Reconnection of quasi-singular current sheets and tearing in the ideal limit

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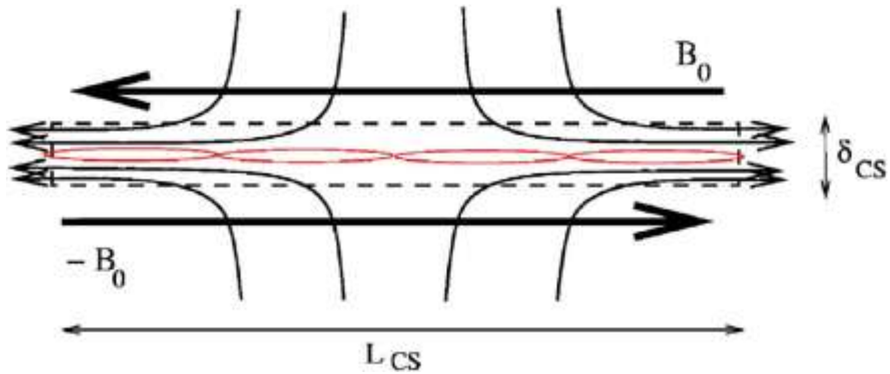
Boulder 21/05/2013

The tearing mode

The typical equilibrium considered is the Harris current sheet (either kept in equilibrium via pressure or with a sheared field component B0z)

$$B_{0x} = \bar{B}\tanh(y/a)$$



$$\gamma(v'' - k^2 v) = \frac{\mathrm{i}k}{4\pi\rho_0}\left[B_{0x}(b'' - k^2 b) - B_{0x}'' b\right]$$

$$\gamma b = \mathrm{i}k B_{0x} v + \eta(b'' - k^2 b)$$

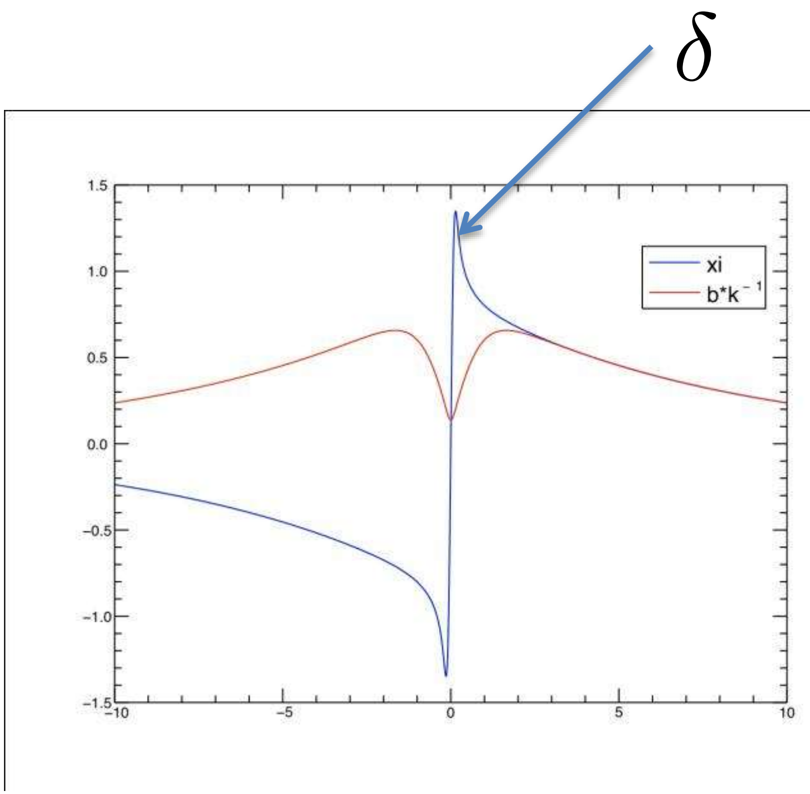
Tearing mode eigenfunctions

$$\Delta' = a\,\frac{b'(\delta/2) - b'(-\delta/2)}{b(\delta/2)}$$

$$\gamma^2 \simeq A\Big(\frac{(ka)}{\tau_a}\Big)^2\Big(\frac{\delta}{a}\Big)^3\,\Delta'$$

$$\gamma\,\tau_a \simeq 2.3(ka)^{-2/5}\,S^{-3/5}(1-k^2a^2)^{4/5}$$

$$S = \frac{a v_a}{\eta}$$



Introducing the explicit k-dependence

$$kaS^{1/4} \simeq 1$$

one finds the maximal growth rate

$$\gamma\,\tau_a \simeq S^{-1/2}$$

Tearing summarized

$$\gamma_{max} \sim \left(\frac{a^3}{V_a \eta}\right)^{-1/2}$$

Impossibly Slow !

Tearing on Sweet Parker current sheets

If the Sweet-Parker current sheet were unstable to the tearing mode, then, because in all our calculations we have used the scale

$$a = L/\sqrt{S}$$

Where S has been redefined as $S = \frac{Lv_a}{\eta}$ Renormalization then gives

$$\gamma_{max}\frac{L}{V_a} \sim \left(\frac{a}{L}\right)^{-3/2} \times S^{-1/2}$$

Or $$\gamma\tau_a \sim S^{1/4}$$

Which grows without bound with S

In reality though

$$\gamma_{max}\frac{L}{V_a} \sim \left(\frac{a}{L}\right)^{-3/2} \times S^{-1/2}$$

$$\frac{a}{L} \sim S^{-\alpha}$$

$$\gamma_{max}\tau_a \sim S^{3\alpha/2-1/2}.$$

You can not imagine going beyond

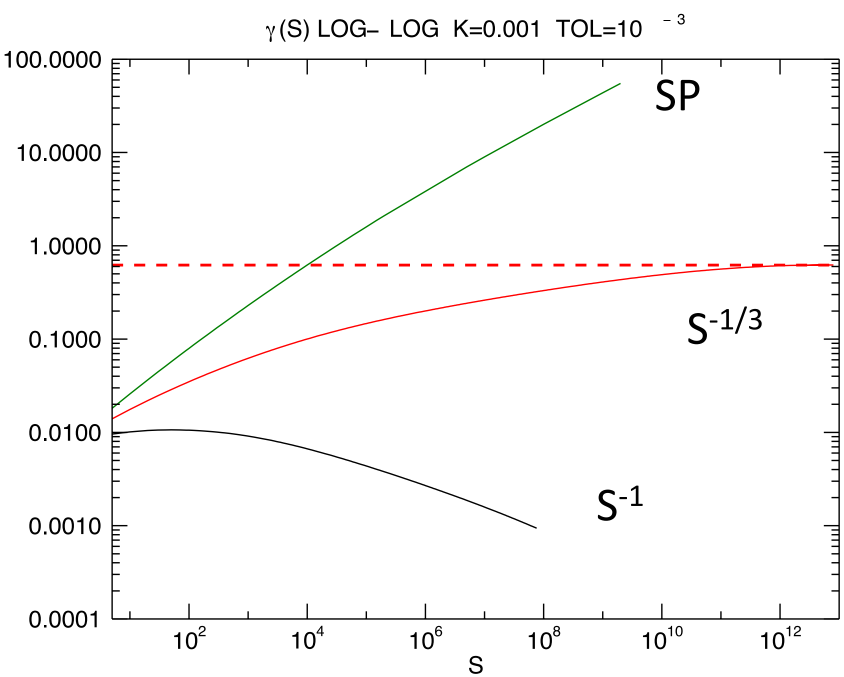
$$\alpha = 1/3$$

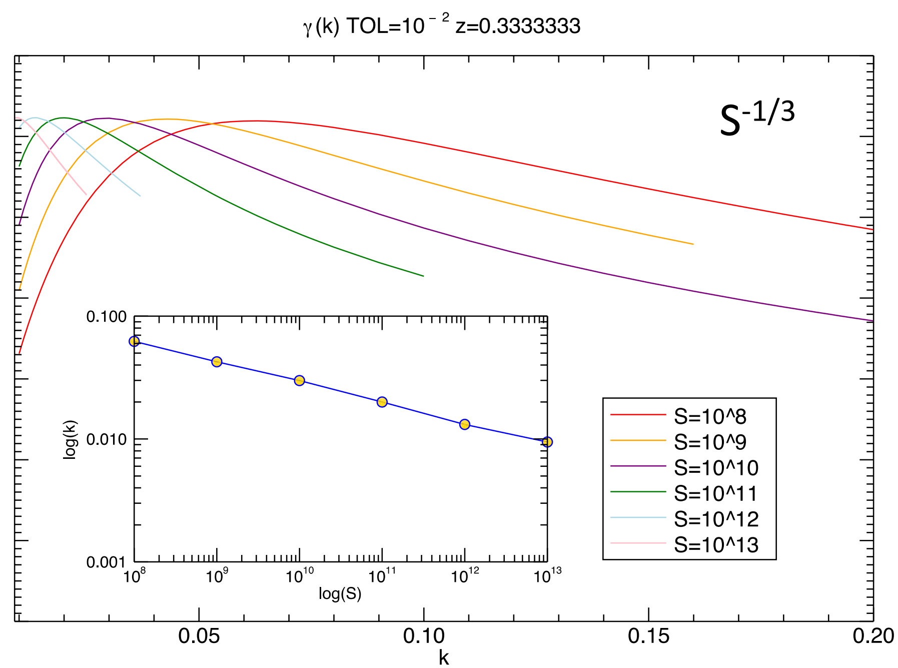
Tearing mode on current sheet with thickness scaling with S

Velli, Matthaeus et al. 2013

Pucci and Velli, 2013

Interesting this is exactly the same scaling for the Resonant absorption singular layer with an S – independent damping





Conclusions - current sheets must be intrinsically turbulent, Kinetics picks up where MHD gives up.

The value of the Dreicer electric field for typical values of coronal plasma parameters $n=10^9$ cm^{-3}, $T=10^6$ K (the Coulomb logarithm L~20) is $E_D=10^{-7}$ statvolt/cm [Mangeney 1999].

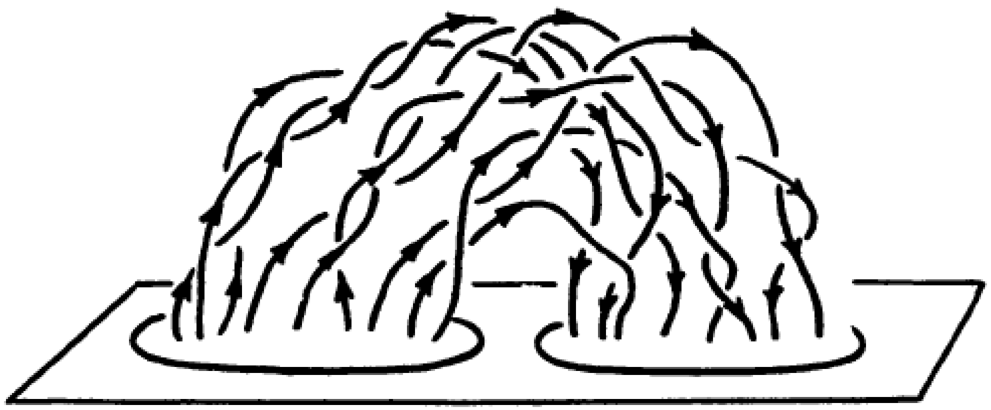
Average heat flux, lost $F \sim 10^6$ erg/cm^2/sec, E=h j.

Height H and surface S the volumetric heating rate is then e = F/H implying that h $j^2 f$= e where f is a filling factor giving the fraction of the volume over which currents effectively dissipate.

Observations of very fine scale structure in the corona would seem to indicate that it must be fairly small, say f~ 0.1.

Then E = h j =(he/f)$^{1/2}$. Taking a volume of height 10^5 km and typical coronal values for the resistivity, one obtains E~ 1 statvolt/cm……

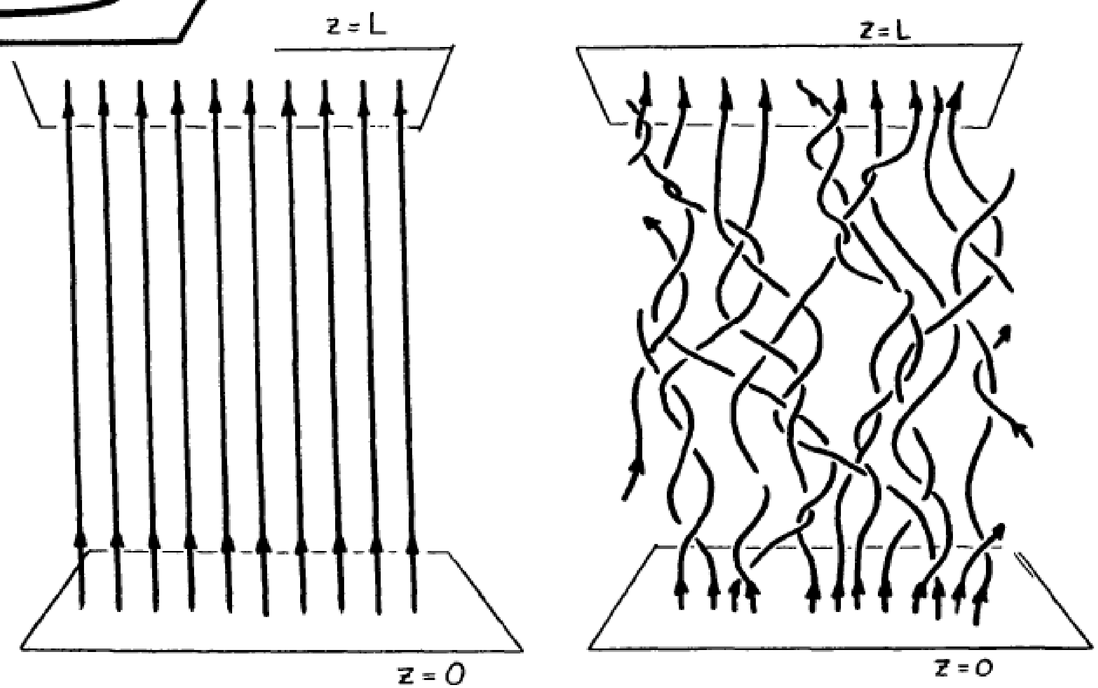
Heating the confined corona



"Reality"

Parker

Model



Conclusions

MHD Turbulence

Current Sheets formation and dissipation
Magnetic reconnection
Nanoflares

are not independent physical processes.Current Sheets are the dissipative structures of MHD Turbulence, which self-consistently account for all these phenomena.

Current Sheets do not generally result directly from a "geometrical" misalignment of neighboring magnetic field lines stirred by their footpoint motions, they are the result of a nonlinear cascade in a self-organized system.

Beyond a numerical threshold total dissipation is independent of the Reynolds number.

Towards a scaling: turbulence?

velocity v among a forest of fixed vertical flux bundles. The upper end of the flux bundles are fixed at some large height L.

$$W = vF\ ,$$

B^2

e bipolar field above the photosphere in

n Θ, of course, and we can estimate th

the necessity to do work W at the heat

y observations, viz. $W = 10^7$ ergs/

t $B_\perp = B \tan\Theta$. The tension in the wa

undle pulls back against the forward moti

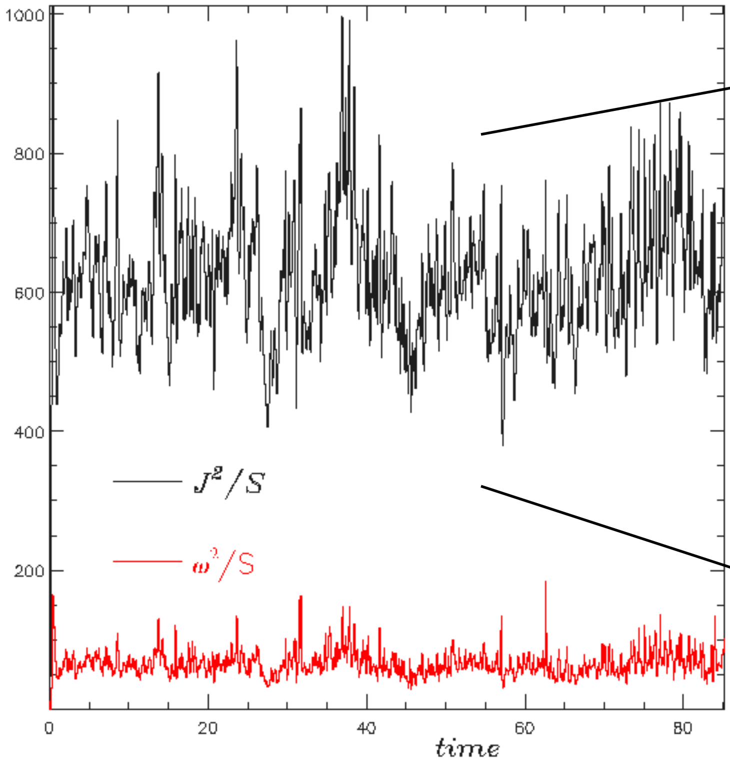
point with a force per unit horizontal area g

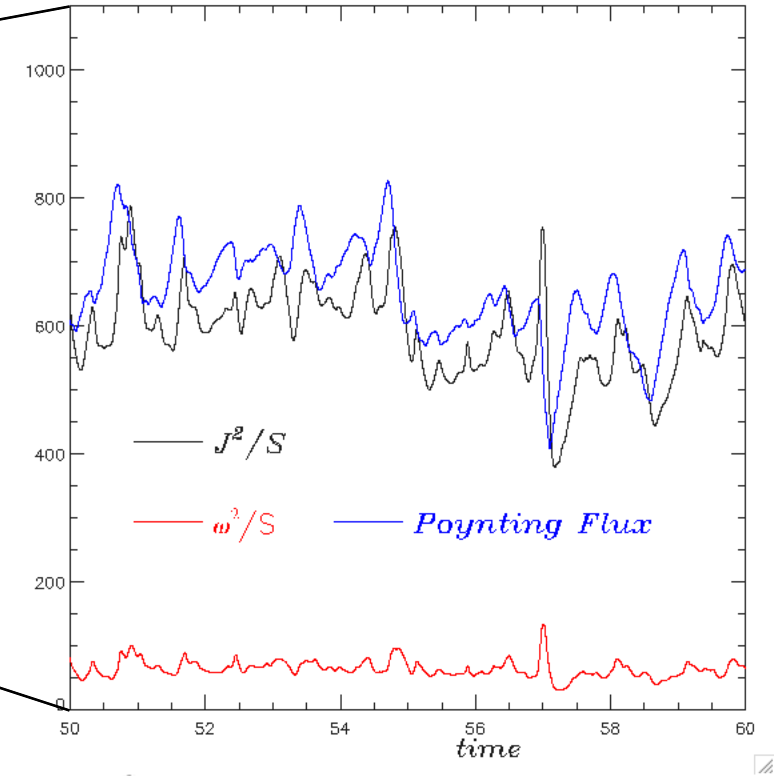
xwell stress

we estimate the magnetic free energy available around a local TD. Consider a volume with transverse scale l and extending a distance $l/\tan\Theta$ along the inclined local field. The associated volume is $l^3/\tan\Theta$ and the available magnetic energy density is $B_\perp^2/8\pi$. Thus with $\tan\Theta = 0.1$, $B_\perp = 10$ Gauss, and $l = 10^2$ km, the free energy is of the order of 4×10^{22} ergs. If a burst of reconnection were to dissipate one tenth of the available energy, the result would be a flaring event of 4×10^{21} ergs. So we might expect to observe bursts of energy over some range of the order of $10^{20} - 10^{24}$ ergs. The coronal temperature in the region would fluctuate up and down on some small transverse scale. The temperature rise would take place during the short life of the burst of reconnection - the nanoflare - and the subsequent cooling time at a density $N = 10^{10}$ /cm3 is estimated at perhaps 30 minutes.

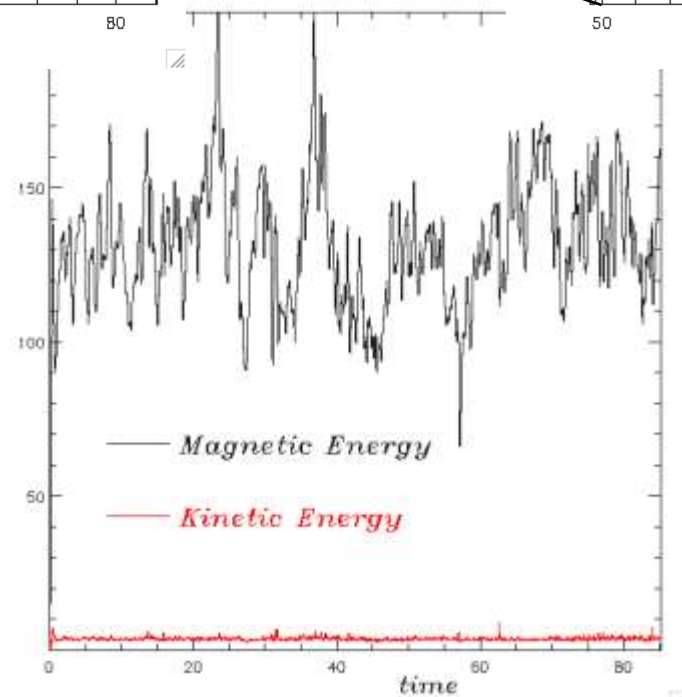
$B \cdot B$

$$F_H = q\ B^2/4\pi\ \delta v,$$





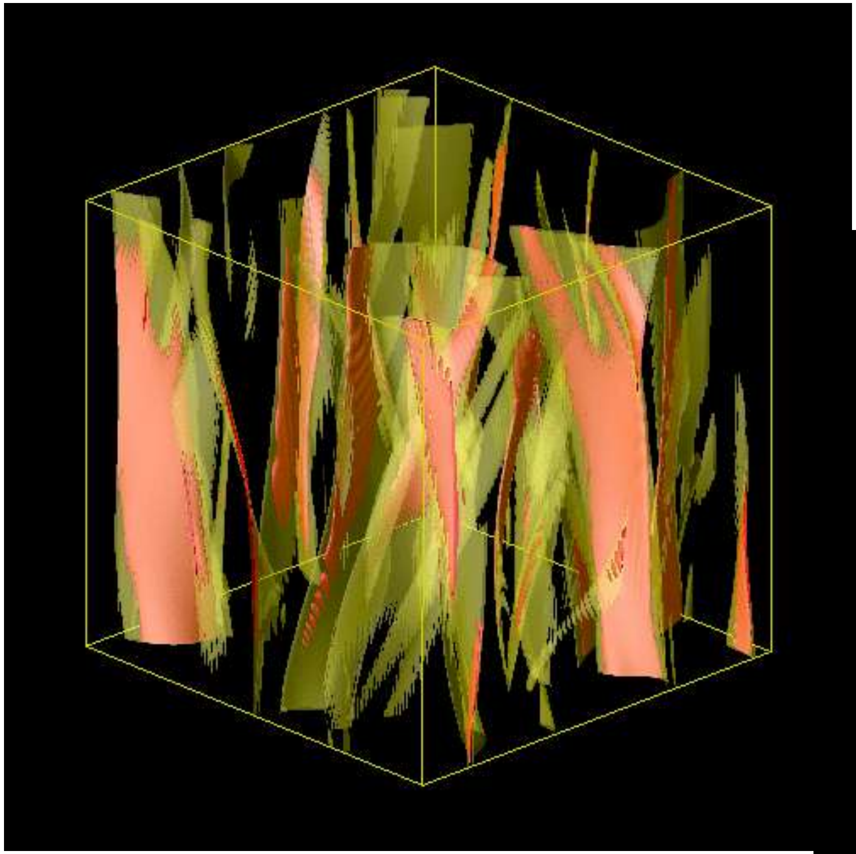
Chaotic time-series

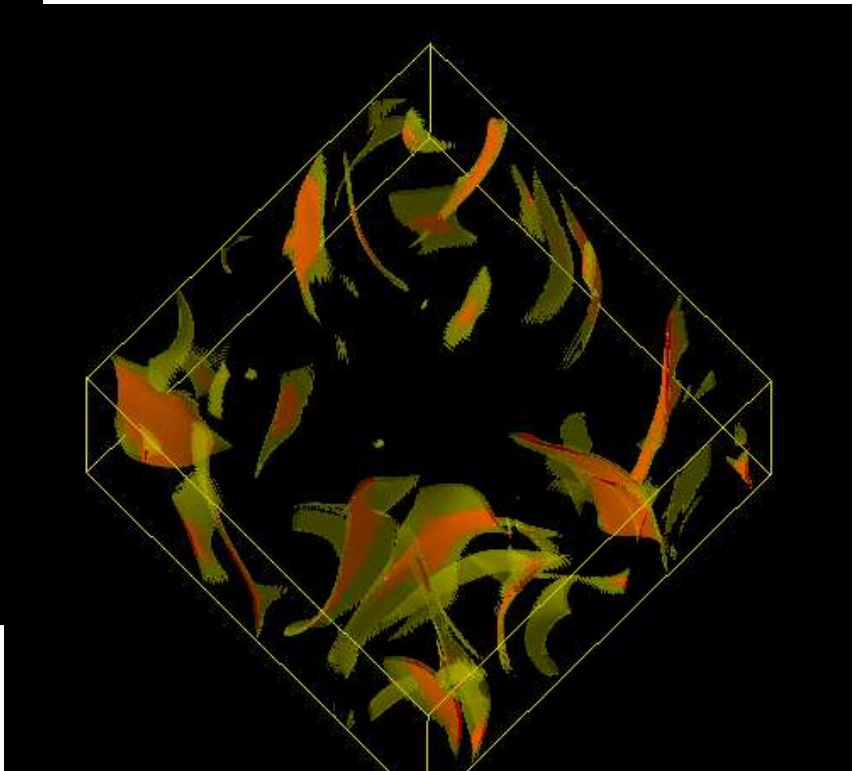


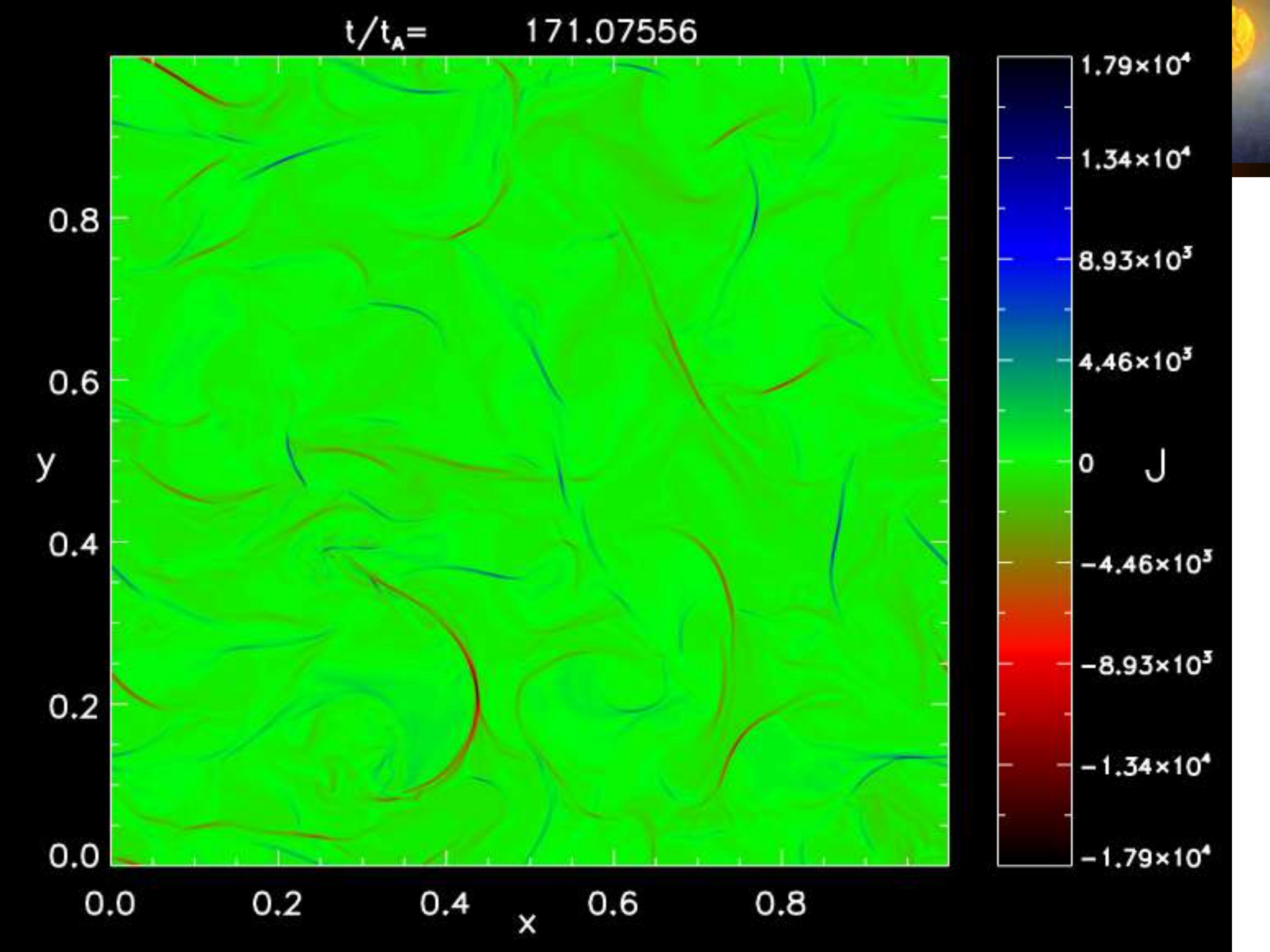
Full RMHD simulations

J²/R

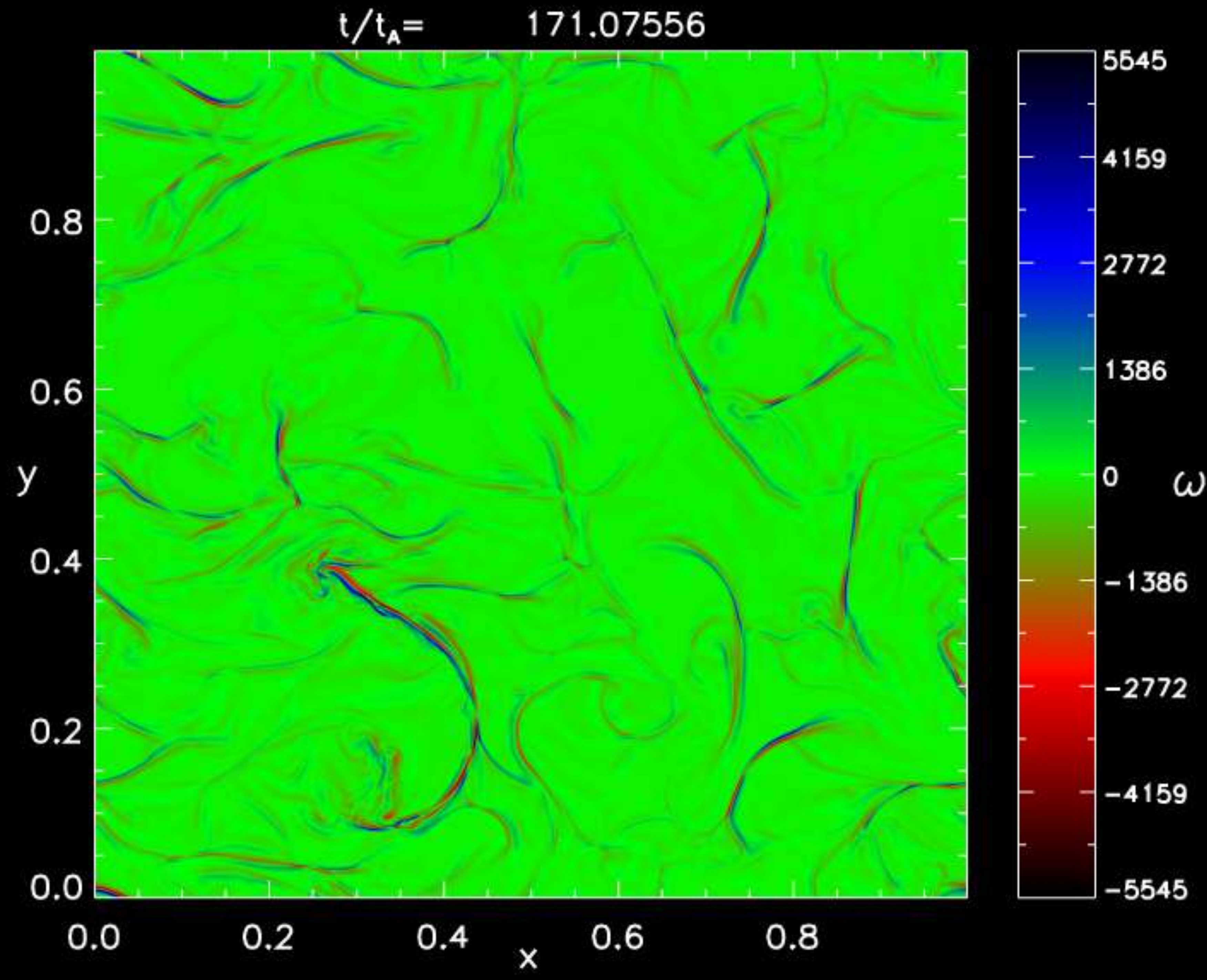
Red = 1000   Yellow = 350   Max = 2.7×10^4 min = 0





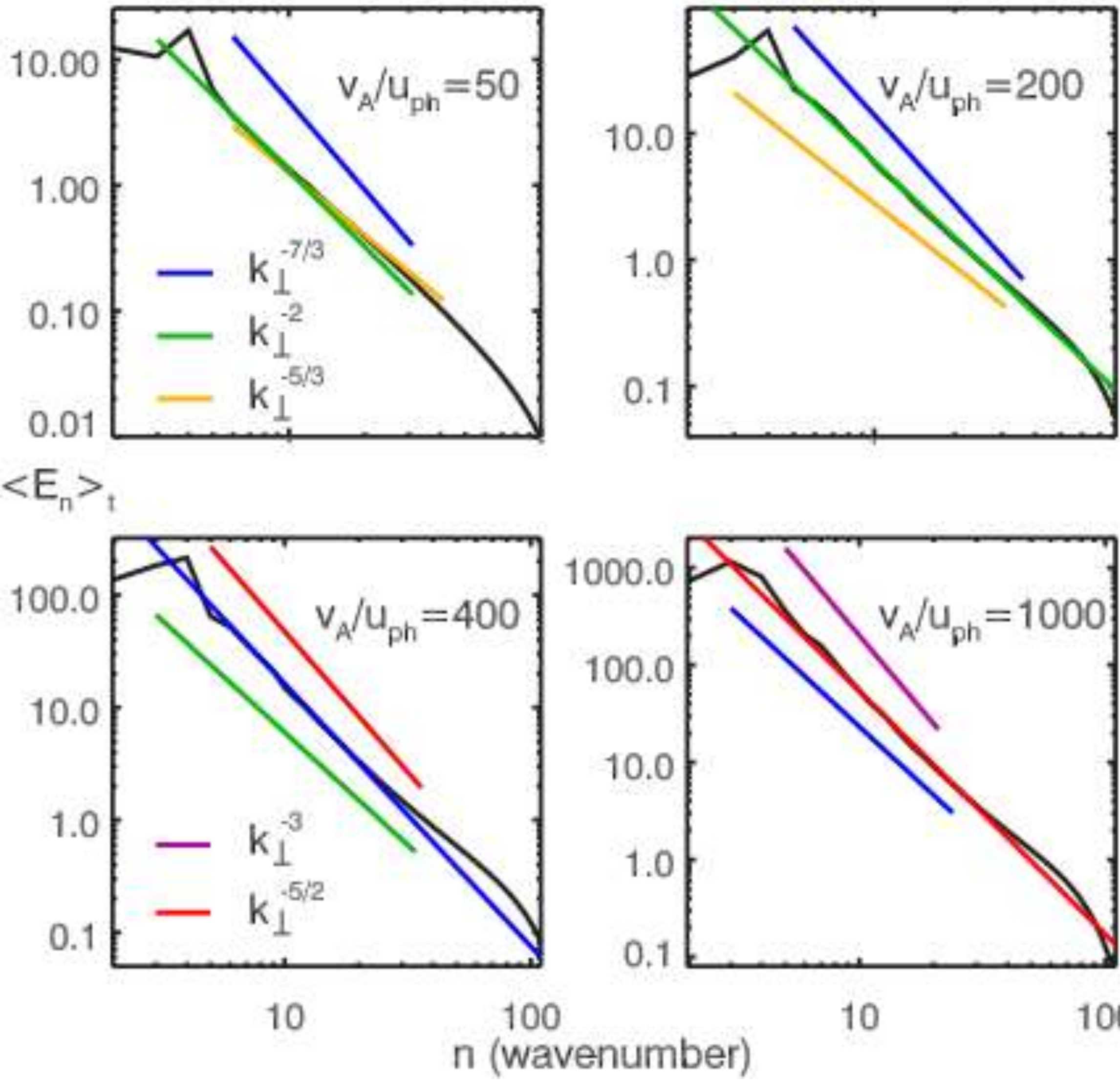


t/t_A= 171.07556
0.0
0.2
0.4
0.6
0.8
x
y
1.79×10^4
1.34×10^4
8.93×10^3
4.46×10^3
0
J
−4.46×10^3
−8.93×10^3
−1.34×10^4
−1.79×10^4



t/t_A= 171.07556
0.0
0.2
0.4
0.6
0.8
y
x
5545
4159
2772
1386
0
-1386
-2772
-4159
-5545
ω

Perpendicular spectra

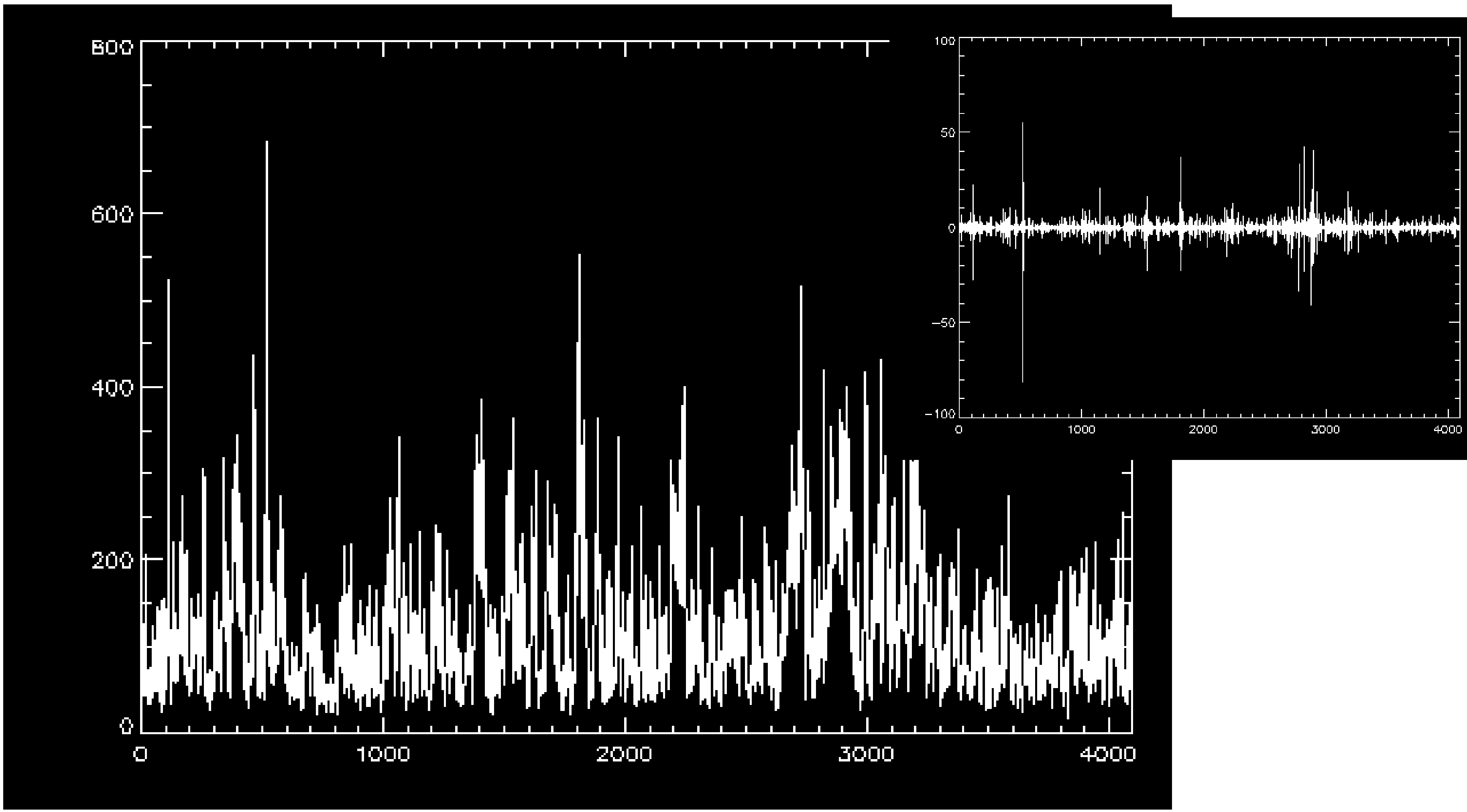


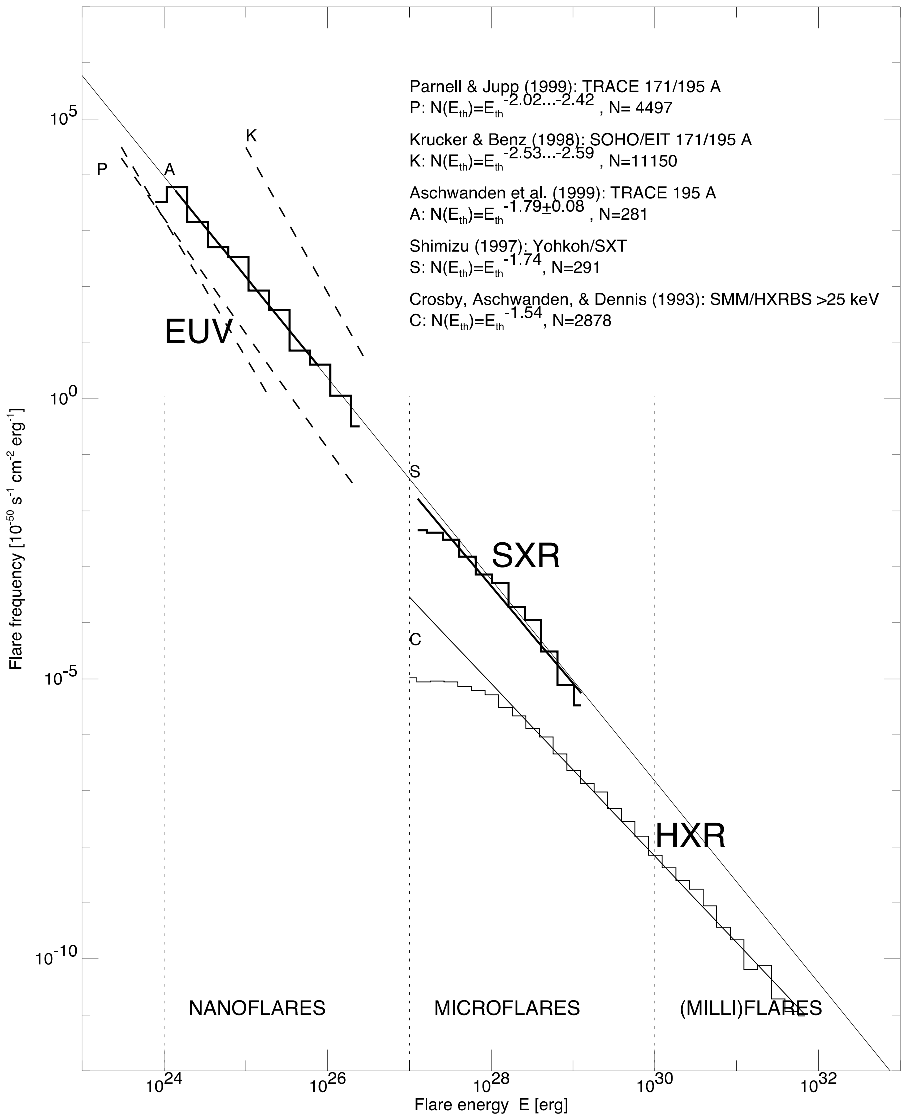
Perpendicular spectra as a function of the control parameter $f = lV_a/L\, u_{ph}$

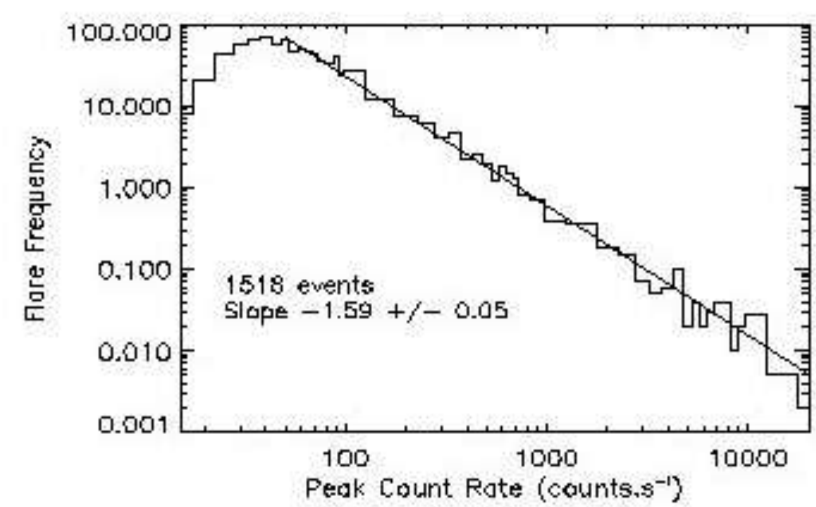
$$\epsilon \sim \rho \frac{z^3}{l_\perp}\left(\frac{L/B_0}{l_\perp/z}\right)^\alpha$$

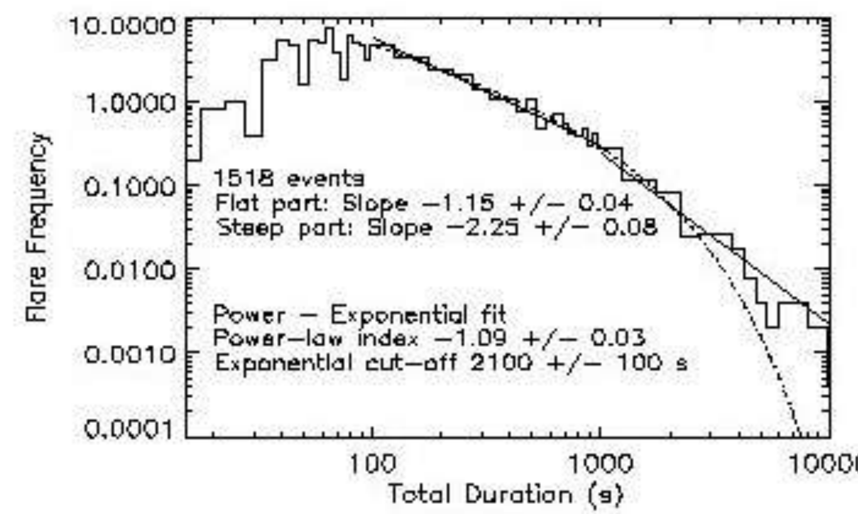
$$E_k \sim k^{-\frac{5+3\alpha}{3+\alpha}}$$

2D Models first showed that the time-series of squared current (dissipation) was intermittent and to lead to discrete events with power law statistics (Einaudi et al. ‘96, Georgoulis et al. ‘98)




Parnell & Jupp (1999): TRACE 171/195 A
P: $N(E_{th})=E_{th}^{-2.02...-2.42}$, N= 4497
Krucker & Benz (1998): SOHO/EIT 171/195 A
K: $N(E_{th})=E_{th}^{-2.53...-2.59}$, N=11150
Aschwanden et al. (1999): TRACE 195 A
A: $N(E_{th})=E_{th}^{-1.79\pm0.08}$, N=281
Shimizu (1997): Yohkoh/SXT
S: $N(E_{th})=E_{th}^{-1.74}$, N=291
Crosby, Aschwanden, & Dennis (1993): SMM/HXRBS >25 keV
C: $N(E_{th})=E_{th}^{-1.54}$, N=2878
EUV
SXR
HXR
NANOFLARES
MICROFLARES
(MILLI)FLARES
Flare frequency [10^{-50} s^{-1} cm^{-2} erg^{-1}]
Flare energy E [erg]


1518 events
Slope −1.59 +/− 0.05
Flare Frequency
Peak Count Rate (counts.s^{-1})


1518 events
Flat part: Slope −1.15 +/− 0.04
Steep part: Slope −2.25 +/− 0.08
Power − Exponential fit
Power−law index −1.09 +/− 0.03
Exponential cut−off 2100 +/− 100 s
Flare Frequency
Total Duration (s)

Reconnection at high S : the "speed of reconnection problem"