Helicity and Large Scale Dynamos; Lessons From Mean Field Theory and Astrophysical Implications

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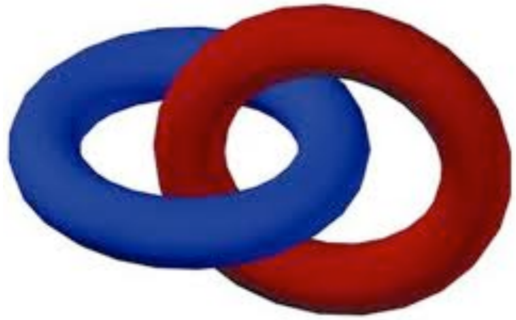
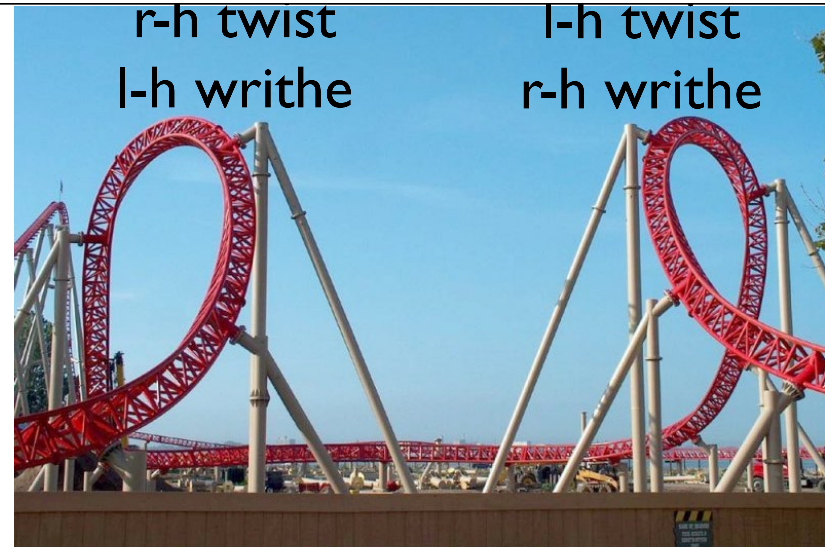
- 21st century theory based on incorporating mag. hel conservation has predictive power
  - quenching and role of fluxes
  - rethinking the visualization of MHD
  - coupling of stellar interiors with their coronae; fluxes; observational implications
- resilience of helical fields to turbulent diffusion
- what is role of reconnection for large scale dynamos?
- potential connections to MRI systems and non-locality of transport in
  - coupling of disks with their coronae and jets
- Lessons for LES: use principles from MFT to inform SGS models and e.g. increase R_m
  - desired to improve scale separation and alleviate ambiguities of Rm dependence
  - multi-scale, multi-physics: dynamos-> coronae -> winds, jets
  - replacing mean field transport coefficients with integral kernels/filtering (nonlocality)

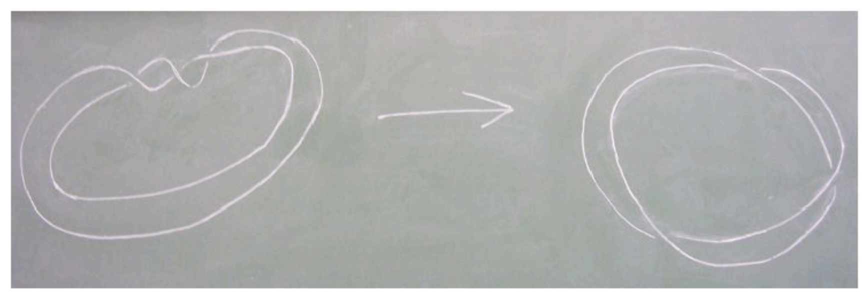
Magnetic Helicity



$$K \equiv \int \mathbf{A} \cdot \mathbf{B} \, dV = 2\phi\psi$$



- measures linkage, twist, and writhe of flux bundles
  - linkage of tubes each of flux ϕ has helicity $2\phi^2$
  - tube of flux ϕ with single full twist has helicity ϕ^2;
  - two linked tubes each of flux *$\phi/2$ and* each with one right handed twist: 2 *$(\phi^2/4)$ (link)* + 2*$(\phi^2/4)$ (twists)* $= \phi^2$
- better conserved than magnetic energy in most circumstances
- energy minimized when magnetic helicity on largest scale



Types of Dynamos

- **Small Scale Dynamo (SSD)**
  - magnetic energy amplification of weak seed field by velocity flows; growth primarily at or below forcing scale.
- **Large Scale Dynamo (LSD)**
  - system initially **kinetic energy dominated**; magnetic energy and magnetic flux amplified on scales large compared to kinetic energy forcing; system initially; Requires large-scale field-aligned electromotive force;
  - e.g. kinetic helicity driven (classic Parker-type dynamo) OR small scale magnetic instability driven (shear + magnetic instability/buoyancy)
- **Large Scale Magnetic Relaxation Dynamo (MRD)**
  - system initially **magnetic energy dominated**, but injection of small scale magnetic twists lead to amplification of large scale field. Requires large-scale field-aligned electromotive force
  - large scales can be modeled as large scale dynamo with EMF LSD in sense that large scale field

20th Century Texbook LSD Theory

- practical approach to modeling LSDs in turbulent rotators; requires mean field aligned electromotive force(EMF) $\overline{\mathcal{E}} \cdot \overline{\mathbf{B}} = \left(\overline{\mathbf{v} \times \mathbf{b}}\right) \cdot \overline{\mathbf{B}}$
- employs global symmetries and employs a pseudoscalar, typically "kinetic helicity" to supply EMF
- Understanding how MFDT saturates is thus not predictable with 20th century theory; quenching is put in "by hand"
- **20th century MF dynamo theory DOES NOT CONSERVE MAGNETIC HELICITY**

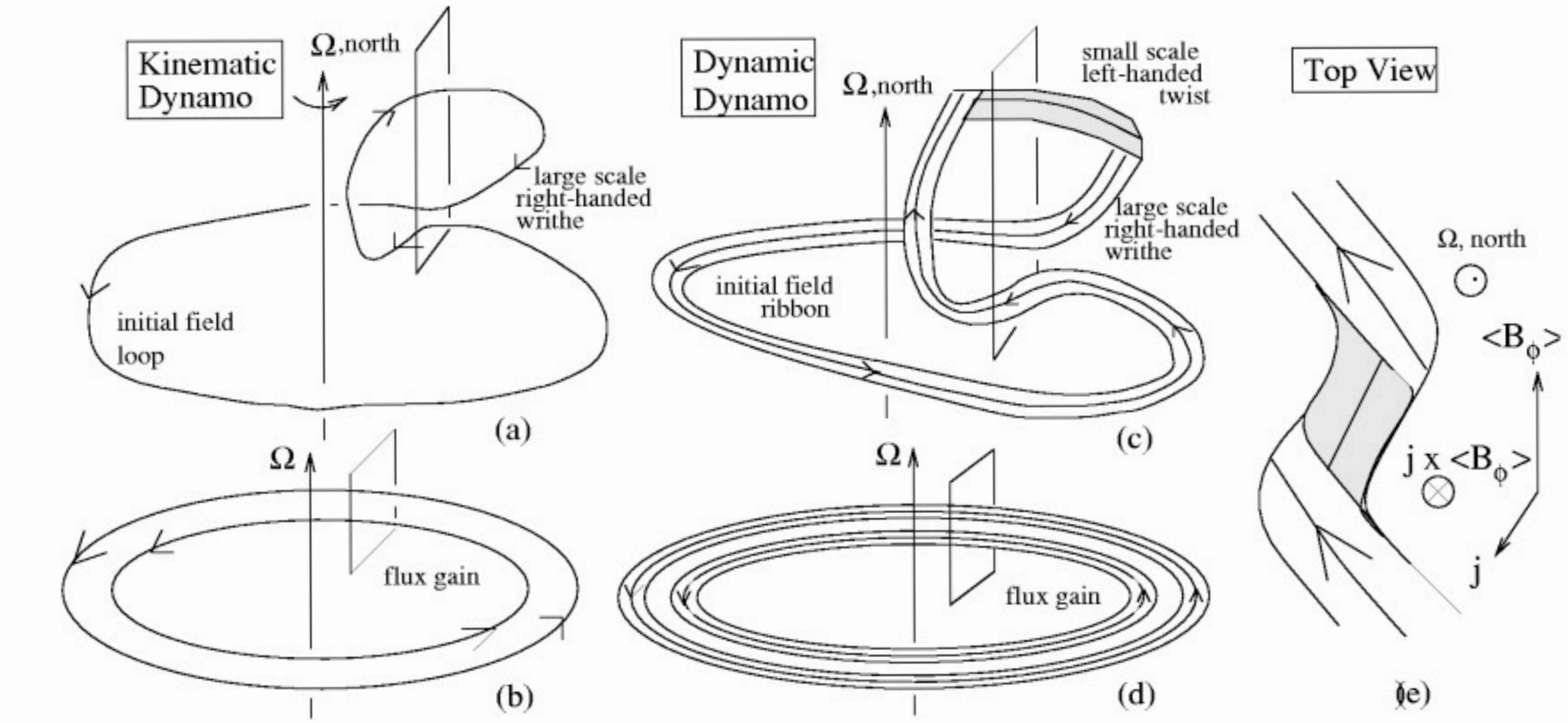
21st Century MFD Theory

- Using EDQNM, Pouquet et al. (1976) demonstrated growth of large scale helical magnetic field; driver is difference between kinetic and current helicities
- Re-interpreted as MFDT (e.g. Blackman & Field 02 Blackman Brandenburg 02; note equations in Kleeorin Ruzmaikin '82), kinetic helicity driving grows large scale magnetic helicity of one sign and "small scale" magnetic helicity of opposite sign. The latter quenches the LSD, consistent with DNS of Brandenburg (2001) of alpha^2 dynamo.... more papers....
- If instead driven by small scale current helicity, the MFDT grows large scale helical field growth of same sign
- Extension of principles to stratified disks emerging (e.g. Vishniac 2009; Gressel 2010, Käpylä & Korpi 2010..): MRI simulations show LSDs (e.g. Brandenburg et al. 1995; Lesur & Ogilive 2008; Davis et al. 2010; Guan & Gammie 2011) likely with EMF sustained by other terms than traditional kinetic helicity. Anisotropic terms, buoyancy, fluxes...
- Helicity fluxes may sustain EMF and alleviate premature quenching in systems with open boundaries, e.g. Bhattacharjee Hameri 1986, BF 2000; Vishniac Cho 2001; Shukurov et al. 2006... or even within sub-volumes of e.g. sheared systems with periodic boundaries
- **21st century MFDT captures nonlinear saturation seen in minimalist numerical experiments**.
- phase delays, nonlocality, implications for more general computation of transport coefficients (BF02,BB02; Hubbard and Brandenburg 2009; Chamandy et al 2013; Park et al. 2013)

Helical LSD: Revising textbook picture

Blackman & Brandenburg 03

(CME ejects SS helicity)



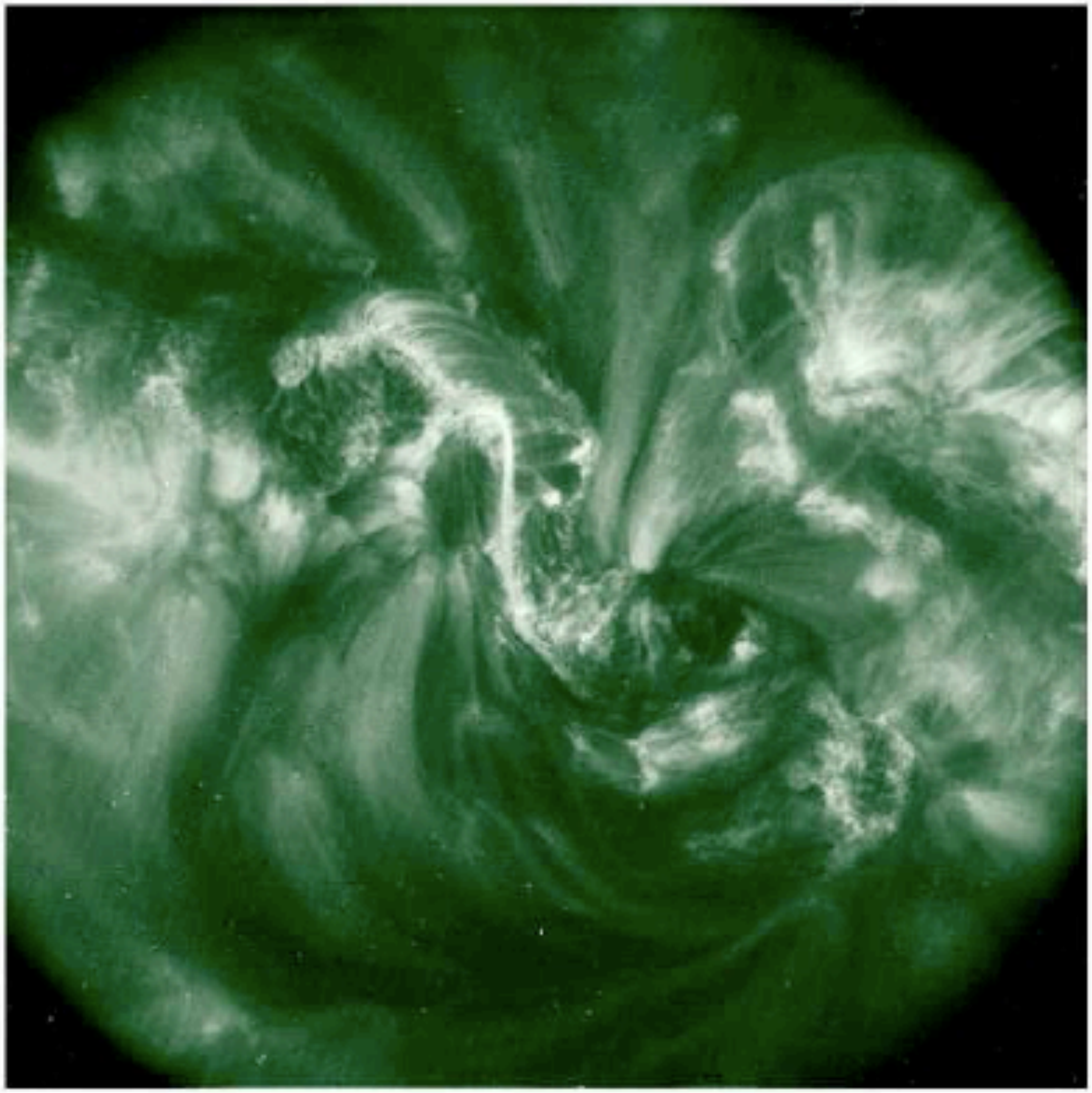
("α-Ω" LSD)

$$\partial_t \overline{\mathbf{B}} = \nabla \times \overline{\mathcal{E}} + \nabla \times (\overline{\mathbf{V}} \times \overline{\mathbf{B}}) + \nu_M \nabla^2 \overline{\mathbf{B}}$$

$$\overline{\mathcal{E}} = \overline{\mathbf{v} \times \mathbf{b}} = \alpha \overline{\mathbf{B}} - \beta \nabla \times \overline{\mathbf{B}} + \nabla \cdot \overline{F}_h$$

$$\alpha = -\frac{\tau}{3}(\overline{\mathbf{v} \cdot \nabla \times \mathbf{v}} - \overline{\mathbf{b} \cdot \nabla \times \mathbf{b}})$$

TRACE Image of a Sigmoid (Gibson et al. 2003) (195 Å)



Relevant solar coronal field properties

- Some evidence that writhe and twist have opposite signs and invariant with respect to solar cycle (Rust and Kumar, 1994, Gibson 2003, Pevstov et al. 2007, Hau & Zhang 2011)

- twist-- l.h. north r.h. south; writhe-- r.h. in north, l.h. in south;

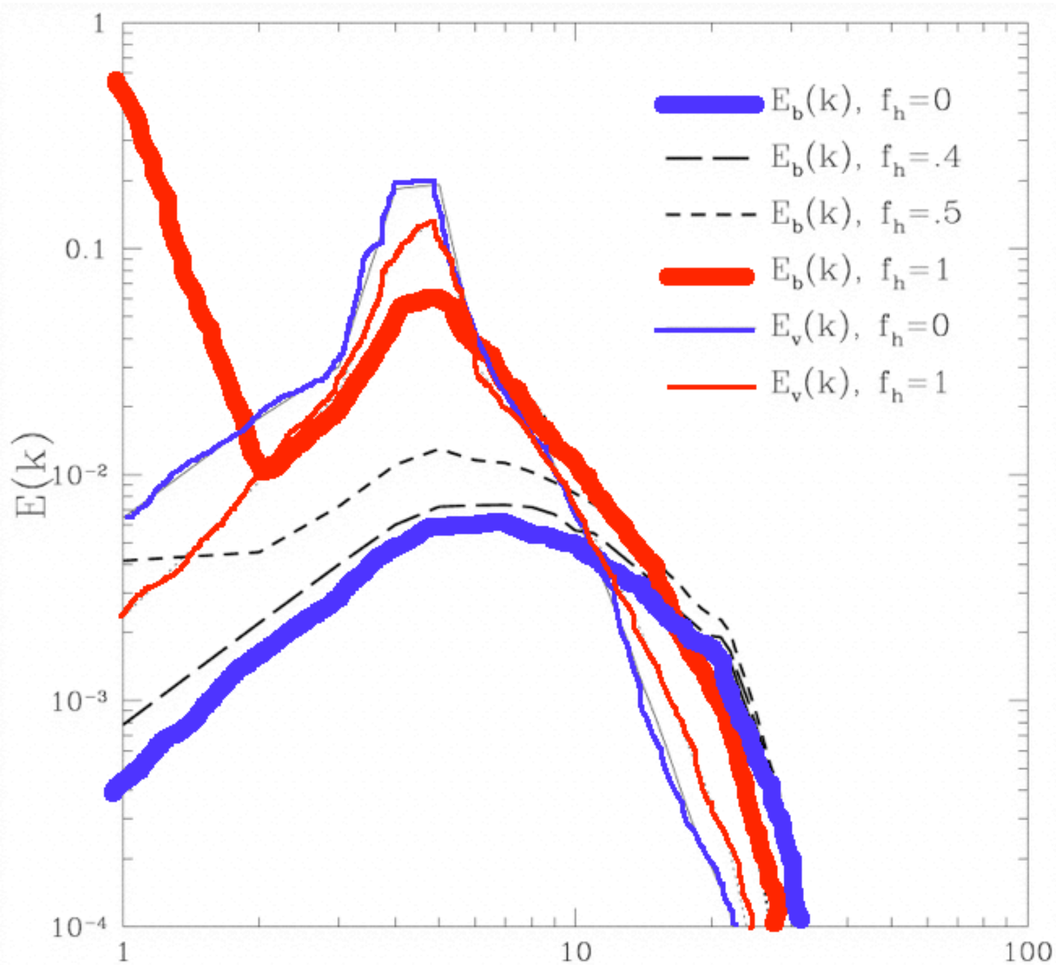
- Hao and Zhang (2011) and Gosain et al. (2013) show for solar cycle 24 that strong fields in active regions show (twist) current helicity in strong fields is negative in north, weak fields are positive

- evidence in solar wind that the relative signs of large and small scale helicity reverse sign with distance from sun (e.g. Brandenburg et al. 2011, Ulysses) may be explicable in terms of expected signs of helicity fluxes

- subtleties: e.g. Hα Filaments exhibit “dextral” (r.h.) twist in North and “sinstral” (l.h.) in south (Martin and McAllister 1994) **BUT**: r.h. Hα filaments supported by l.h. fields and vice versa (Rust 1999)

- Chae 04; Shuck 05; Lim et al. 07+; LCT track footpoint motions to measure helicity injection

α^2 helical dynamo simulations in periodic box



(e.g Meneguzzi et al. 1981; Brandenburg 01; Maron & Blackman 2003, Graham et al. 2011.. Park & B 2012)

- Simplest helical dynamo: drive with isotropic forcing with and without kinetic helicity at k=5; periodic box
- **Thick Blue**: saturated mag energy spectrum (non-helical kinetic forcing) $\langle \mathbf{v} \cdot \nabla \times \mathbf{v} \rangle = 0$
- **Thick Red**: saturated mag energy spectrum (helical forcinng) $\langle \mathbf{v} \cdot \nabla \times \mathbf{v} \rangle = k_f \langle \mathbf{v} \rangle^2$
- Kinetic helicity affects both large (k< 5) and small scale magnetic energy spectrum

Explaining Saturation of α^2 dynamo

$$\partial_t(\overline{\mathbf{A}}\cdot\overline{\mathbf{B}}) = 2\overline{\mathcal{E}}\cdot\overline{\mathbf{B}} - 2\nu_M\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}$$

$$\partial_t(\overline{\mathbf{a}\cdot\mathbf{b}}) = -2\overline{\mathcal{E}}\cdot\overline{\mathbf{B}} - 2\nu_M\overline{\mathbf{j}\cdot\mathbf{b}}$$

closure allows phase lag

$$\partial_t\overline{\mathcal{E}} = \overline{\partial_t\mathbf{v}\times\mathbf{b}} + \overline{\mathbf{v}\times\partial_t\mathbf{b}}$$

$$= \frac{1}{3}(\overline{\mathbf{b}\cdot\nabla\times\mathbf{b}} - \overline{\mathbf{v}\cdot\nabla\times\mathbf{v}})\overline{\mathbf{B}} - \frac{1}{3}\overline{\mathbf{v}^2}\nabla\times\overline{\mathbf{B}} - \frac{\overline{\mathcal{E}}}{\tau} + (boundary\ terms)$$

Large scale magnetic helicity

$$\partial_t H_1^M = 2k_1\underbrace{\frac{\tau}{3}(k_2^2H_2^M - H_2^V)}_{\alpha}H_1^M - 2\beta k_1^2H_1^M - 2\nu_M k_1^2H_1^M + (\nabla\cdot\langle Q_1\rangle)$$

Small scale magnetic helicity

$$\partial_t H_2^M = -2k_1\underbrace{\frac{\tau}{3}(k_2^2H_2^M - H_2^V)}_{\alpha}H_1^M + 2\beta k_1^2H_1^M - 2\nu_M k_2^2H_2^M + (\nabla\cdot\langle Q_2\rangle)$$

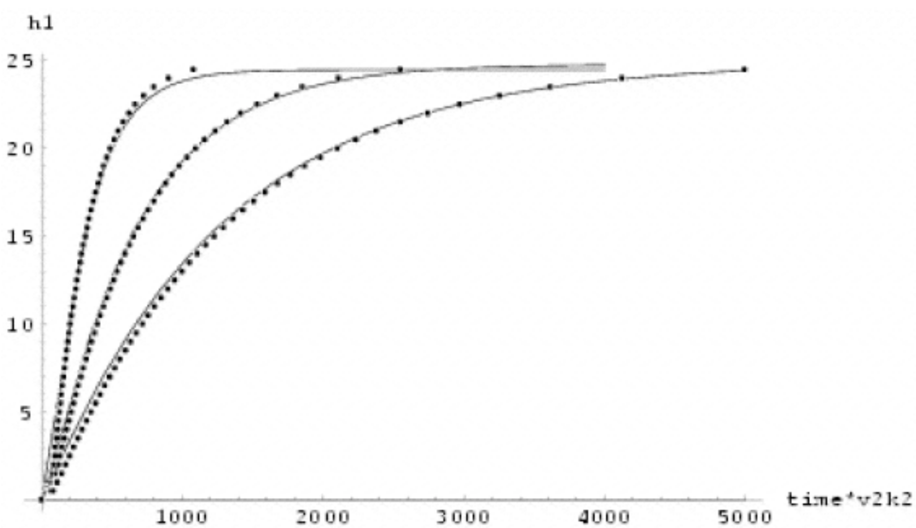
small scale kinetic helicity (forced)

$$\partial_t H_2^V = 0$$

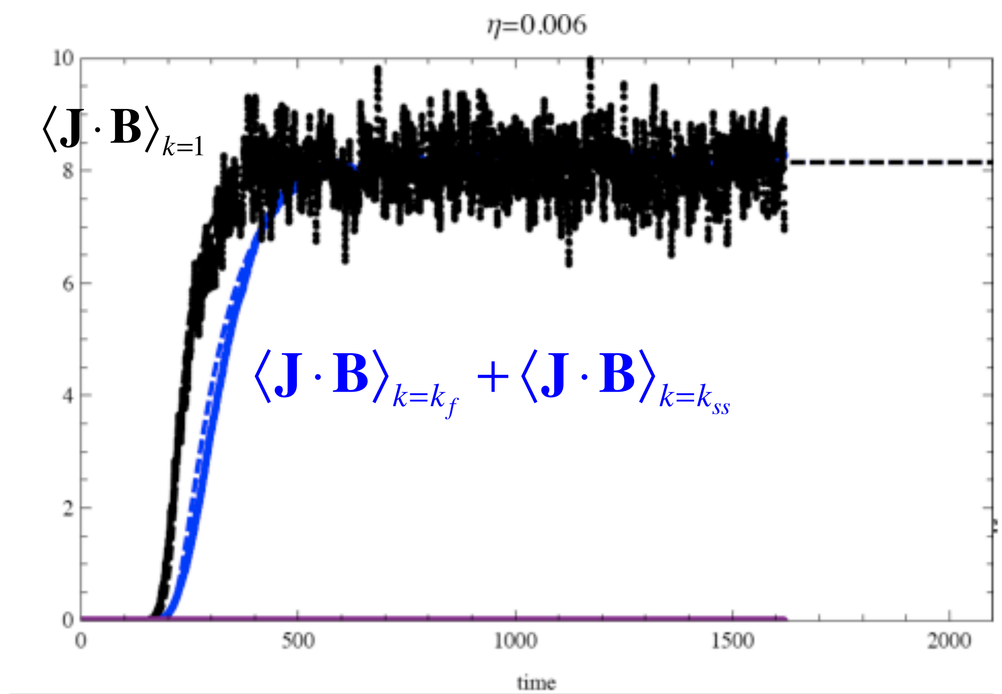
α^2 Dynamo in Periodic Box: Saturation theory vs. simulations

Magnetic helicity of large scale field for R_M =150,250, 500: Solid line: 2 scale model (Blackman & Field 2002); Dotted lines: empirical fit to sims. of Brandenburg (2001)

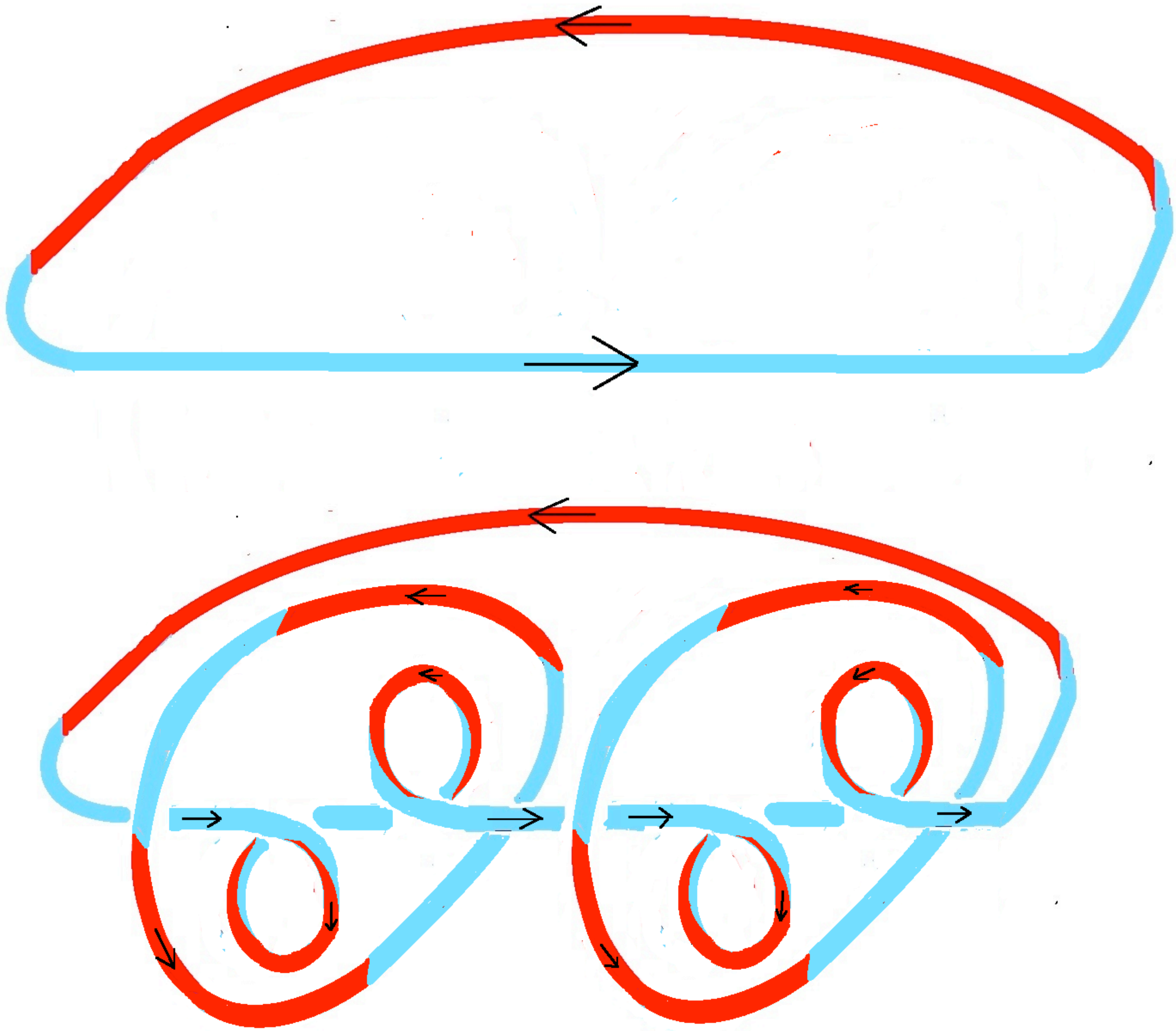
Current helicity of large scale field (black) and small scale (blue) different R_M ~ 80 Dotted= theory; Solid = simulations (Park & Blackman 2011)



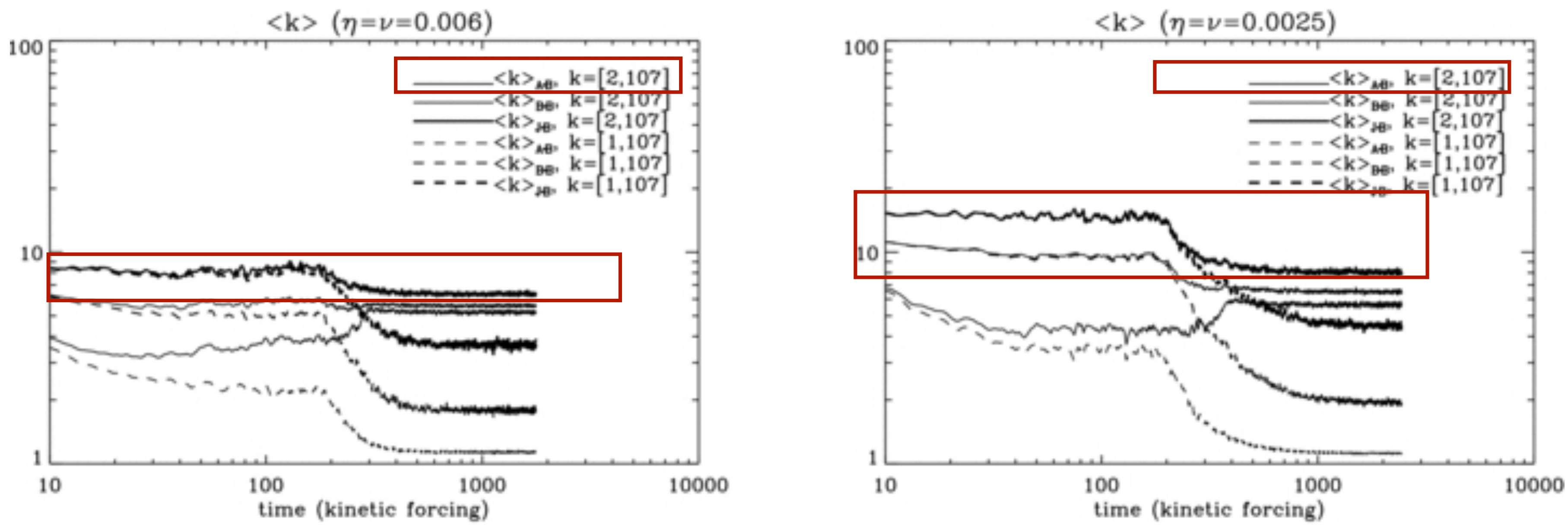
- At end of kinematic regime: $\overline{\mathbf{B}}^2 \sim f_h v_2^2 \left(\frac{k_1}{k_2}\right)$
- At end of resistive regime: $\overline{\mathbf{B}}^2 \sim f_h v_2^2 \left(\frac{k_2}{k_1}\right)$



α^2 Large Scale Dynamo



Example of R_M "ambiguity" from α^2 Large Scale Dynamo sims



(a)

(b)

Park K , and Blackman E G MNRAS 2012;423:2120-2131

$$k_{\mathbf{J}\cdot\mathbf{B}} = \frac{\sum_{2}^{k_{dis}} k\langle \mathbf{J}\cdot\mathbf{B}\rangle_k}{\sum_{2}^{k_{dis}} \langle \mathbf{J}\cdot\mathbf{B}\rangle_k}$$

- Weak dependence of small scale on resistivity; what happens for very large R_M?
- Also: not equal to forcing scale k=5
- partly motivates 3 scale model over 2 scale model

Galactic dynamos with small scale helicity fluxes (Shukurov et al. 2006; Sur et al 2007):

$$\mathcal{E} = \alpha\overline{B} - \eta_t\overline{J}$$

$$\alpha = \alpha_K + \alpha_m$$

$$\alpha_m = \tfrac{1}{3}\rho^{-1}\overline{\tau j \cdot b}$$

$$\frac{\partial \overline{B}_r}{\partial t} = -\frac{\partial}{\partial z}\left(\overline{U}_z\overline{B}_r + \mathcal{E}_\phi\right) + \eta\frac{\partial^2 \overline{B}_r}{\partial z^2},$$

$$\frac{\partial \overline{B}_\phi}{\partial t} = -\frac{\partial}{\partial z}\left(\overline{U}_z\overline{B}_\phi - \mathcal{E}_r\right) + \eta\frac{\partial^2 \overline{B}_\phi}{\partial z^2} + q\Omega_0\overline{B}_r,$$

$$\frac{\partial \alpha_m}{\partial t} = -2\eta_t k_0^2\left(\frac{\mathcal{E}\cdot B}{B_{eq}^2} + \frac{\alpha_m}{R_m}\right) - \boxed{\nabla\cdot\left(\alpha_m\overline{U}\right)}$$

$$\overline{B}_r = \overline{B}_\phi = 0 \quad \text{at } z = \pm h.$$

$$\boxed{C_U = \frac{U_0}{\eta_t k_1},} \quad C_\Omega = \frac{\Omega_0}{\eta_t k_1^2}, \quad C_\alpha = \frac{\alpha_0}{\eta_t k_1}$$

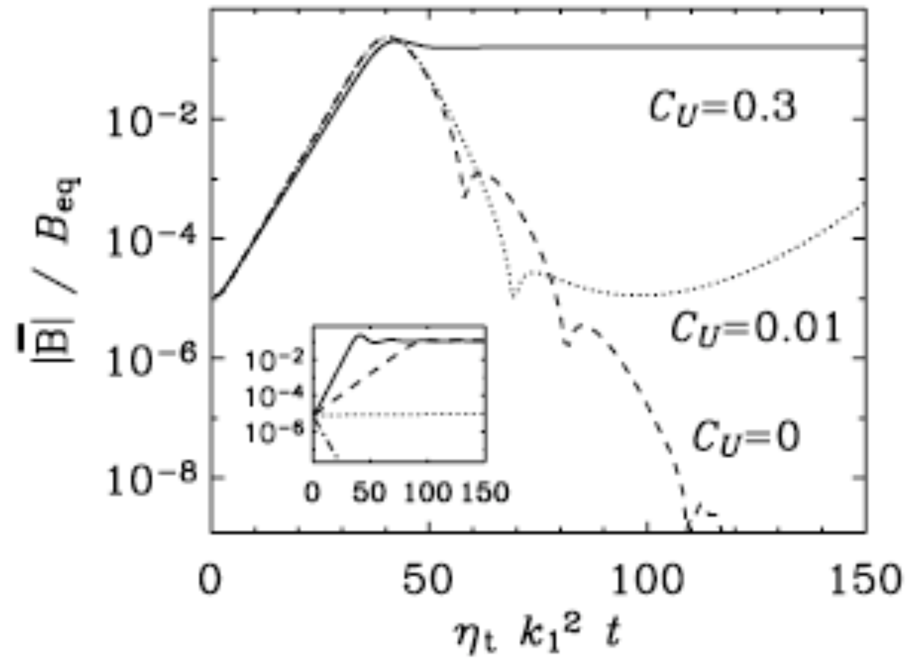
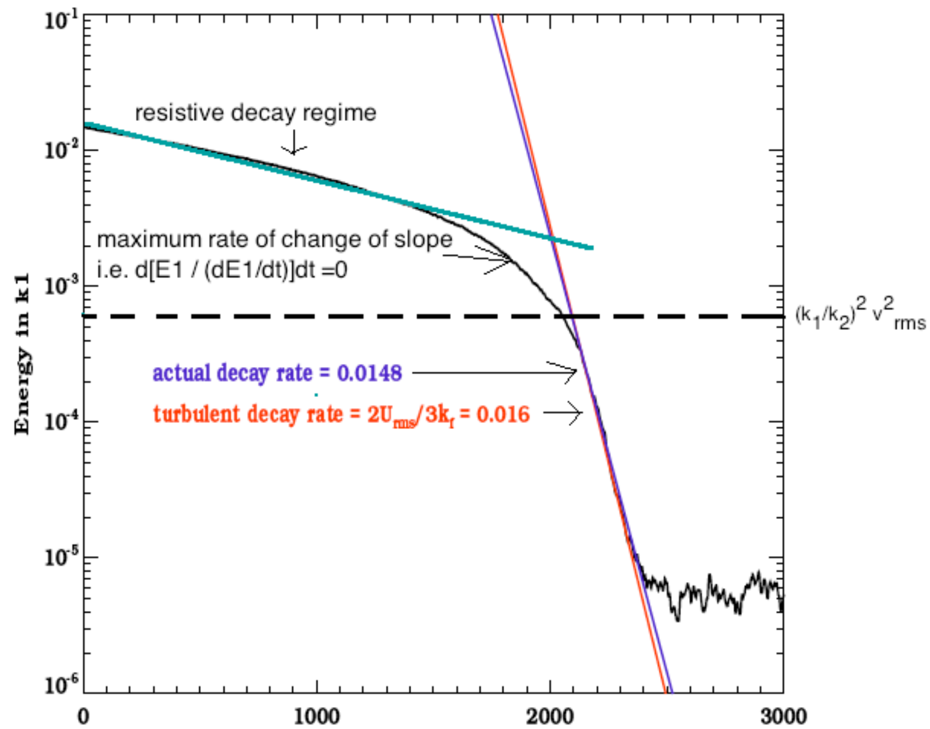


Fig. 1. Evolution of the field strength at $z = 0$ obtained by solving Eqs. (4)–(7) with vertical advection (solid line, $C_U = 0.3$) and without it (dashed line, $C_U = 0$), for $C_\Omega = -2$, $C_\alpha = 1$ and $R_m = 10^5$. The dynamo is neutrally stable at $C_\alpha = 0.26$ for $C_U = 0.3$ and $C_\Omega = 2$. The dotted curve, obtained for $C_U \ll 1$, shows that even weak advection can affect the long-term evolution of magnetic field. For $C_U = 0$, nonlinear effects make the α profile flatter at small $|z|$; this causes an oscillatory decay of the field. The inset shows similar results for C_U = 0.1 (solid), 1.5 (dashed), 2 (dotted) and 3 (dash-dotted).

Helical Fields are Resilient to Turbulent Diffusion

- Implications
  - primordial fields could survive turbulent diffusion but look for helicity reversals across midplane
  - Helical fields in jets need not imply magnetic domination
  - helical fields easier to accrete for jets



Blackman &Subramanian 2013;
Bhat et al. 2013)

- Resolution = 256^3
- Initial magnetic field : Beltrami fields : $\mathbf{B} = (sinz, cosz, 0)$; Helicity = +1, at k1 (in k-space)=1. Amplitude = 0.03. The choice of the amplitude is a bit arbitrary, just low enough to keep the initial Brms from going to a much higher value as compared to the urms.
- Initial velocity field = 0.
- Forcing : non-helical forcing at k_f=5, amplitude = 0.03.
- Resitivity, η and viscosity, ν = 2e-4

Connections between LSD, Helicity, MRI & accretion disks

- Shear driven MRI in stratified disk drives sustains both LSD and SSD (Brandenburg et al. 1995, Lesur & Ogilvie 2008, Davis et al. 2010; Guan et al. 2011; Oishi and MacLow 2011; Simon et al. 2012)
- don’t need <v.curl v> since other terms that depend on anisotropy or helicity flux can sustain $\overline{\mathcal{E}}\cdot\overline{\mathbf{B}}$. All LSDs involve $\overline{\mathcal{E}}\cdot\overline{\mathbf{B}}$ and one must analyze hemispheres separately to assess. (Oversights have been made by integrating over full box)
- Even if ratio of total stress to pressure converges, the scale and spectrum of this stress is important and helicity is likely important for influencing contributions at large scale. If large scale dominates, them MRI is not really supplying a local viscosity but nonlocal transport.
- From simulations there is incomplete evidence for convergence of energy and stress spectra as function of resolution and domain size. Simon et al. (2012) quasi- converge but with caveats (need to test variation of vertical domain, increased resolution, role of b.c., initial conditions, and their largest runs don’t quite converge with respect to location of peak)
- Likely only fields above a critical scale (such that buoyancy beats diffusion) escape to corona. LSD in disk may determine the amount of magnetic energy above this critical scale.
- Some magnetic structures dissipate in corona, some open up to infinity facilitating jets. May involve transfer of magnetic helicity via magnetic relaxation dynamo; analogous to RFP/Spheromak and solar corona. (connections to Miller and Stone 00; Uzdensky & Goodman 08)
- Large scale helical fields are resilient to diffusion, and would be in principle easier to advect.
- global sims: (e.g. DeVilliers et al. 03; Fromang & Nelson 2006; Beckwith et al. 2010 Sorathia et al. 2010; Penna et al. 2010; Romanova et al 2012); Magnetic helicity dynamics perhaps relevant in modeling physics of the structures that form and/or survive diffusion.
- Desire mean field model of accretion that incorporate local + non-local transport, aided by the above principles

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