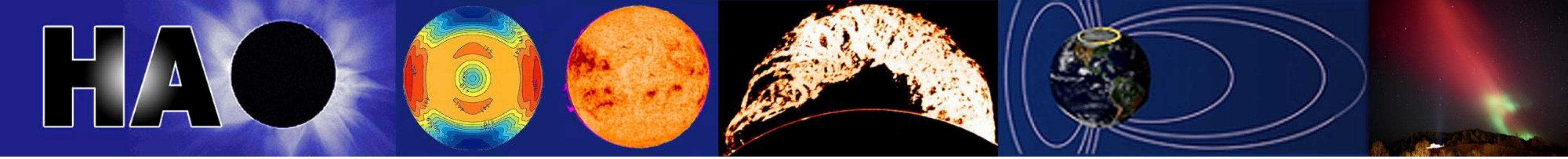

A Convective Dynamo Simulation of a Cyclic Solar Dynamo

Yuhong Fan
National Center for Atmospheric Research

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NCAR

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NSF

- Finite-difference Spherical Anelastic MHD (FSAM) code solves the following anelastic MHD equations in a partial spherical shell domain:

$$\nabla \cdot (\rho_0 \mathbf{v}) = 0,$$

$$\rho_0 \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right] = 2\rho_0 \mathbf{v} \times \mathbf{\Omega} - \nabla p_1 + \rho_1 \mathbf{g} + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla \cdot \mathcal{D}$$

$$\rho_0 T_0 \left[\frac{\partial s_1}{\partial t} + (\mathbf{v} \cdot \nabla)(s_0 + s_1) \right] = \nabla \cdot (K \rho_0 T_0 \nabla s_1) - (\mathcal{D} \cdot \nabla) \cdot \mathbf{v} + \frac{1}{4\pi} \eta (\nabla \times \mathbf{B})^2 - \nabla \cdot \mathbf{F}_{\mathrm{rad}}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}),$$

$$\frac{\rho_1}{\rho_0} = \frac{p_1}{p_0} - \frac{T_1}{T_0},$$

$$\frac{s_1}{c_p} = \frac{T_1}{T_0} - \frac{\gamma - 1}{\gamma} \frac{p_1}{p_0},$$

$$\mathbf{F}_{\mathrm{rad}} = \frac{16 \sigma_s T_0^{\ 3}}{3 \kappa \rho_0} \nabla T_0,$$

Summary of numerical schemes used by the FSAM code to solve the above equations:

— uses a staggered spatial discretization on a r-θ–ϕ grid

— A modified TVD Lax-Friedrichs scheme (Rempel et al. 2009) is used for evaluating the fluxes of all the advection terms in the momentum and entropy equations

— Ensure angular momentum conservation by solving the ϕ-component of the momentum equation in angular momentum conservative form.

— Use the constrained transport (CT) scheme on the staggered grid to ensure the divergence free condition for the magnetic field is satisfied. The CT scheme is used in conjunction with an upwinded evaluation of both v and B based on the Alfv́en wave characteristics for computing the v × B electric field (Stone & Norman 1992).

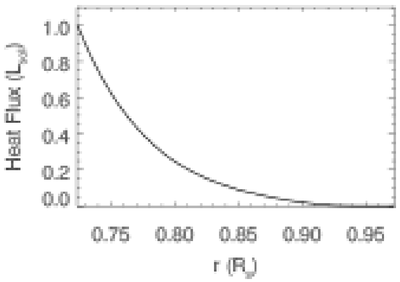
— two-step predictor-corrector time stepping

— Solving the elliptic equation for p_1 at every sub-time step to ensure divergence free of momentum

- FFT in the ϕ-direction → a 2D linear system for each azimuthal order m
- The 2D linear equation (in θ and ϕ) for each azimuthal order m is solved with the generalized cyclic reduction scheme of Swartztrauber (NCAR's FISHPACK).
- In MPI domain decomposition, need to do global transpose from 2D decomposition in r and θ to 1D decomposition in azimuthal order m, and back during the pressure solve.

Simulation setup

- Convection is driven by the radiative diffusive heat flux as a source term in the entropy equation:

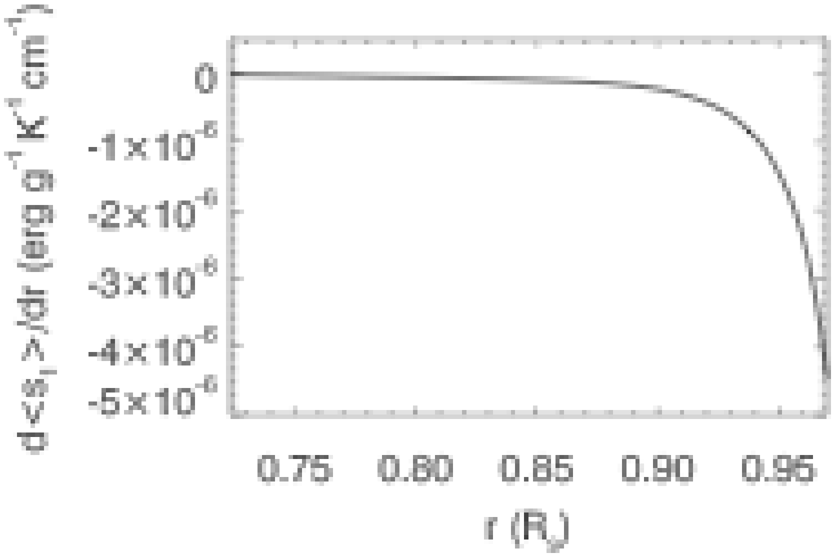


- The boundary condition for s_1:

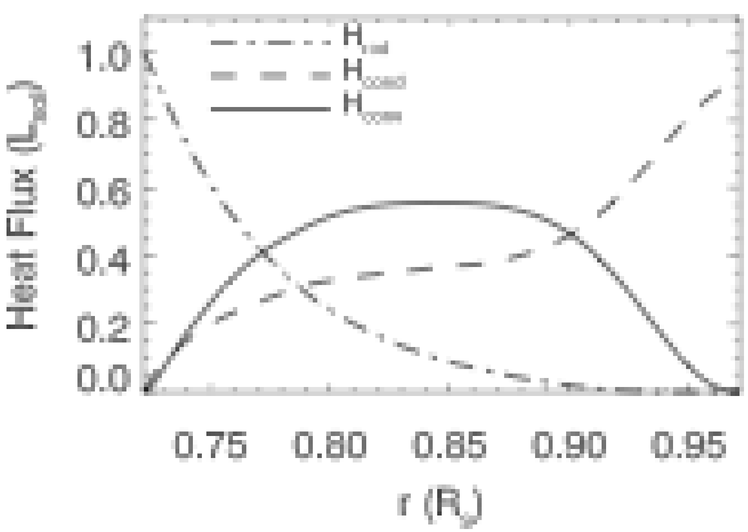
- The velocity boundary condition is non-penetrating and stress free at the and the top, bottom and θ–bondaries

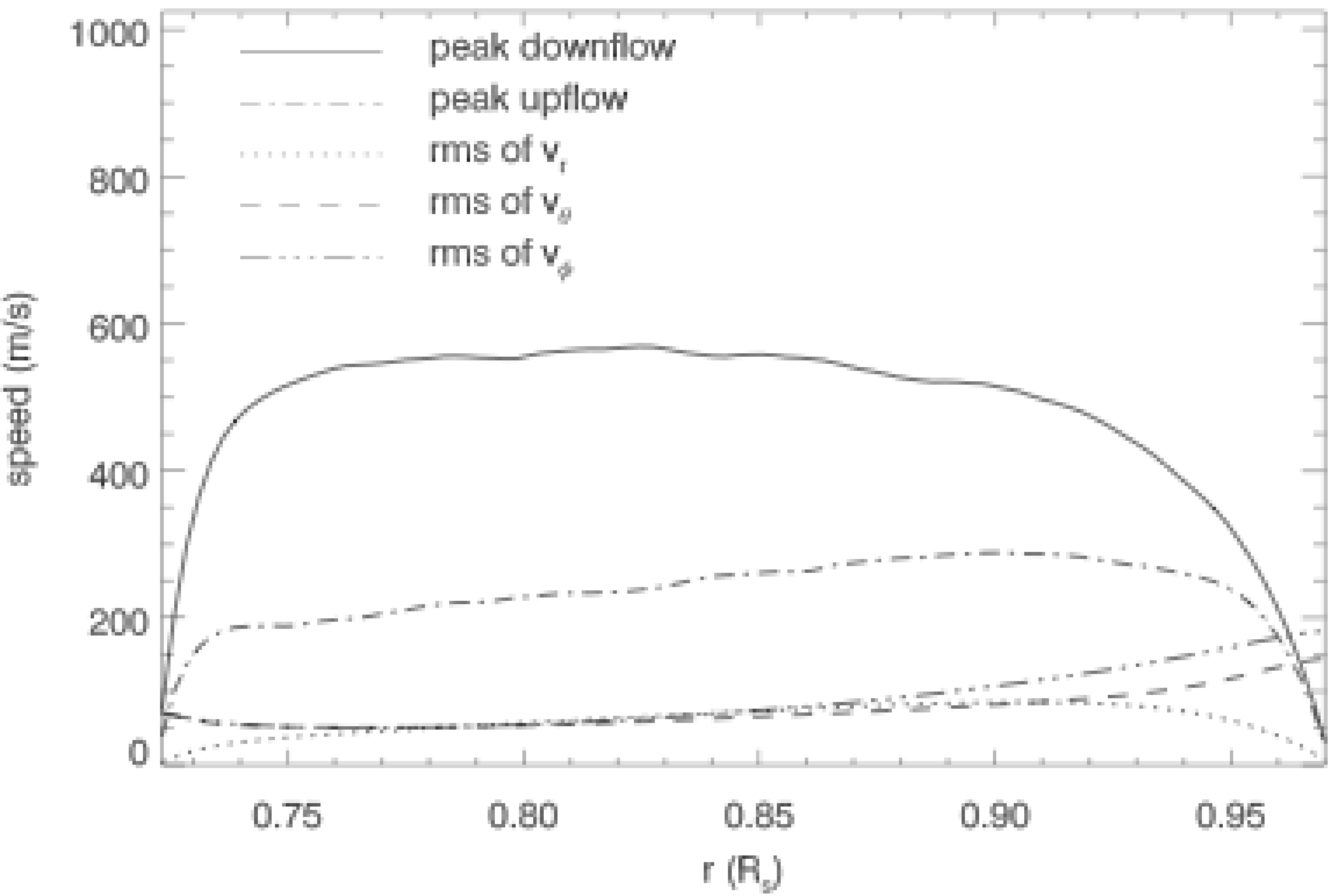
- For the magnetic field: perfect conducting walls for the bottom and the θ–boundaries; radial field at the top boundary

- Angular rotation rate for the reference frame , net angular momentum relative to the reference frame is zero.



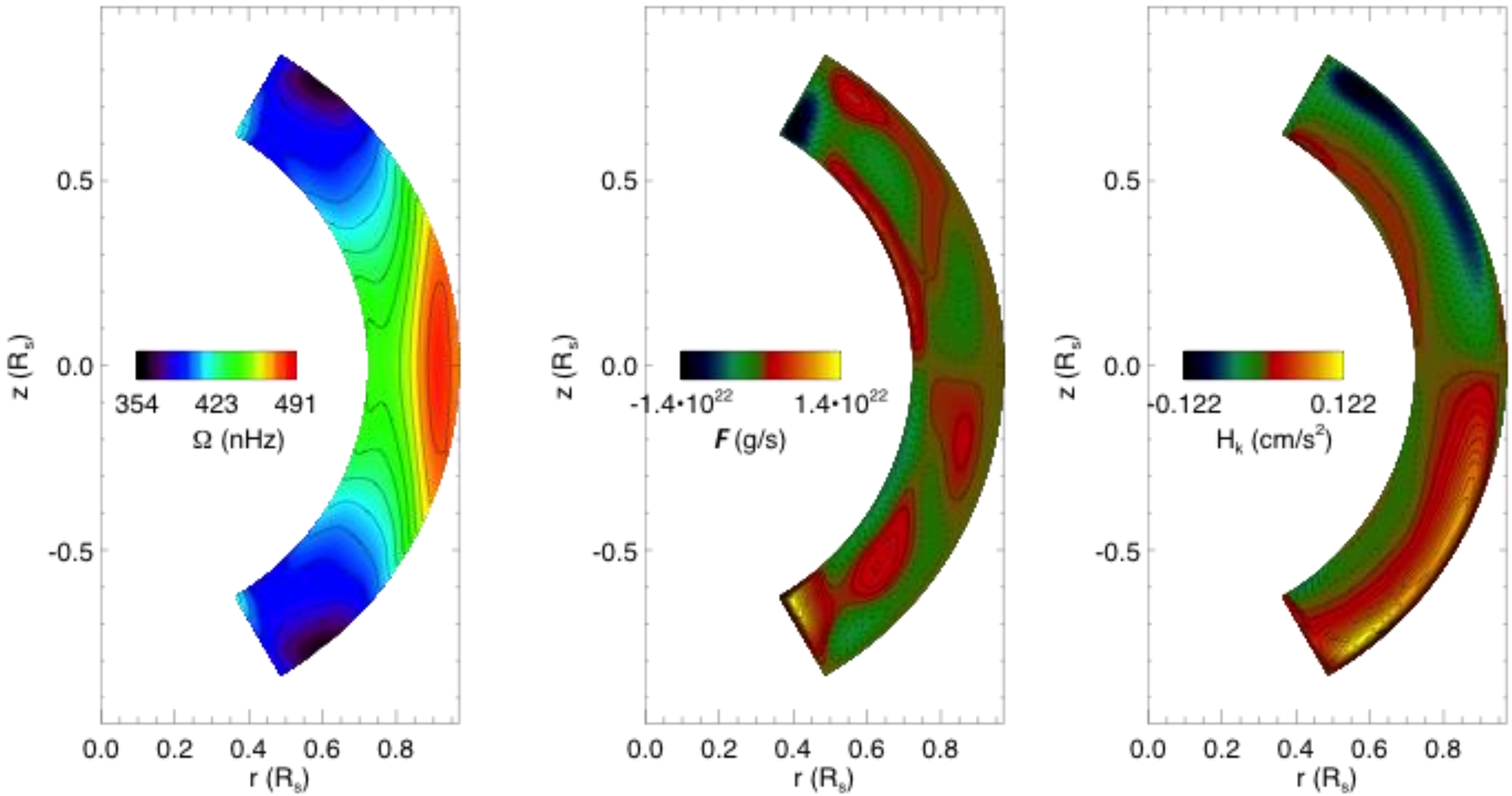
In Model S of JCD, the entropy gradient at $0.97R_s$ is ~10^{-5} erg g^{-1} K^{-1} cm^{-1}

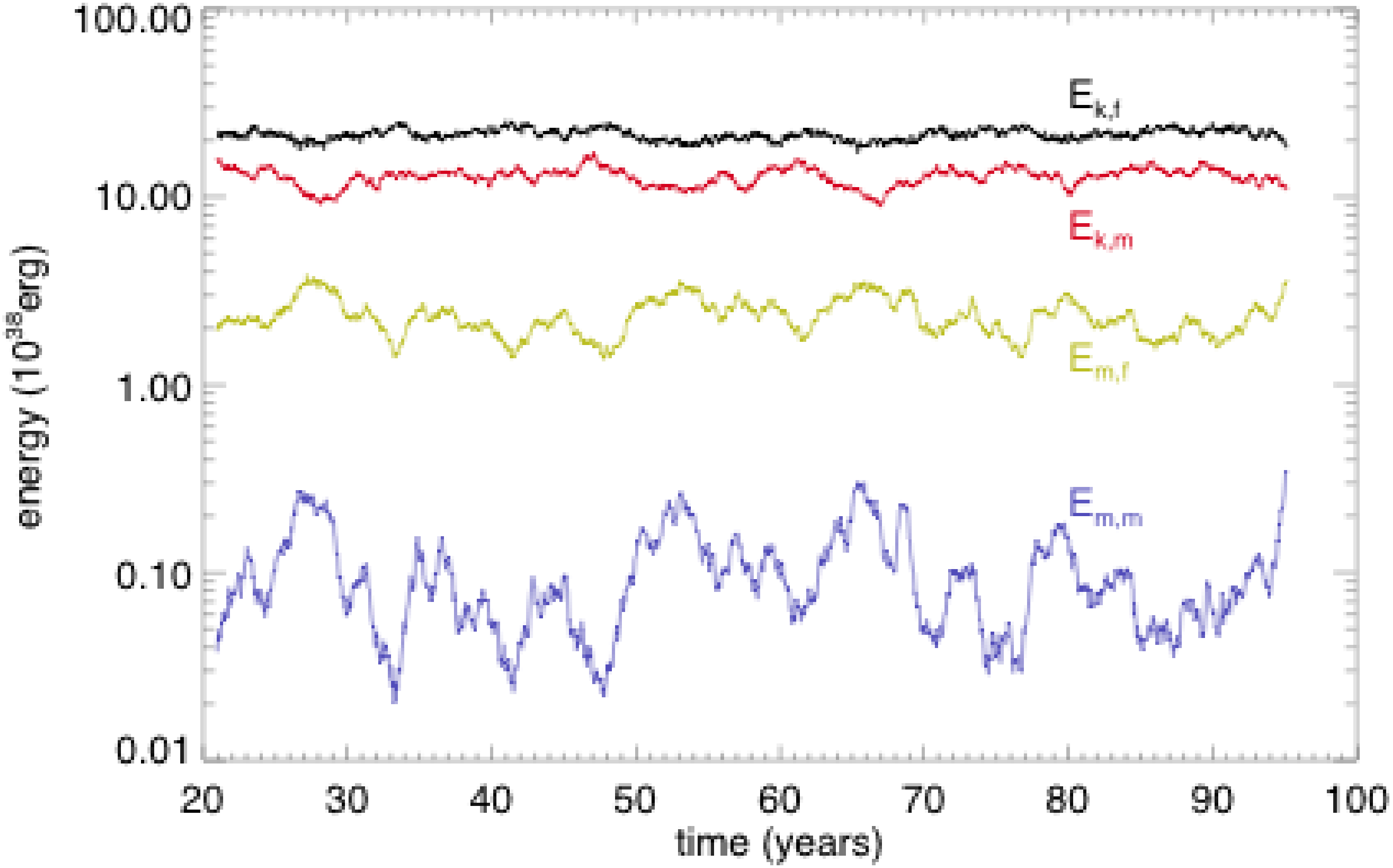




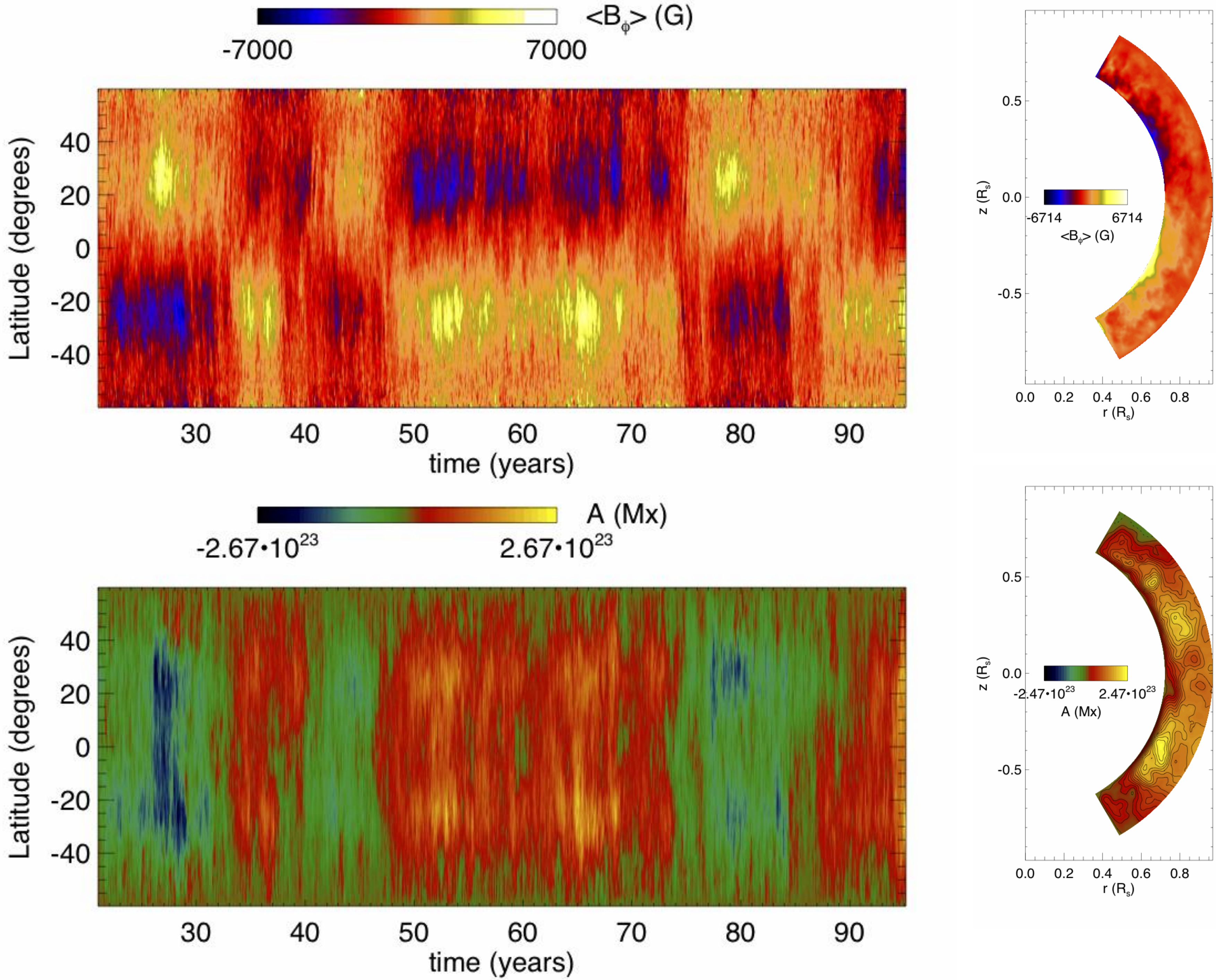
peak downflow
peak upflow
rms of v_r
rms of v_θ
rms of v_ϕ
speed (m/s)
1000
800
600
400
200
0
0.75
0.80
0.85
0.90
0.95
r ($R_\odot$)

Mean flow properties

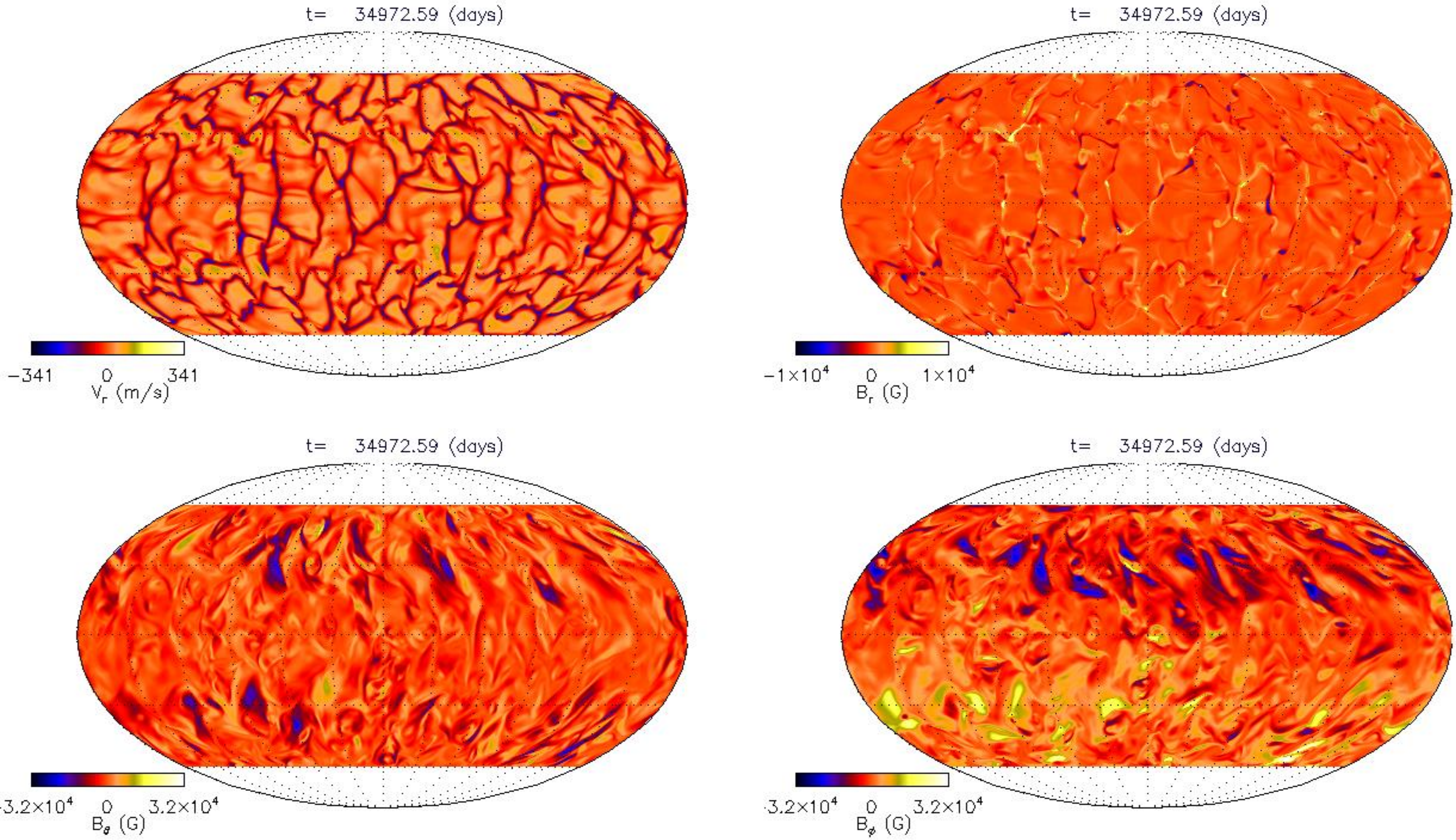


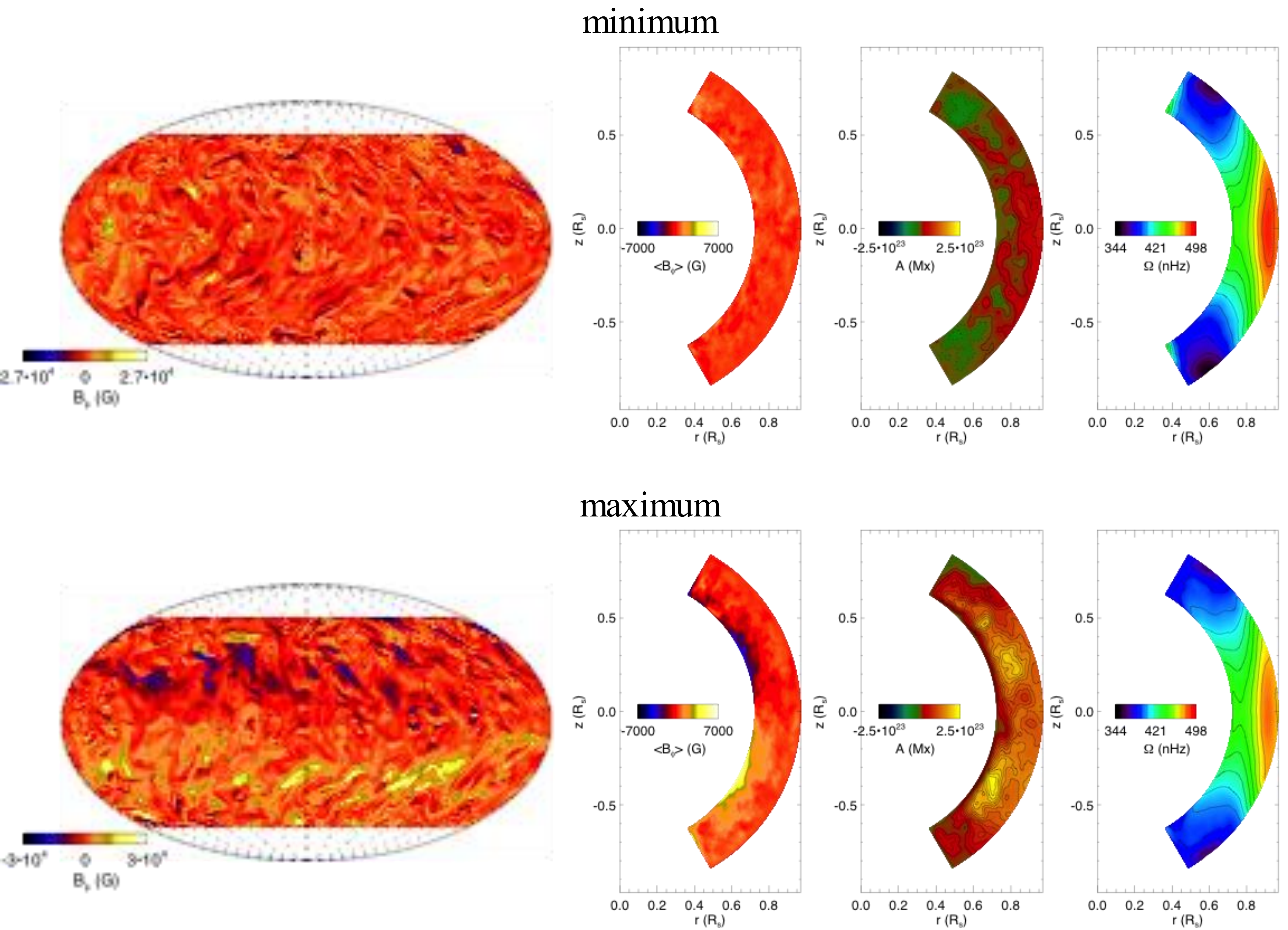


100.00
10.00
1.00
0.10
0.01
energy (10^{38}erg)
20
30
40
50
60
70
80
90
100
time (years)
$E_{k,f}$
$E_{k,m}$
$E_{m,f}$
$E_{m,m}$

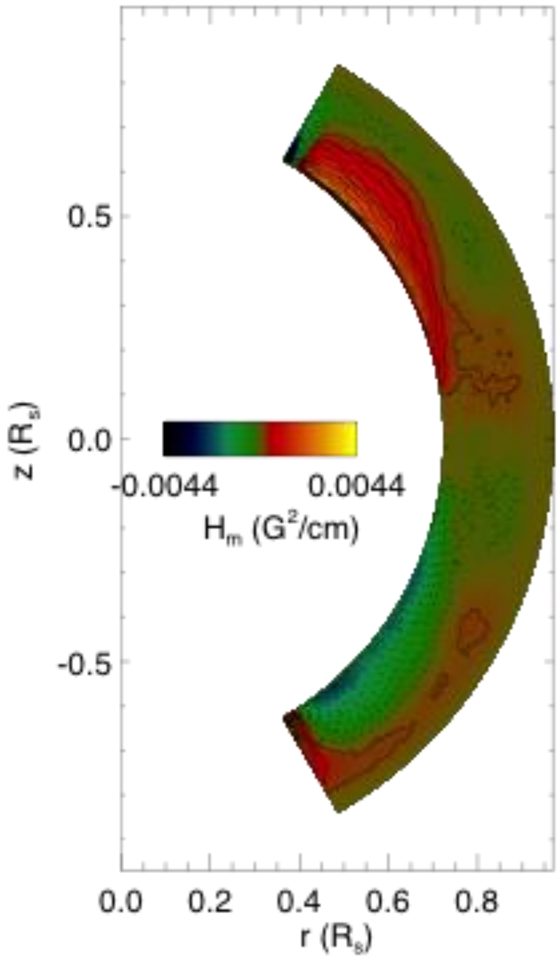


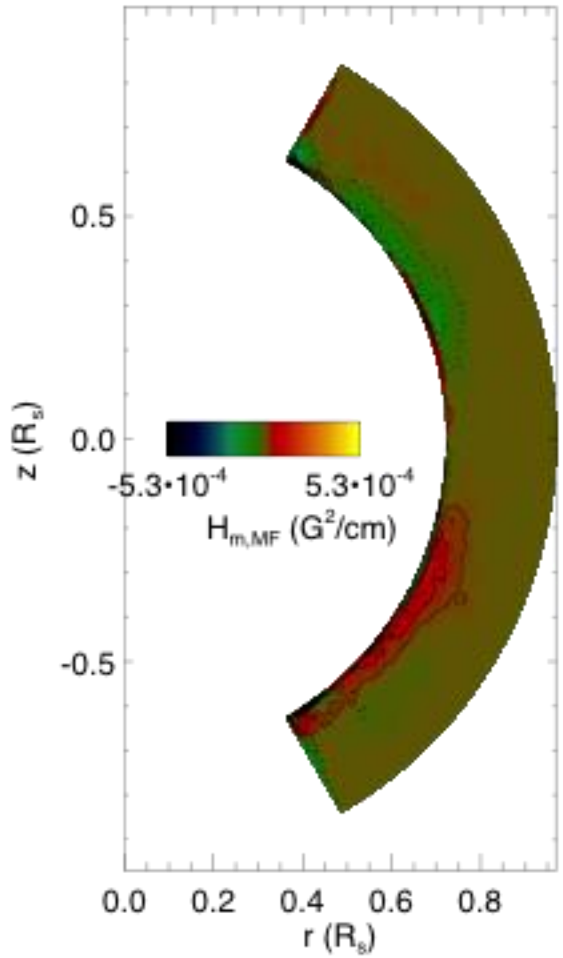
$\langle B_\phi \rangle$ (G)
-7000
7000
Latitude (degrees)
time (years)
z (R_s)
r (R_s)
-6714
6714
$\langle B_\phi \rangle$ (G)
A (Mx)
$-2.67 \cdot 10^{23}$
$2.67 \cdot 10^{23}$
Latitude (degrees)
time (years)
z (R_s)
r (R_s)
$-2.47 \cdot 10^{23}$
$2.47 \cdot 10^{23}$
A (Mx)


t= 34972.59 (days)
-341 0 341
V_r (m/s)
t= 34972.59 (days)
-1×10^4 0 1×10^4
B_r (G)
t= 34972.59 (days)
-3.2×10^4 0 3.2×10^4
B_θ (G)
t= 34972.59 (days)
-3.2×10^4 0 3.2×10^4
B_φ (G)


minimum
maximum
-2.7·10⁴
0
2.7·10⁴
-3·10⁴
3·10⁴
B_φ (G)
-7000
7000
<B_φ> (G)
-2.5·10²³
2.5·10²³
A (Mx)
344
421
498
Ω (nHz)
z (R_s)
r (R_s)
0.5
0.0
-0.5
0.0
0.2
0.4
0.6
0.8

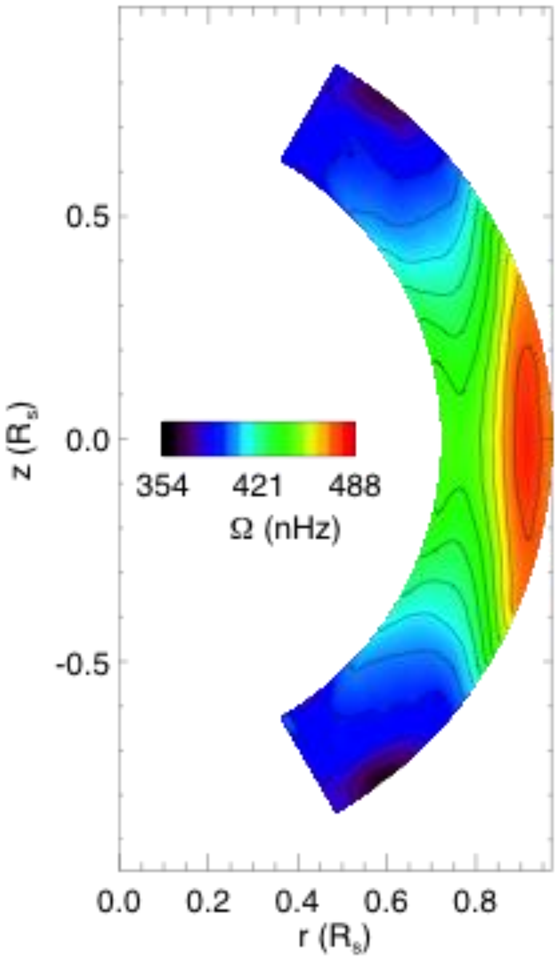
Current helicity

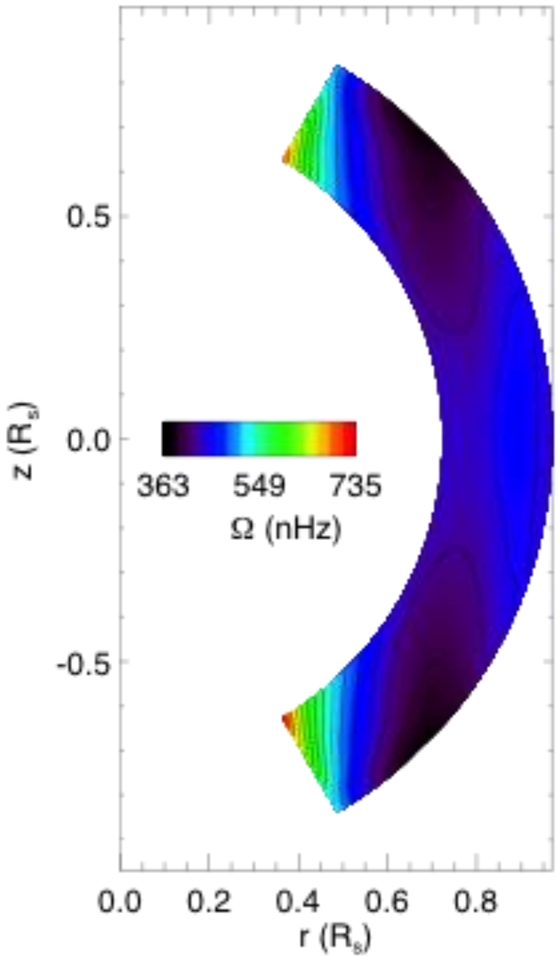


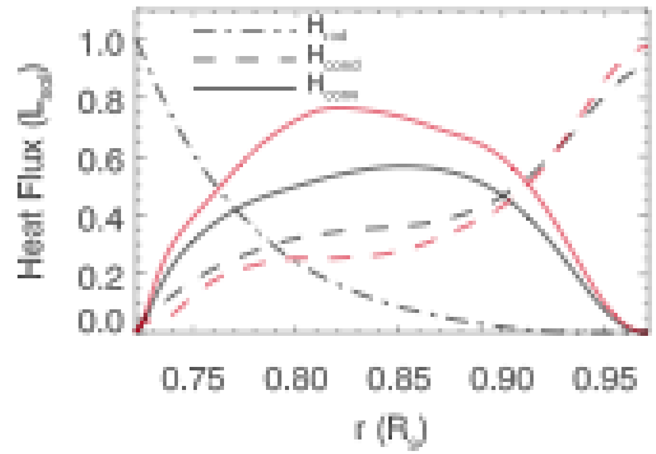


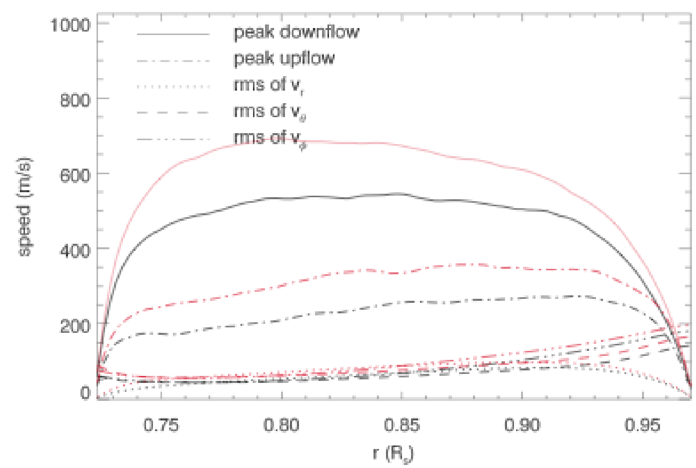
Effect of magnetic fields on differential rotation

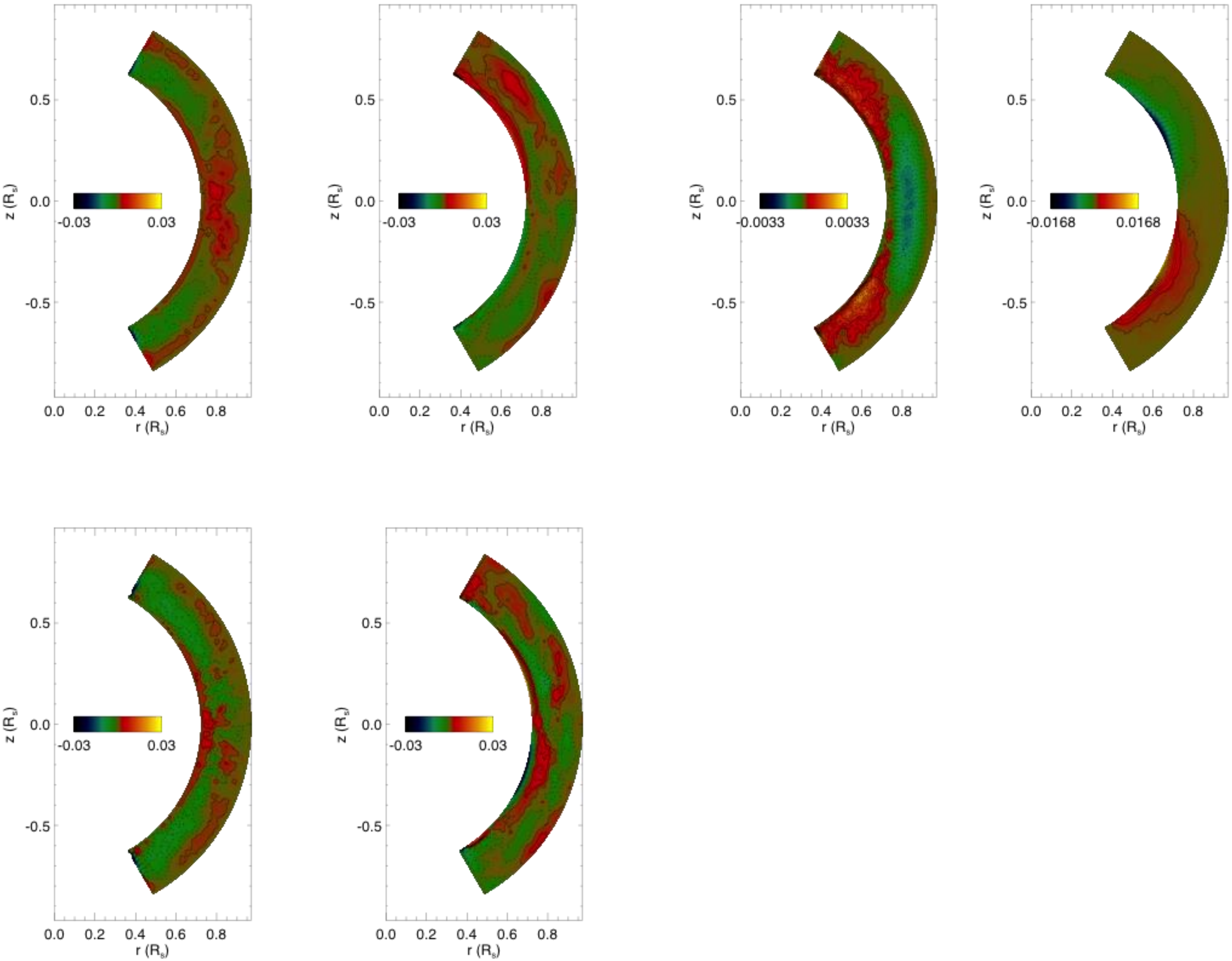
Set B to zero →





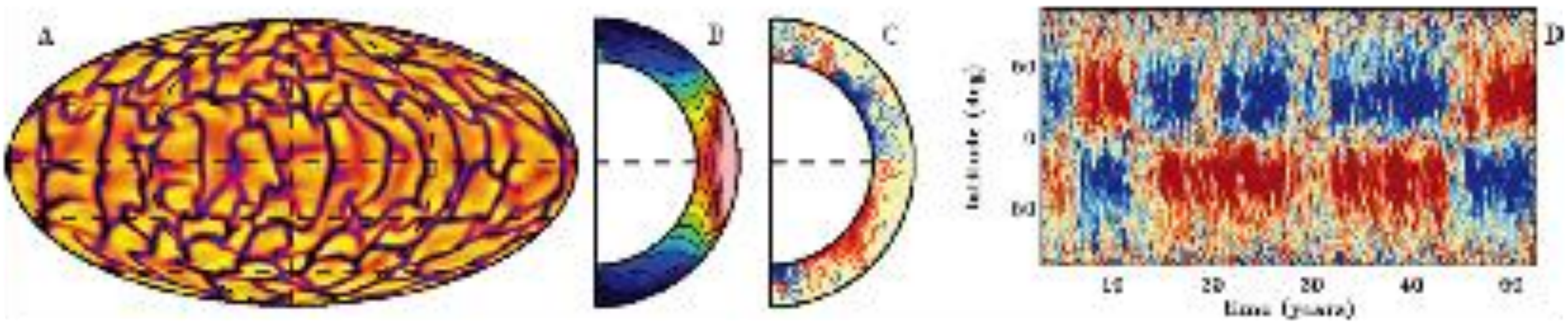




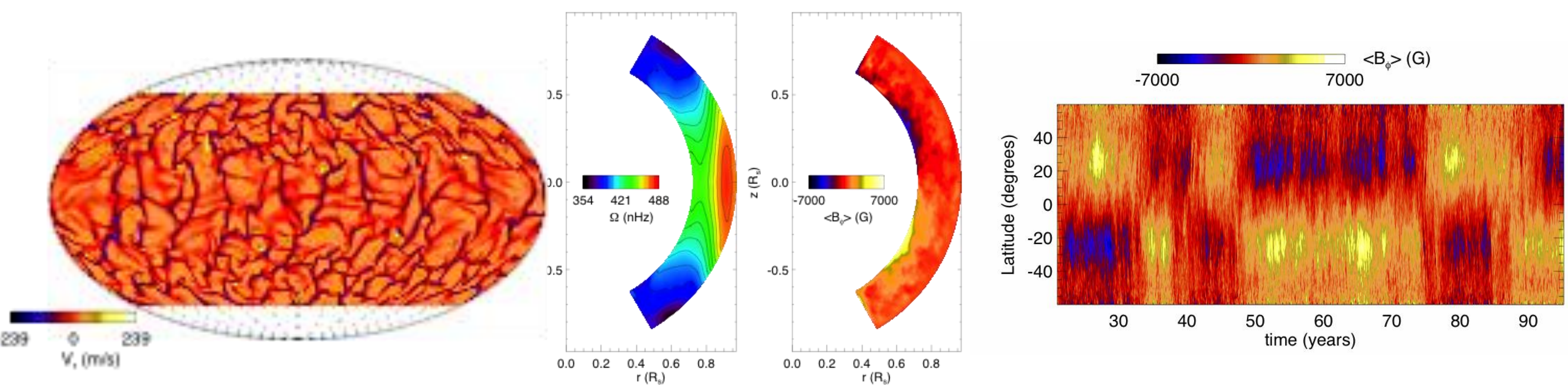

z (R_s)
r (R_s)
-0.03
0.03
-0.0033
0.0033
-0.0168
0.0168
0.5
0.0
-0.5
0.0 0.2 0.4 0.6 0.8

Good qualitative agreement with a convective dynamo solution with ASH

ASH solution (Miesch 2011)



FSAM solution



Summary

- MHD simulation of rotating solar convection driven by the radiative heat flux in a partial spherical shell produces a quasi-cyclic large-scale mean field with a period of about 10 years, undergoing irregular polarity reversals

- The mean axisymmetric toroidal magnetic field peaks at the bottom of the convection zone, reaching a value of about 7000 G. Including the fluctuating component, individual channels of strong field reaching 30kG are present.

- The axisymmetric mean field shows a current helicity that is predominantly negative in the northern hemisphere (consistent with the sense of twist of the solar active regions). But the current helicity of the fluctuating component is opposite, and is of much larger amplitude.

- The presence of the magnetic field appears to be important for maintaining the solar-like differential rotation. Without the magnetic field, the convective flows drive a differential rotation with a faster rotating polar-region