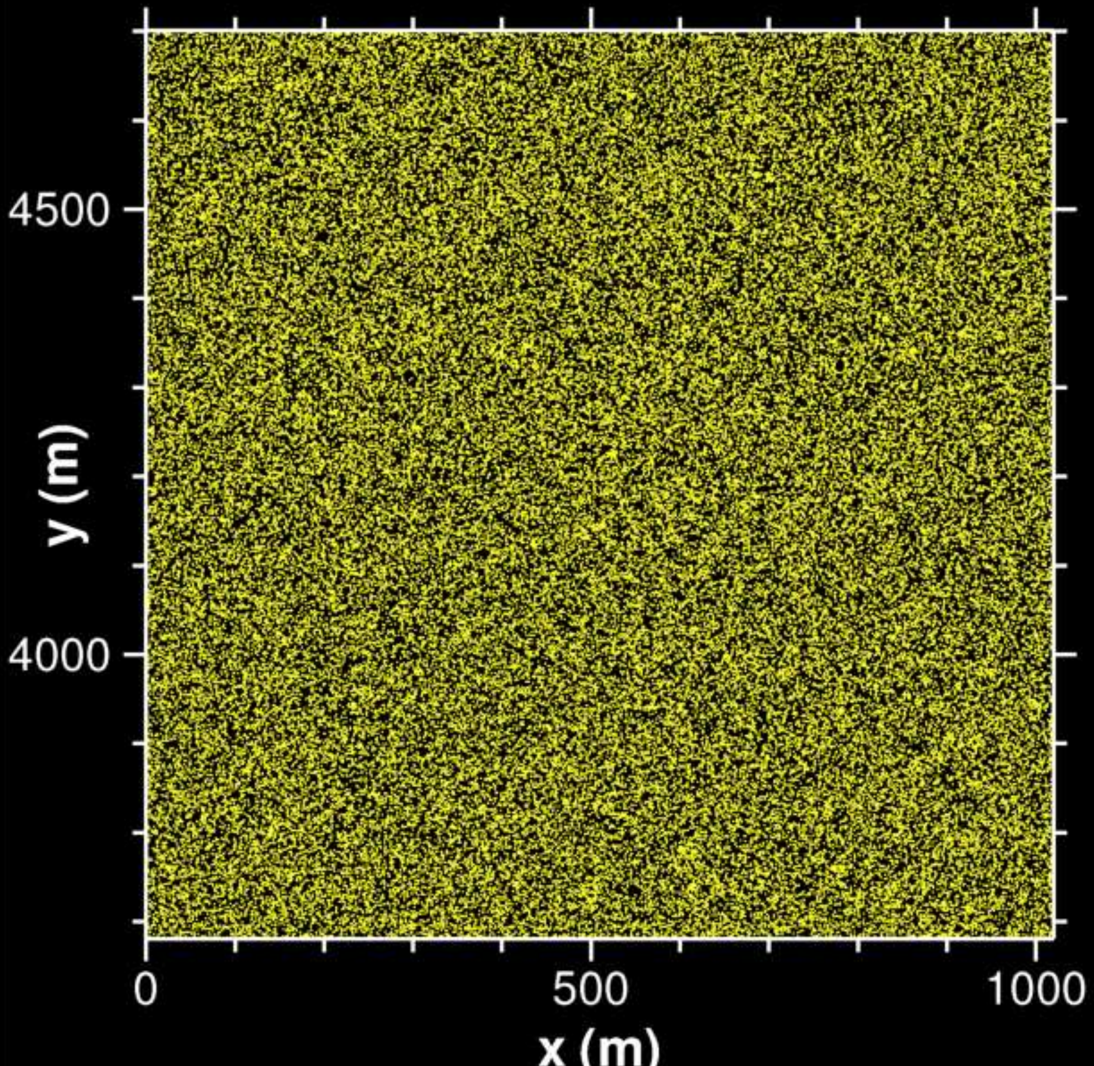
FREE CONVECTION 512^3 W-FIELD

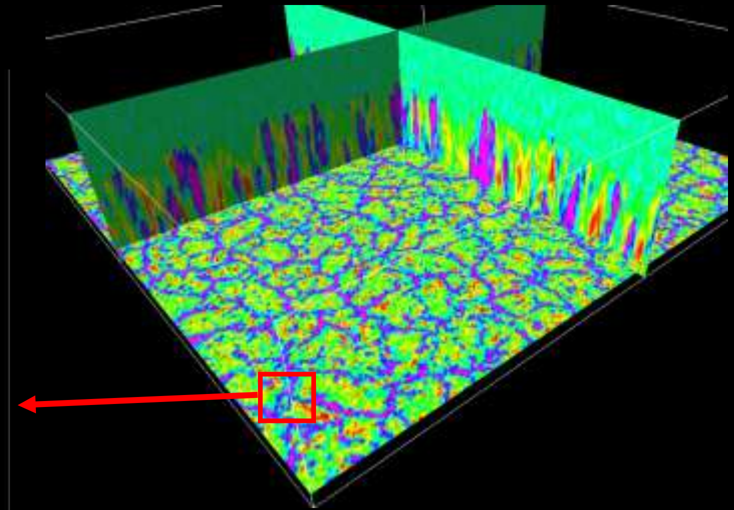


$$\Delta x = \Delta y = 10\ m$$

$$\Delta z = 4\ m$$

LES OF CONVECTIVE PBL, 4096 CPUS, 1024^3 GRIDPOINTS



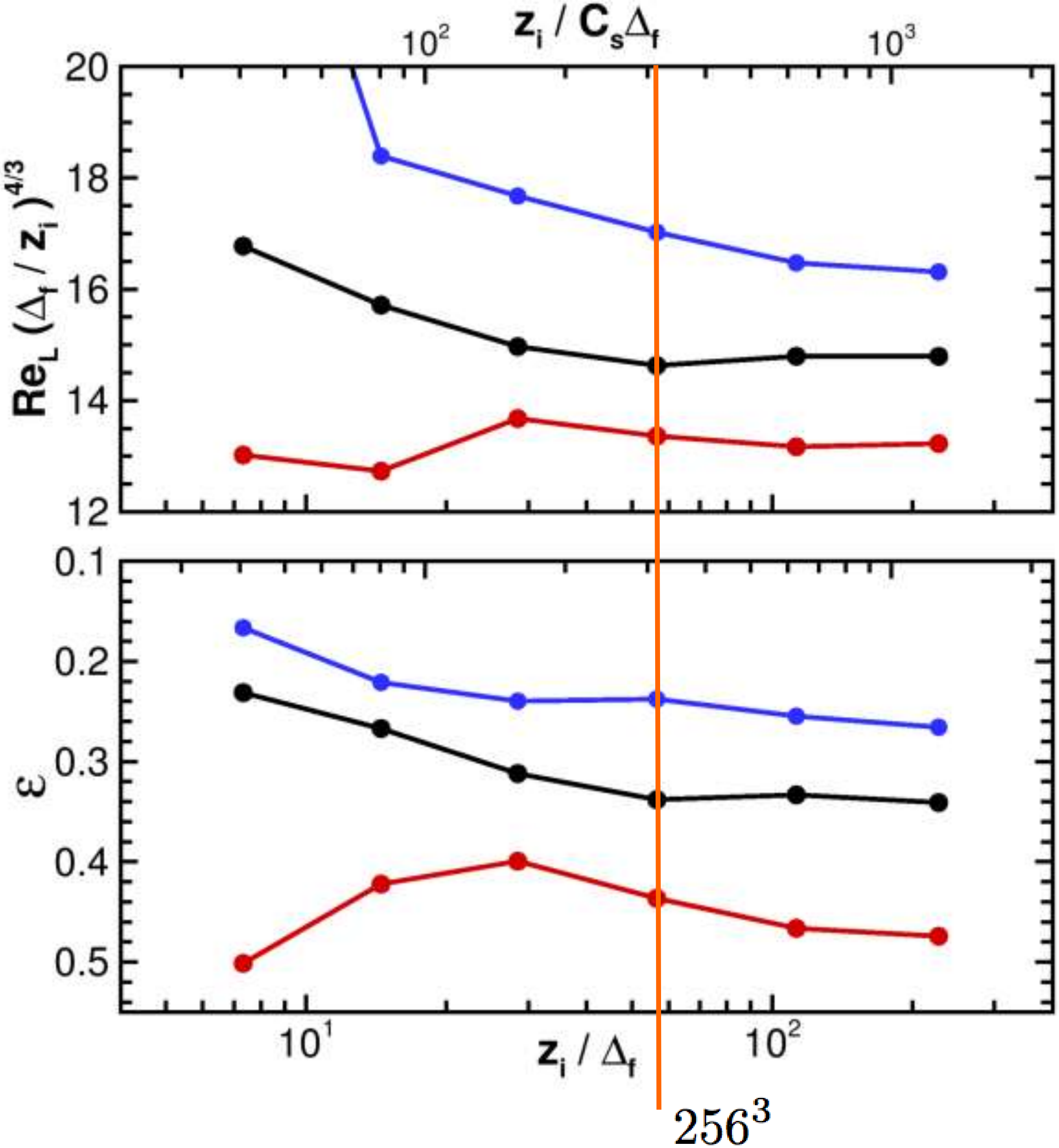


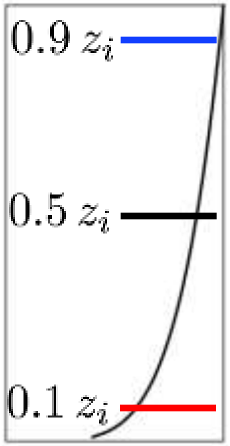
DUST DEVILS in TALAMAKAN DESERT OF CHINA
Courtesy of Don Lenschow NCAR

DO LES STATISTICS CONVERGE WITH MESH REFINEMENT?

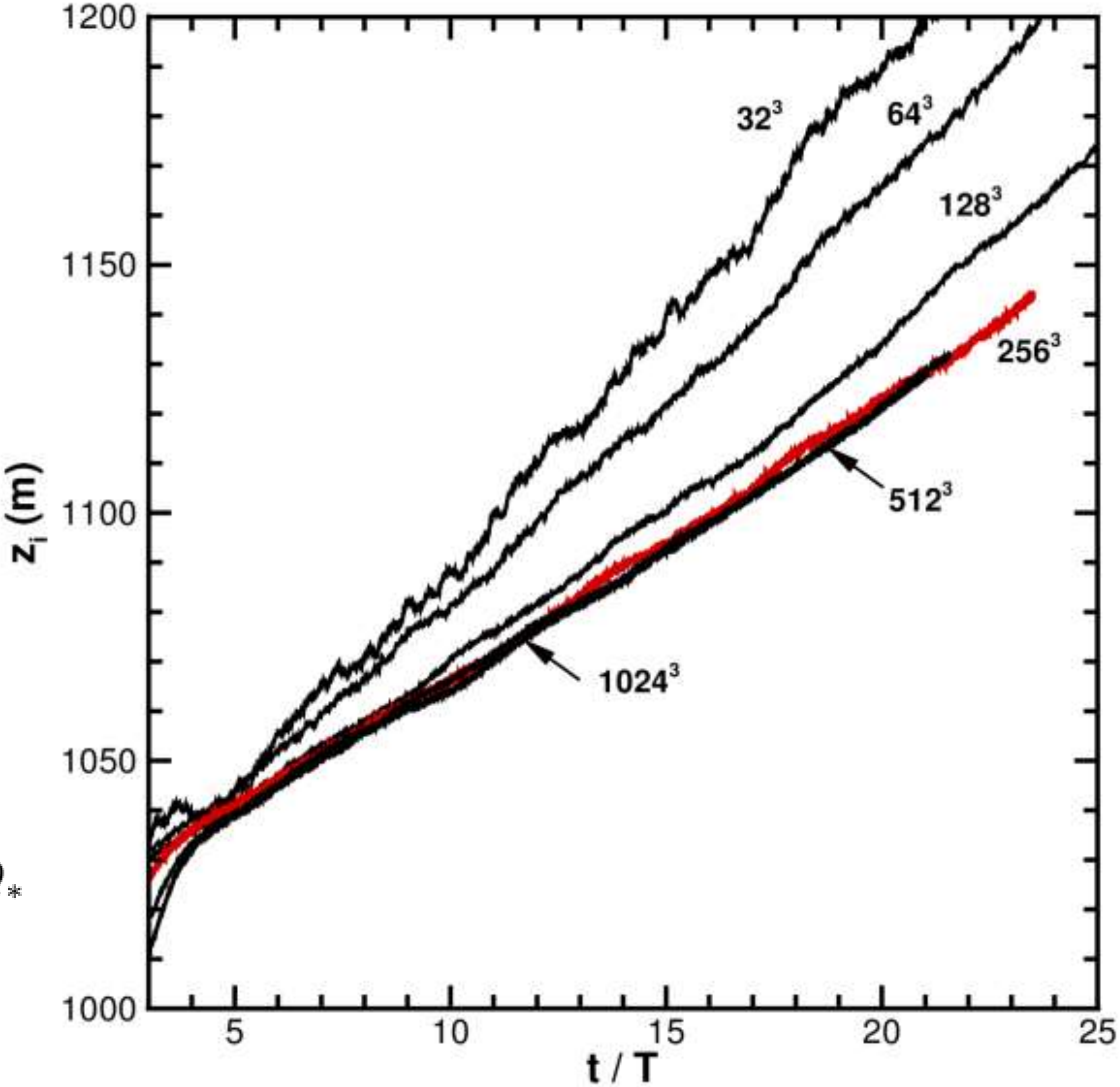
Large eddy
Reynolds number

Dissipation





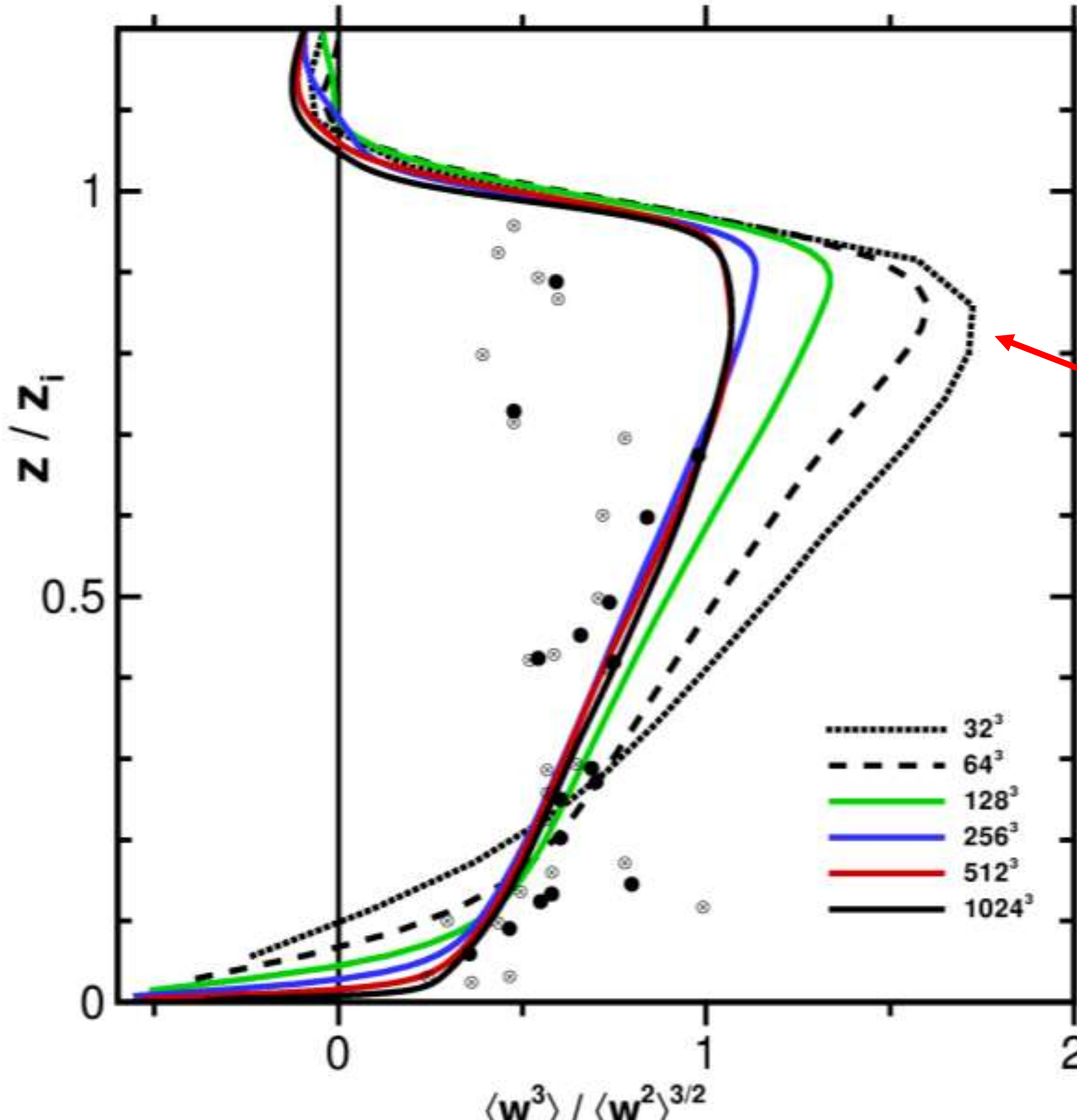
PBL HEIGHT z_i FOR VARYING MESHES



$\langle w\theta \rangle_{min} \sim -0.2Q_*$

Entrainment rate $w_e = dz_i/dt$ decreases with increasing mesh resolution

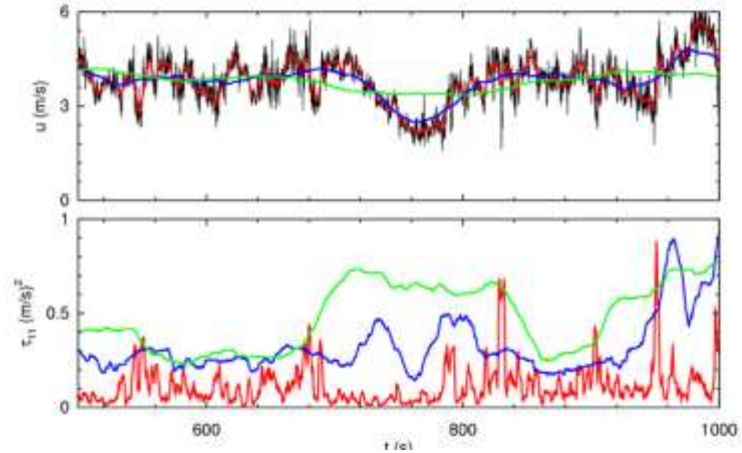
IMPACT OF GRID RESOLUTION ON SKEWNESS



No SGS skewness!

Can we use targeted observations to provide insight as to the nature of SGS motions in high Re PBLs?

HIGH REYNOLDS NUMBER OBSERVATIONS AND LES



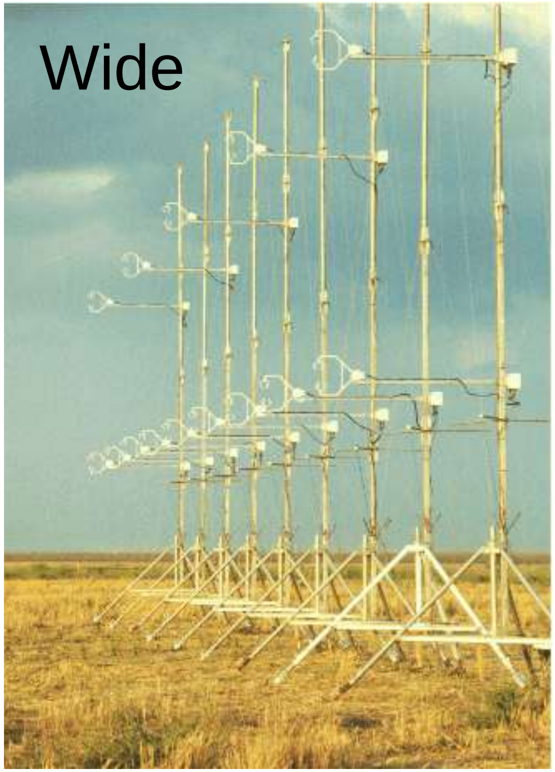
- **SINGLE-POINT MEASUREMENTS**
  - Cannot be used directly to improve LES
- **MULTI-POINT MEASUREMENTS**
  - Span a range of filter widths, $e.g.$, $\mathcal{O}$(m) to $\mathcal{O}$(100m)
  - Ideally 3-D, time varying "volume" of turbulence and scalars in canonical flows with shear, stratification, near boundaries, ...
  - Horizontal Array Turbulence Study field campaigns, HATS (2000), OHATS (2004), CHATS (2007), AHATS (2008)

HATS CONFIGURATIONS

$\sim 36\ cases$

$-1.2 < z/L < 1.6$

$0.15 < \Lambda_w/\triangle_f < 15$



RATIONALE FOR EXPERIMENTAL DESIGN

- Allows *spatial* filtering of flow field and decomposition into resolved and subfilter scale velocities $(\overline{U_i}, u_i)$:

$$U_i = \overline{U_i} + u_i \equiv \int U(x'_j) G(x_i, x'_j) dx'_j + u_i$$

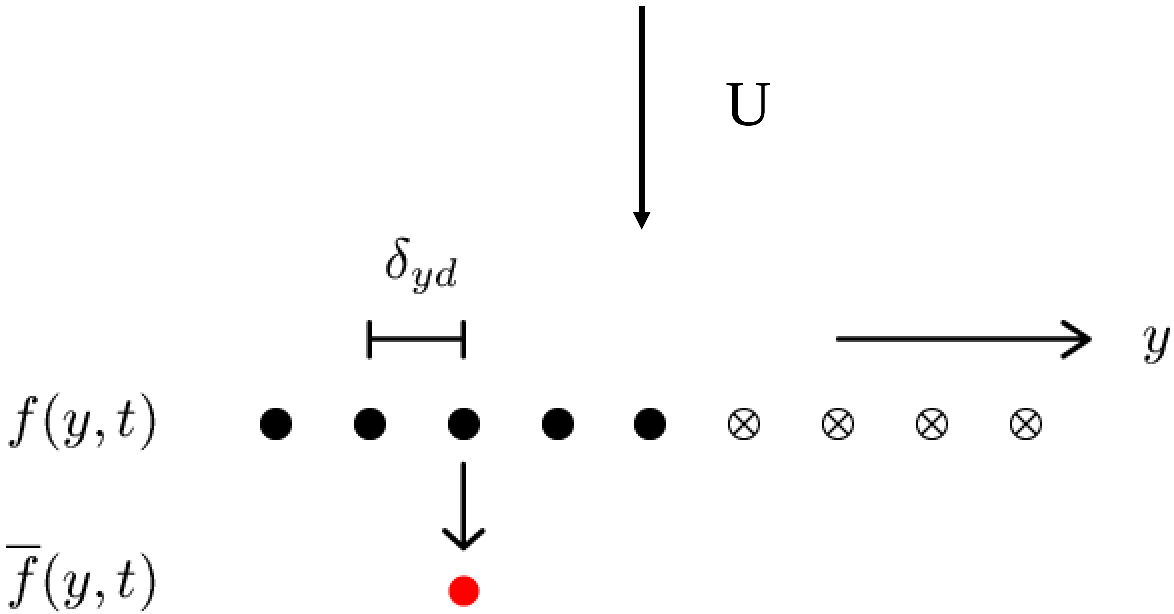
- Allows construction of SFS fluxes:

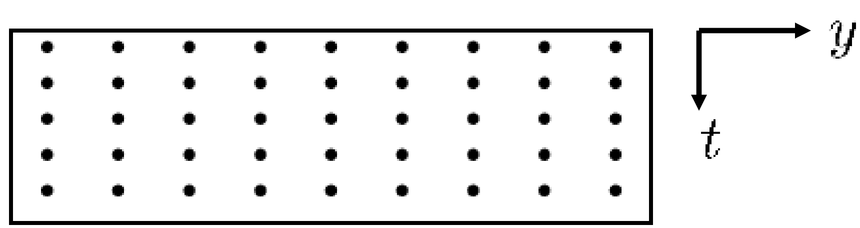
$$\mathcal{T}_{ij} = \overline{U_i U_j} - \overline{U_i}\ \overline{U_j}$$

- Allows measurement of resolved gradients $\partial\overline{U_i}/\partial x$, $\partial\overline{U_i}/\partial y$ and $\partial\overline{U_i}/\partial z$

- Allows expansion of SFS fluxes $\mathcal{T}_{ij}$ into Leonard, Cross, and Reynolds terms which requires *double* spatial filtering, *e.g.*, $\overline{\overline{U_i} u_j}$

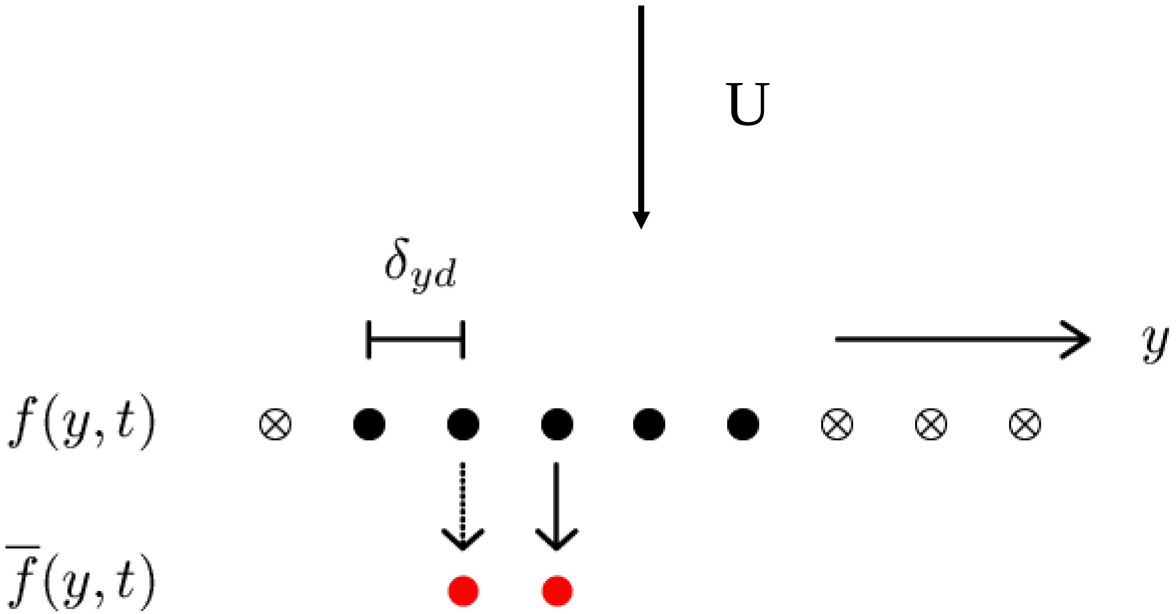
AN EXAMPLE OF LATERAL (Y) FILTERING





``2D plane of turbulence”

AN EXAMPLE OF LATERAL (Y) FILTERING



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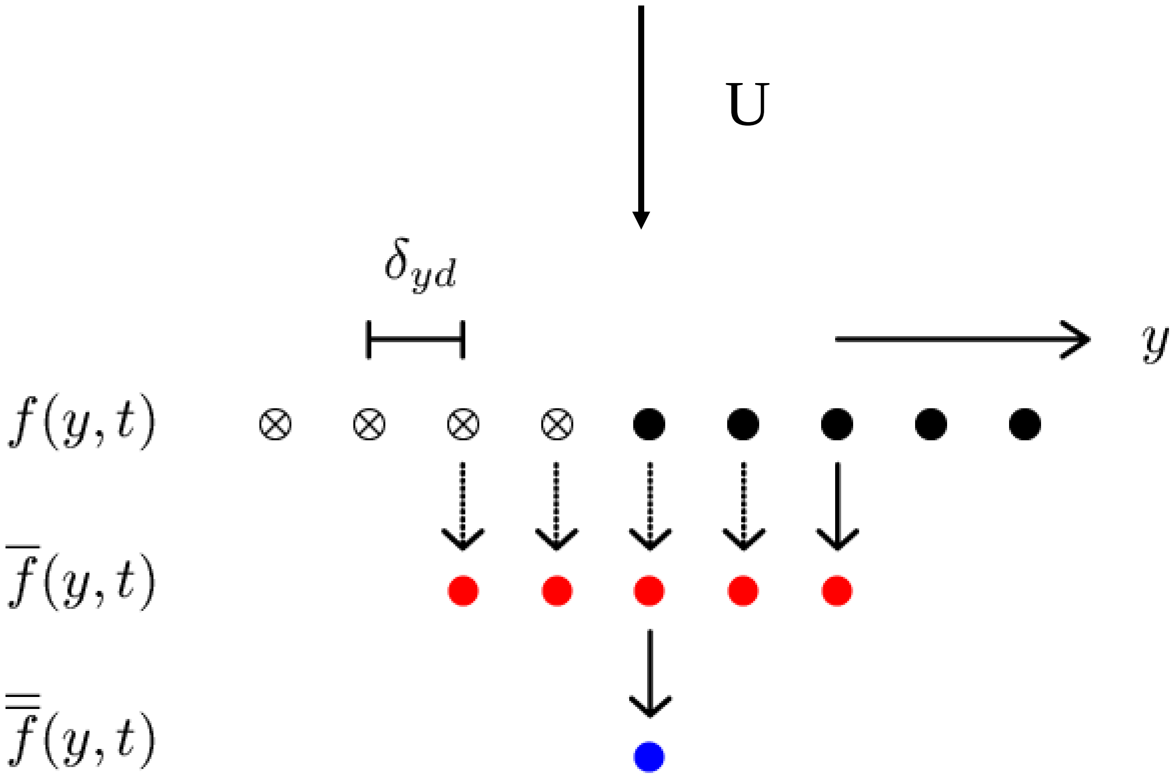
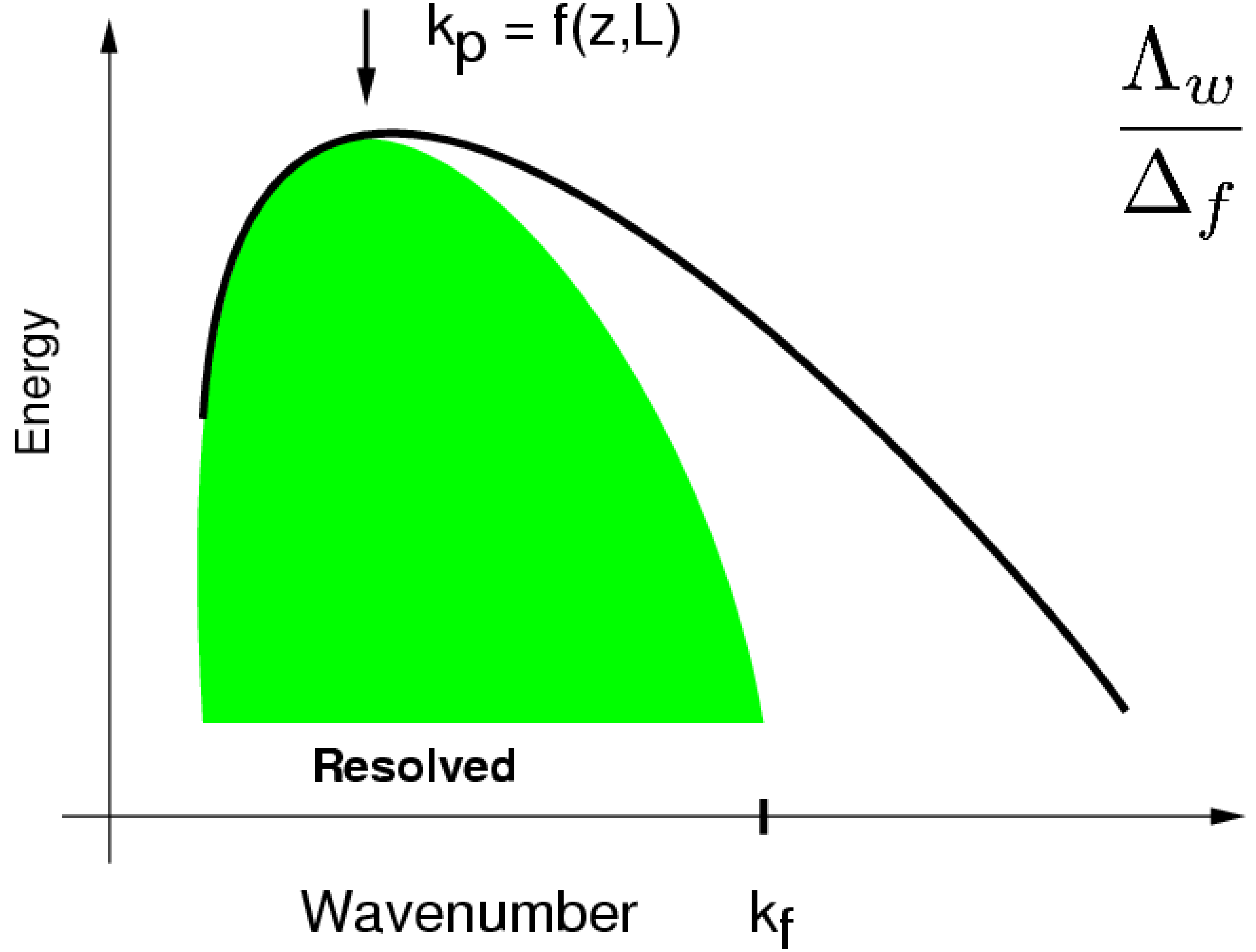
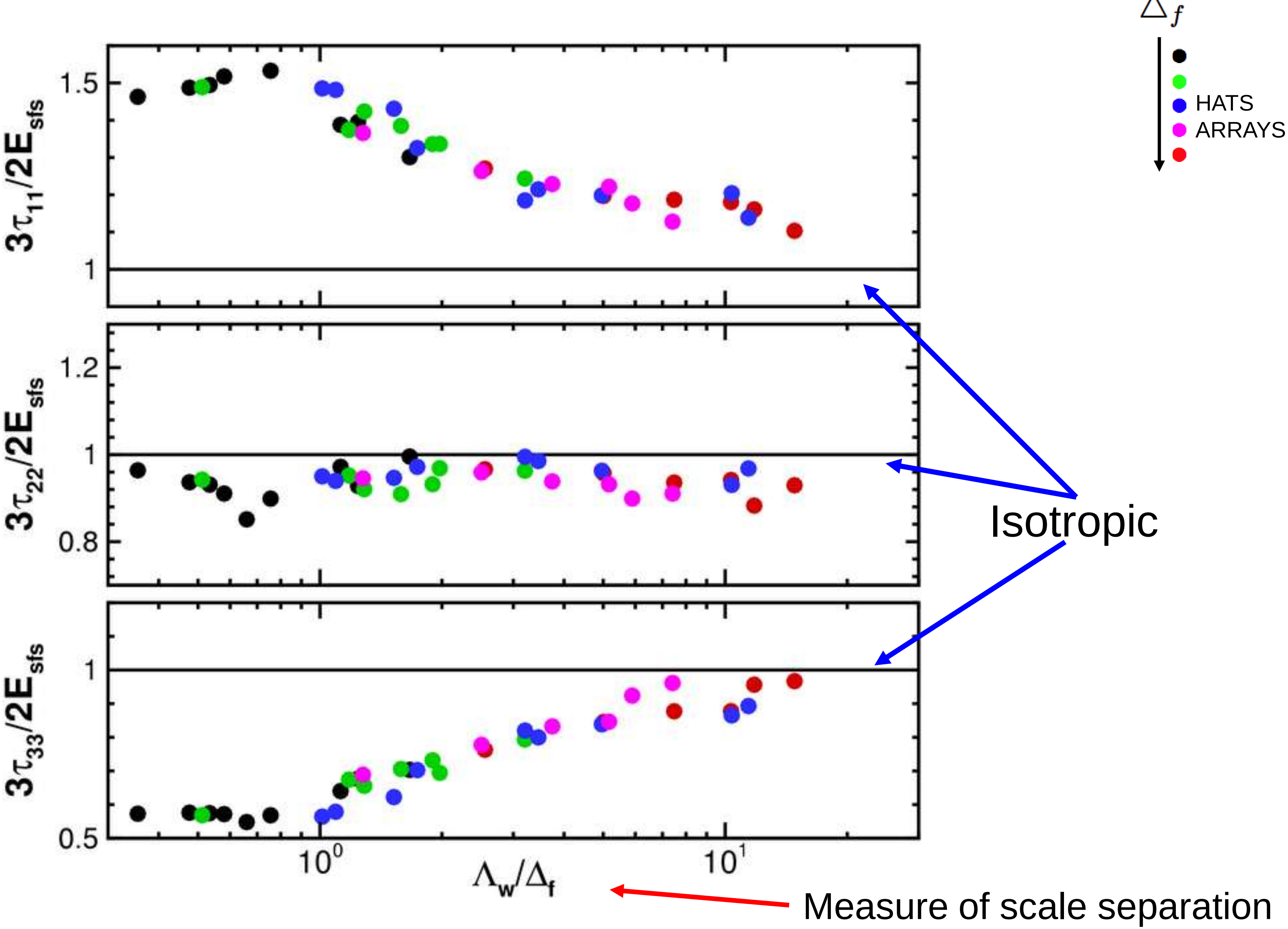


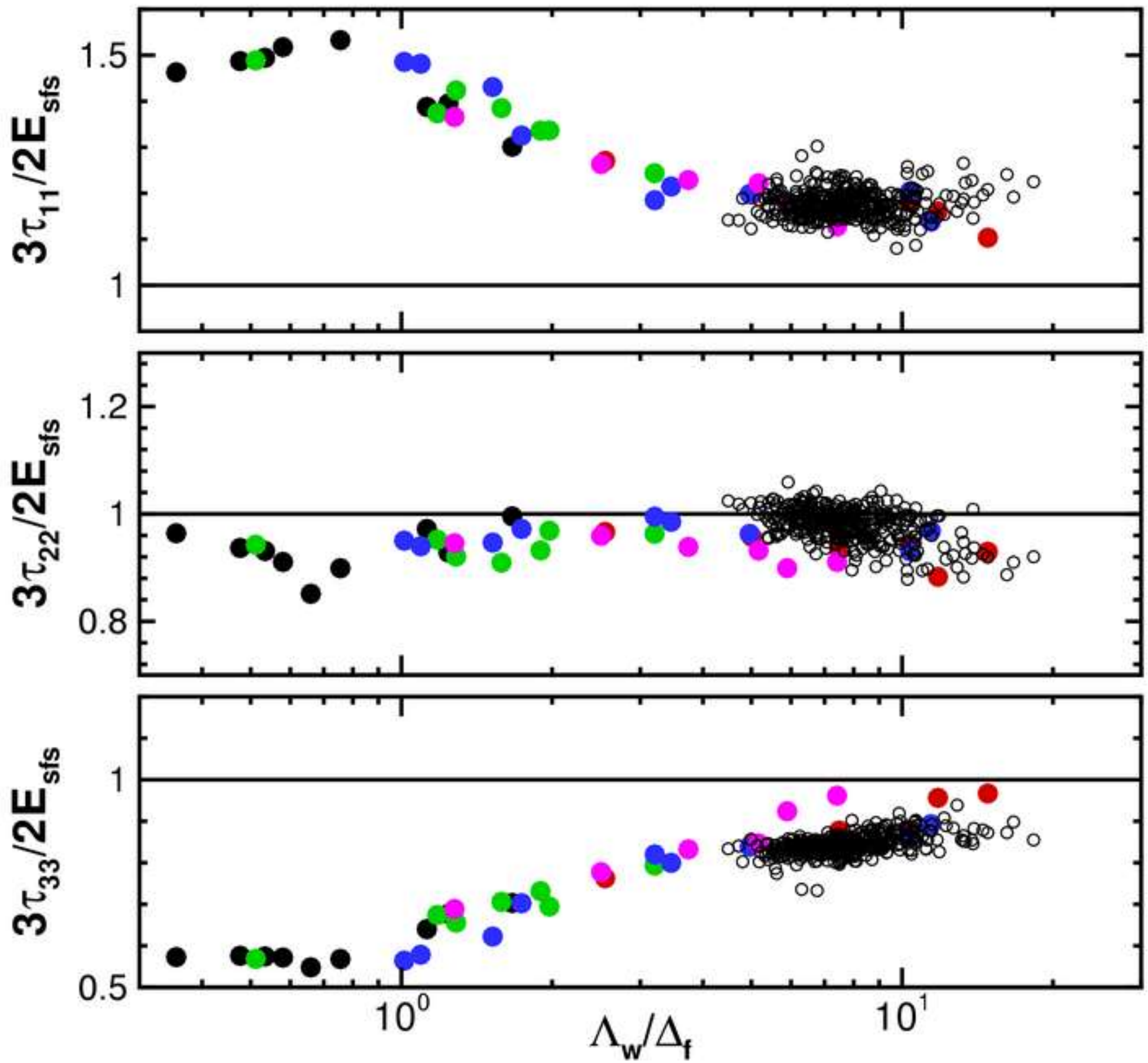
SPECTRAL PEAK AND FILTER CUTOFF WAVENUMBERS



SFS VELOCITY VARIANCES $\tau_{ij} = \overline{u_i u_j} - \overline{u}_i\,\overline{u}_j$



SFS VELOCITY VARIANCES



RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- What are the parent equations for the Smagorinsky model?

RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- **What are the parent equations for the Smagorinsky model?**
  - Lilly (1967), Deardorff (1973), Wyngaard (2004), Hatlee & Wyngaard (2007) *Germano (1992)*

$$
\begin{aligned}
\frac{D\tau_{ij}}{Dt} \quad = \quad & \frac{2}{3}e\left(\frac{\partial\overline{u}_i}{\partial x_j}+\frac{\partial\overline{u}_j}{\partial x_i}\right) \\
& -\left[\tau_{ik}\frac{\partial\overline{u}_j}{\partial x_k}+\tau_{jk}\frac{\partial\overline{u}_i}{\partial x_k}-\frac{1}{3}\delta_{ij}\tau_{kl}\left(\frac{\partial\overline{u}_k}{\partial x_l}+\frac{\partial\overline{u}_l}{\partial x_k}\right)\right] \\
& -\frac{1}{\rho}\left[\overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}-\overline{p}\left(\frac{\partial\overline{u}_i}{\partial x_j}+\frac{\partial\overline{u}_j}{\partial x_i}\right)\right] \\
& + \text{transport} + \text{buoyancy production}
\end{aligned}
$$

Isotropic production

Pressure destruction

Anisotropic deviatoric production

RATE EQUATIONS FOR SUBGRID DEVIATORIC STRESS

- **What are the parent equations for the Smagorinsky model?**
  - Lilly (1967), Deardorff (1973), Wyngaard (2004), Hatlee & Wyngaard (2007)

$$
\begin{aligned}
\cancelto{0}{\frac{D\tau_{ij}}{Dt}} = \; & \frac{2}{3} e \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) \\
& \cancelto{0}{- \left[\tau_{ik} \frac{\partial \overline{u}_j}{\partial x_k} + \tau_{jk} \frac{\partial \overline{u}_i}{\partial x_k} - \frac{1}{3} \delta_{ij} \tau_{kl} \left(\frac{\partial \overline{u}_k}{\partial x_l} + \frac{\partial \overline{u}_l}{\partial x_k} \right) \right]} \\
& - \frac{1}{\rho} \left[\overline{p \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)} - \overline{p} \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) \right] \\
& + \cancelto{0}{\text{transport}} + \cancelto{0}{\text{buoyancy production}}
\end{aligned}
$$

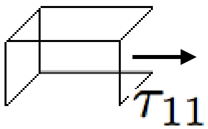
Rotta model

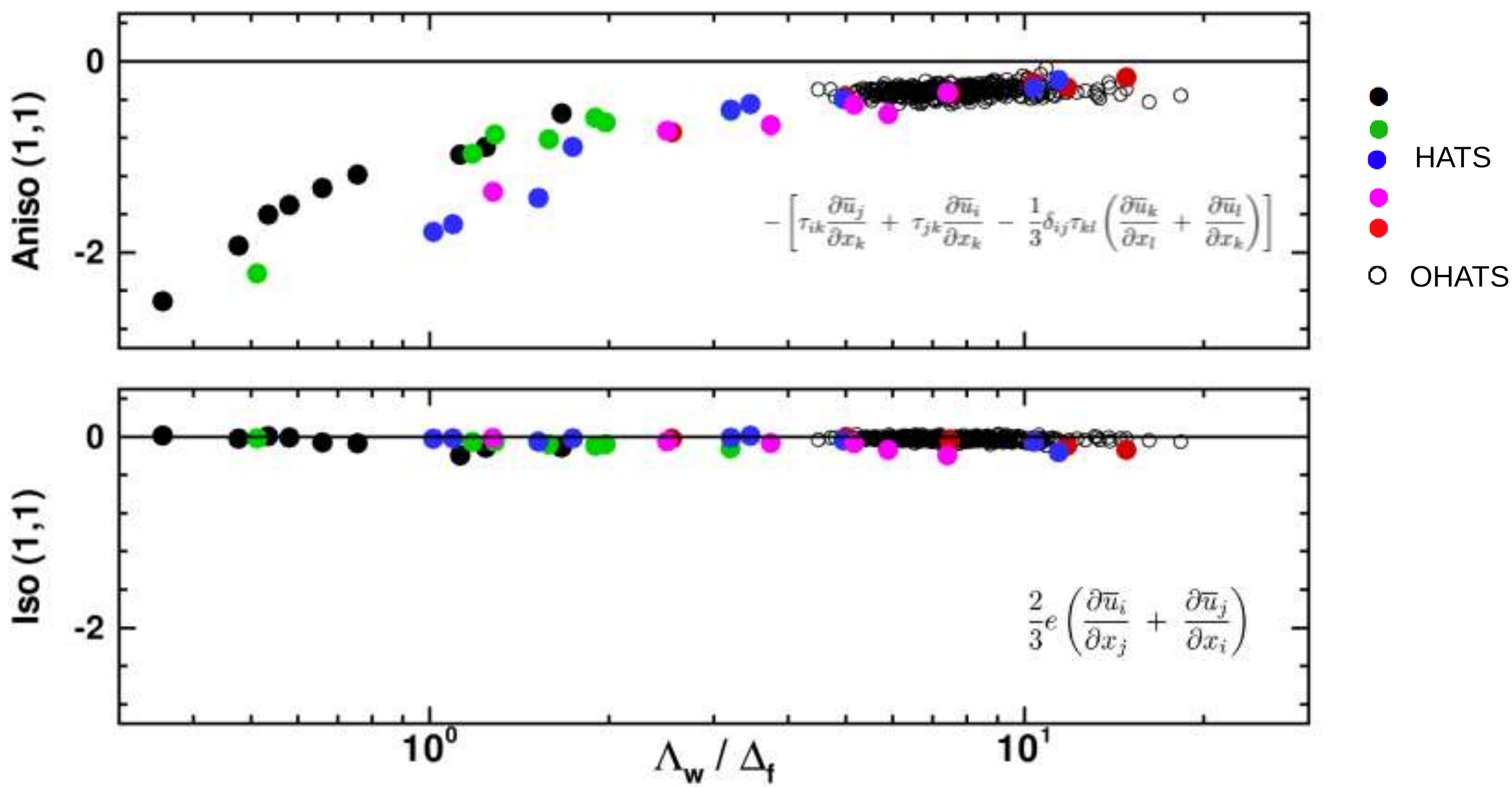
$$
\boxed{\frac{\tau_{ij}}{T} = \frac{2}{3} e \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right)}
$$

Time scale

$$
T = c \frac{\Delta_f}{\sqrt{e}}
$$

PRODUCTION OF SUBFILTER SCALE FLUX τ_{11}



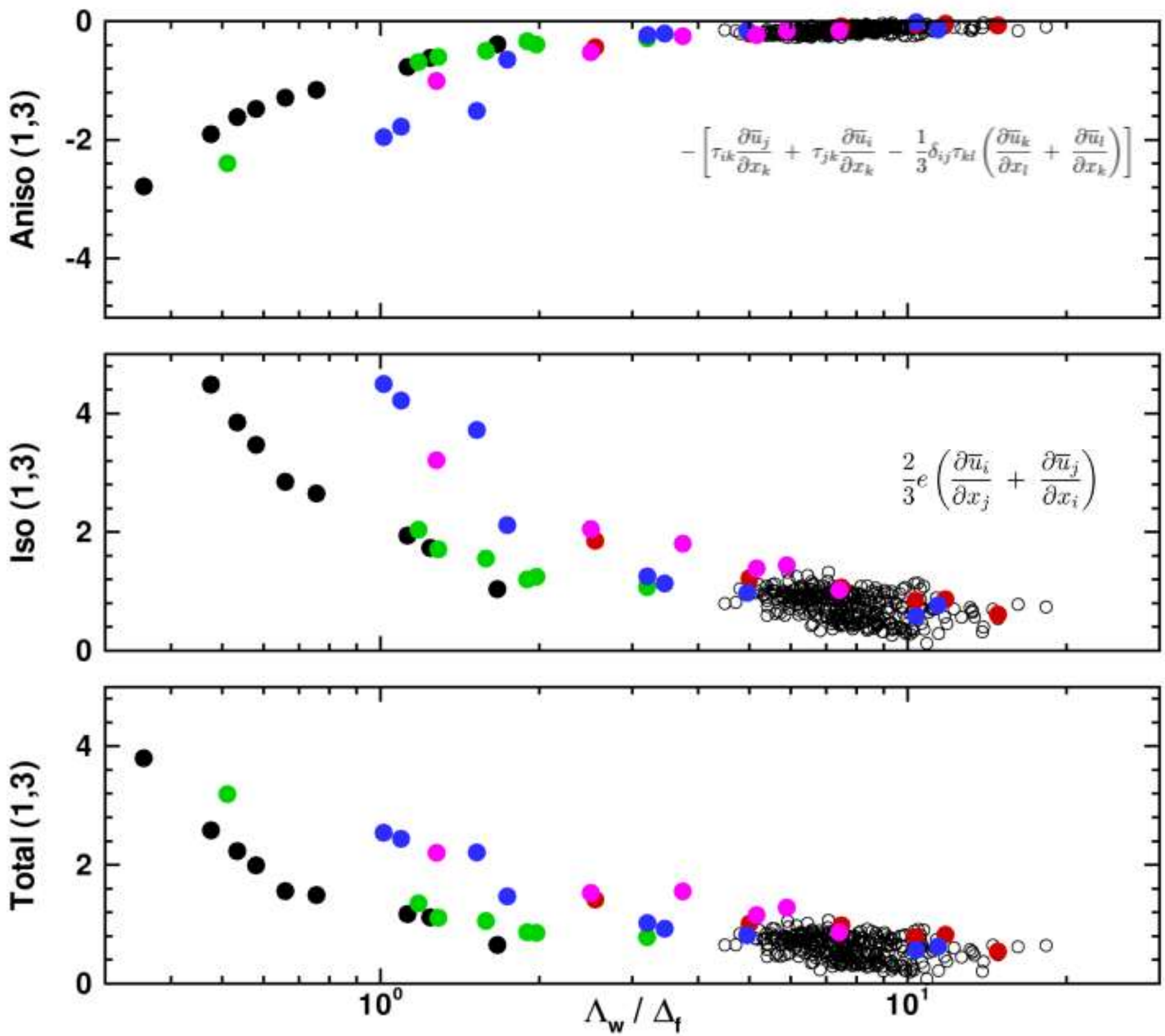


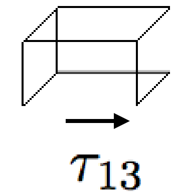
RANS

Ideal LES

Measure of scale separation

PRODUCTION OF SUBFILTER SCALE FLUX τ_{13}



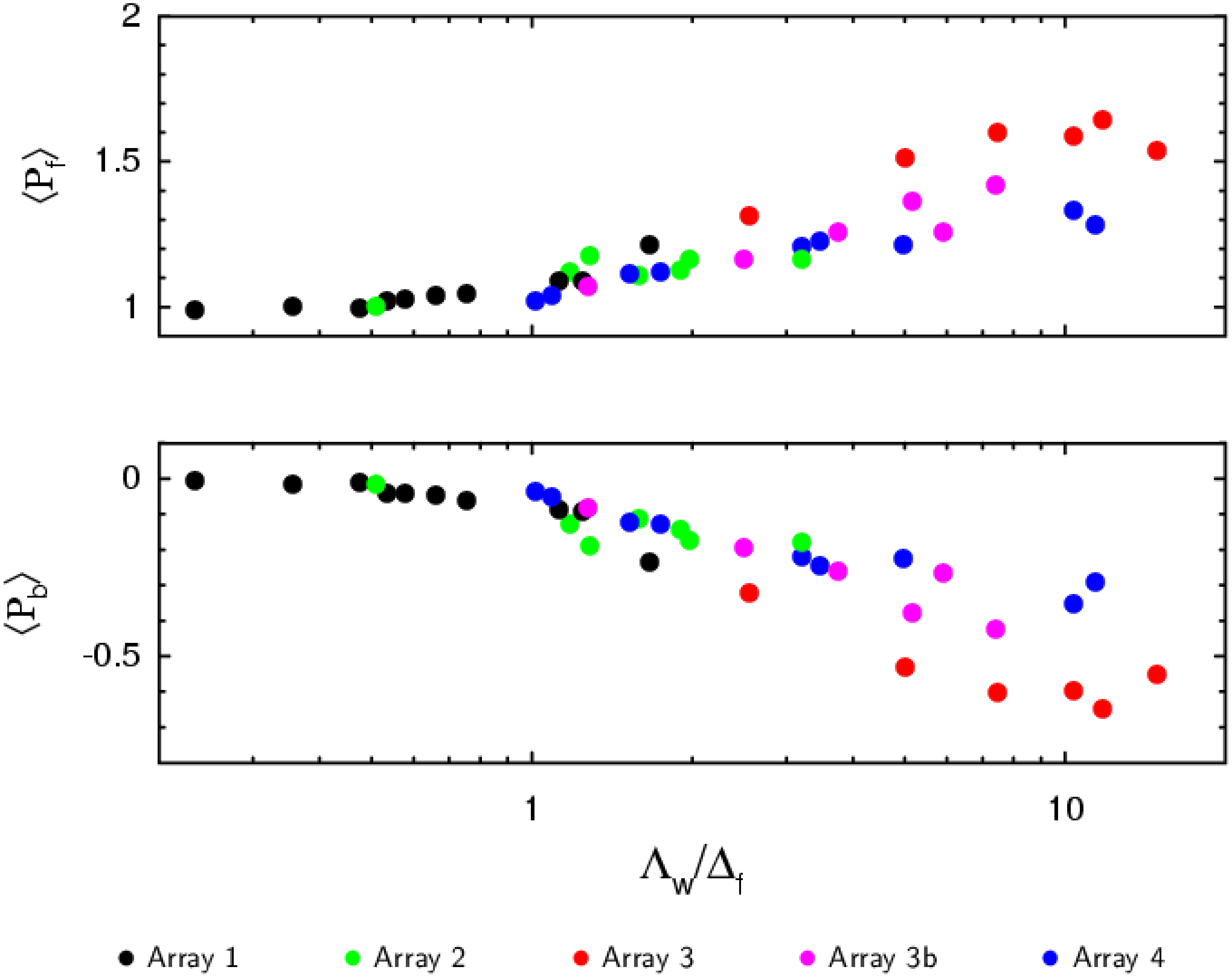


Decomposition of SFS Production into Forwardscatter and Backscatter

$$P = -\mathcal{T}_{ij} S_{ij} \qquad P_f = (P + |P|)/2 \qquad P_b = (P - |P|)/2$$

Decomposition of SFS Production into Forwardscatter and Backscatter

$$P = -\mathcal{T}_{ij} S_{ij} \qquad P_f = (P + |P|)/2 \qquad P_b = (P - |P|)/2$$



SUMMARY

- Atmospheric and oceanic boundary-layer dynamics are unique compared to flat-wall boundary layers because of surface waves
  - Winds, currents, drag, variances, dissipation, entrainment, ...
- Carefully crafted high Re LES neatly exposes the interactions between winds-waves, waves-currents
- LES solutions for means and second-order moments converge with mesh refinement provided $z_i/C_s\triangle_f > 300$ (for daytime convective BL)
  - Solutions exhibit approximate Reynolds-number similarity
  - Entrainment rate decreases with increasing mesh resolution
  - Vertical velocity skewness is an indicator of mesh sensitivity
- Measurements of subgrid-scale variables show SGS (eddy viscosity) parameterizations used in LES are inadequate when the ratio $\Lambda/\triangle_f \sim \mathcal{O}(1)$ or less
  - Anisotropic production of scalar and momentum flux in surface layers is important
- Yes! LES is exceedingly useful, but can be improved

AN LES PROPOSAL TO STUDY TERRA-INCOGNITA DYNAMICS

- Run big with conventional LES! Processor counts $\sim \mathcal{O}(10^4)$ or more
- Design LES with a minimum (tolerable) resolution say $L_{turb}/\triangle_f \sim 20$ then choose N_{points} large to capture key large-scale processes
- To be interesting N_{points} needs to be large enough to capture $L_{turb}/\triangle_f = 1$
- *A-prior* filtering of LES databases at Terra-Incognita resolutions, say $L_{turb}/\delta_f = [10, 1, 0.5, 0.1]$ where $\delta_f > \triangle_f$
- Others are doing this in the atmosphere and ocean, $e.g.$, [*Moeng and Arakawa*(2012)], Baylor Fox-Kemper $et\ al.$(2012) on *Yellowstone*
- Dissect the *Horizontal Array Turbulence Studies* carried out in the atmospheric surface layer