How anisotropic can you get? models and observations

W H Matthaeus

What are causes of anisotropy?

- Mean magnetic field
- Expansion
  - uniform expansion→radial preferred direction
- Shear (velocity, magnetic, density…)
  - as in Rogallo models
- Rotation
  - Taylor-Proudman thm, etc

These can compete with each other!

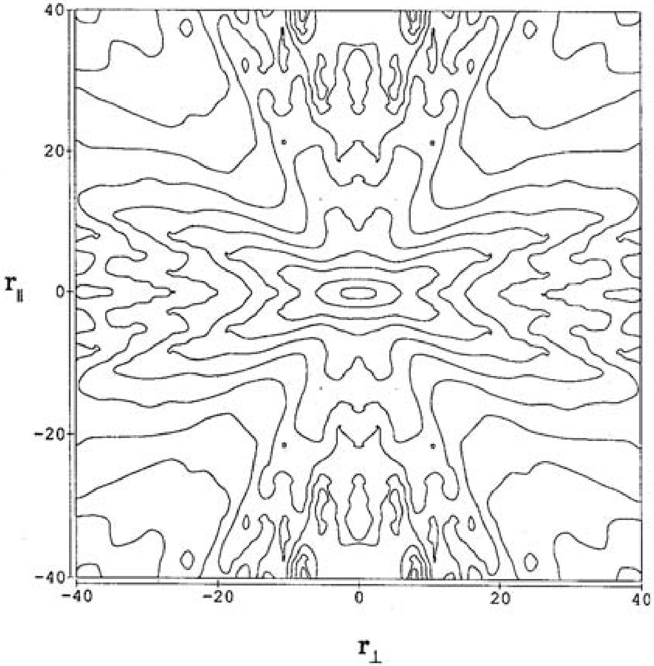
Spectral/correlation anisotropy

- Theory
- Simulation
- Observation in SW

2D axisymmetric magnetic field correlation fn. from ~2 years Of ISEE-3 data

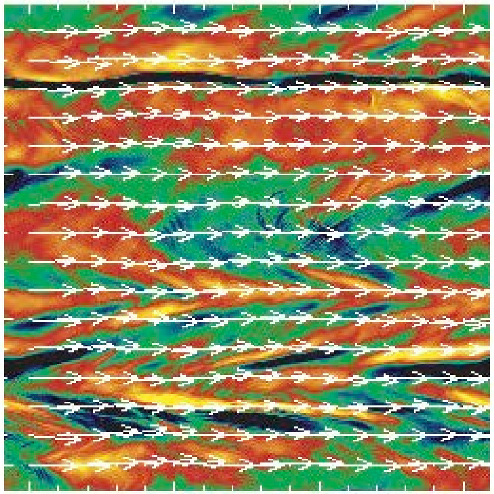
“Maltese Cross”

Mathaeus et al 1990



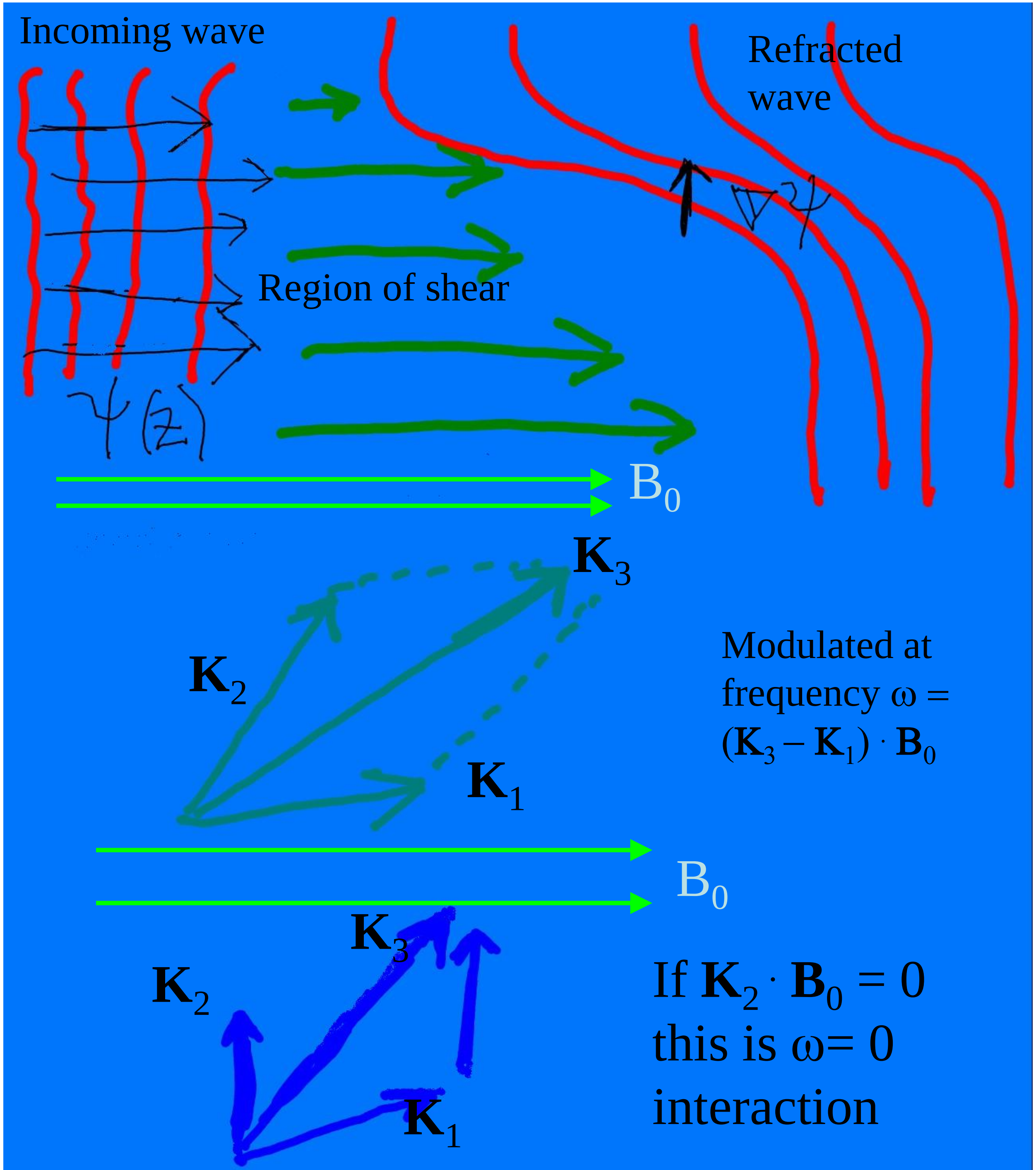
Spectral method simulation with strong B0=10 brms

Parallel Direction →



Perp plane

Dmitruk + whm, 2004



Incoming wave
Refracted wave
Region of shear
ψ(z)
B_0
$\mathbf{K}_3$
$\mathbf{K}_2$
$\mathbf{K}_1$
Modulated at frequency $\omega = (\mathbf{K}_3 - \mathbf{K}_1) \cdot \mathbf{B}_0$
B_0
$\mathbf{K}_3$
$\mathbf{K}_2$
$\mathbf{K}_1$
If $\mathbf{K}_2 \cdot \mathbf{B}_0 = 0$ this is $\omega = 0$ interaction

Oughton et al, Phys Plasmas, 11, 2214 (2004)

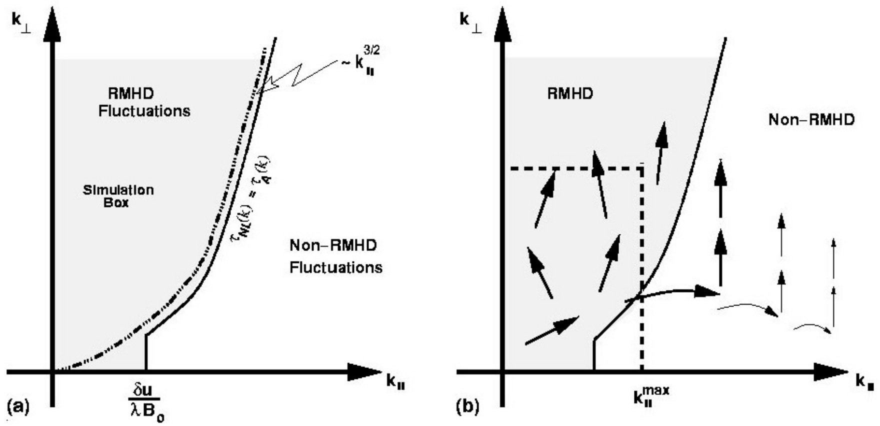
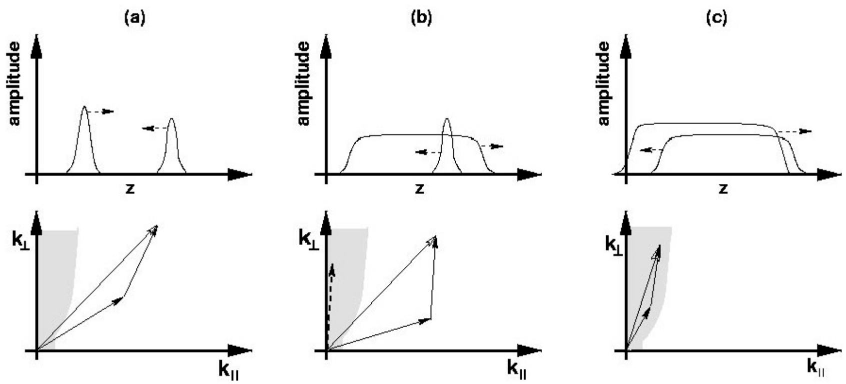


FIG. 1. (a) Cartoon sketch depicting the RMHD and non-RMHD regions, in Fourier space, and their boundary (solid curve) defined by the modal RMHD condition: $\tau_{NL}(\mathbf{k}) = \tau_A(\mathbf{k})$, subject to the fluctuations being of small amplitude. The dash-dot curve represents the asymptotically valid form for the inertial range boundary, $k_\perp \sim (k_\parallel B_0)^{3/2}$. For illustration this has been inap-



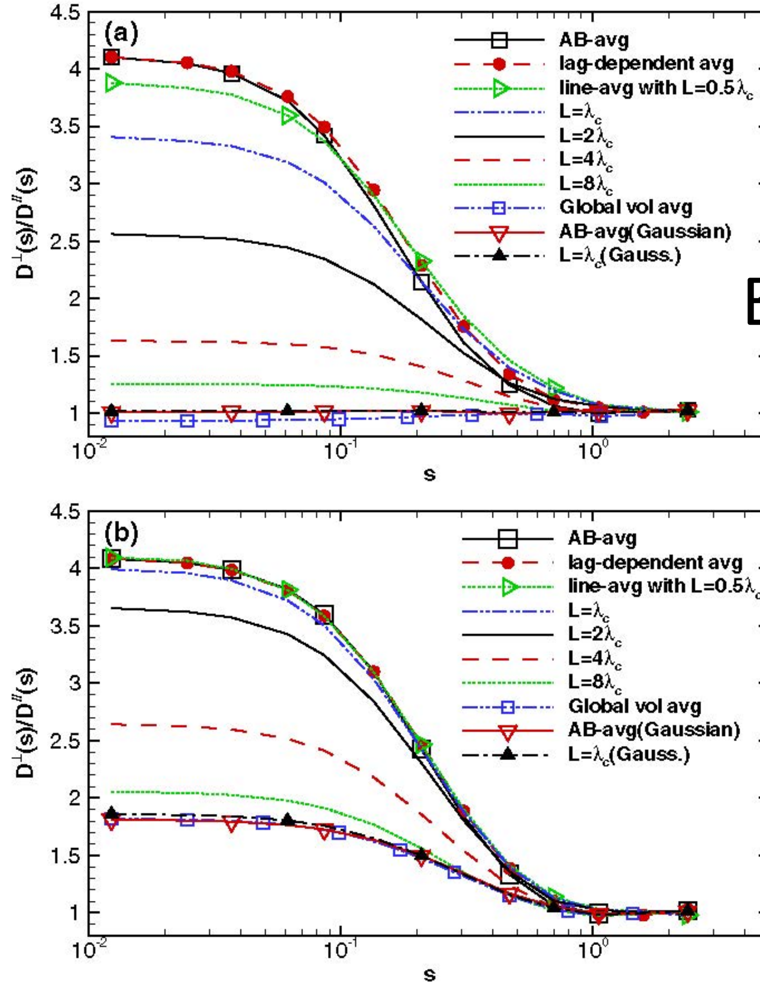
Anisotropy due to mean magnetic field

- global (Robinson Rusbridge, 1970; Montgomery Turner, 1982; Shebalin et al, 1983; Oughton et al, 1994)

- Local (Cho Vishniac, 2000; Milano et al, 2001)
  - Stronger than global
  - Random coordinate system
  - Does not relate directly to spectrum but to higher order statistics

Local and global anisotropy measures in 3D 512^3 MHD

Conditional structure functions, with separation parallel to, or perp to, local magnetic field (Milano et al, Phys Plasma, 2001; Matthaeus et al, 2012)



$B_0 = 0$

$B_0 = 1$

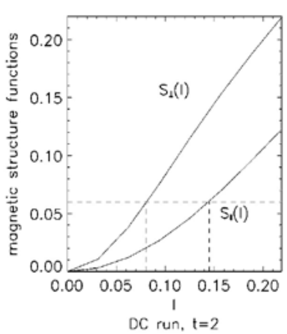
$D(s) = <|\mathbf{B}(\mathbf{x}) - \mathbf{B}(\mathbf{x} + s\, \mathbf{y})|^2>$

$D^{\perp}(s)$: $\mathbf{y} \perp <\mathbf{B}>^*$

$D^{\parallel}(s)$: $\mathbf{y} \parallel <\mathbf{B}>^*$

plots: $D^{\perp}(s)/ D^{\parallel}(s)$

With various definitions of $<\mathbf{B}>^*$



← Individual structure functions
With local "box" average"

Limiting effects

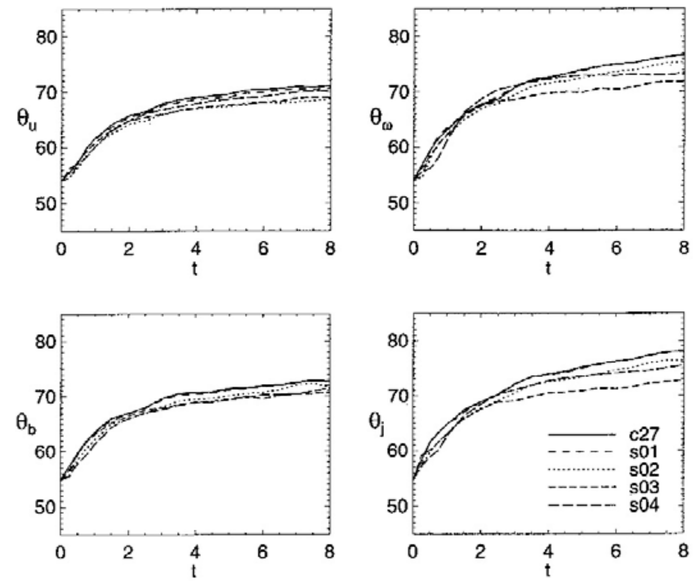
- Competition of drivers
- Specific structure of the energy containing scales
- Parallel spectral transfer
- Compressive couplings
- Sources (shear driving wave particle interactions…)

Comparison of spectral and variance anisotropies

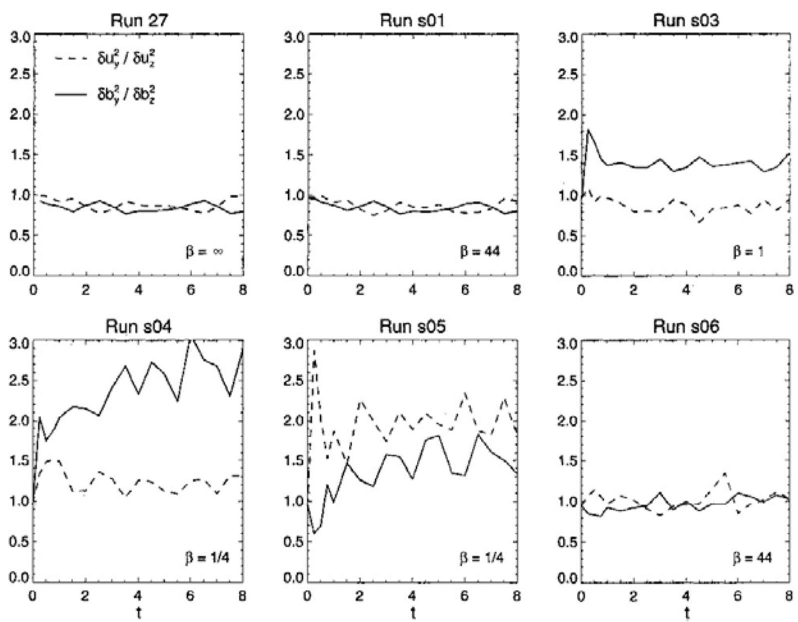
Table 1. Parameters for the Compressible and Incompressible Runs

Run	B_0	$\delta b/B_0$	M_s	β	Anisotropy: Spectral u_T	Spectral u_L	Spectral ρ	Var $\delta u_x^2/\delta u_z^2$
c27	1	1	0	∞	S	—	—	0.87
s01	1	1	0.15	44.4	S	I	M	0.85
s02	1	1	0.5	4	S	I	M	0.82
s03	2	1/2	0.5	1	S	I	S	0.85
s04	4	1/4	0.5	1/4	S	I	S	1.2
s05	4	1/4	0.5	1/4	M	I	M	2.0
s06	1	1	0.15	44.4	S	I	I	1.0

Horizontal spaces separate runs with distinct initial data. Run c27 is incompressible, runs s01-s04 (cat solenoidal (transverse) initial velocity, and runs s05-s06 (category 2) an irrotational (longitudinal) initial $N = 64$ Fourier modes are used in each direction, $\nu = \mu = 1/250$, and the initially excited wavenumbers li and 8. Columns 6-10 summarize the results of the runs in terms of (1) the level of spectral anisotrop (S, strong; M, moderate; I, isotropic), and (2) the average of the variance ratios (for $t \geq 2$). All runs spectral anisotropy in ω, **b**, and **j**.



ure 2. Anisotropy angles as a function of time for **u**, ω, **b**, and **j**. Note the approximate ilarity between the incompressible and compressible results. Angles are in degrees.



- Incompressible MHD does not collapse to "Alfven mode"
- Compressible MHD evolves towards low parallel variance
- Spectral anisotrpy Kperp>Kpar always occurs

Conclusions/points of discussion

- Many factors can cause/limit anisotropy
- Variance anisotropy is not a property of incompressible MHD
- Spectral anisotropy is a function of Reynolds number
- Compressive modes sometimes/always much more isotropic than incomp. Modes

→ IN MODELING: better NOT make extreme assumptions at the onset!

→ IN MODELING: better to not make extreme assumptions about anisotropy except maybe in extreme circumstances

→ IN MODELING: how can local anisotropy be built in? It's a higher order statistic…do we need it ?