Helicity, pseudoinvariants and 1/f noise: can modeling capture these effects?

W H Matthaeus

Helicity, pseudoinvariants and 1/f noise

- Recall inverse cascade scenario
- When do you get 1/f noise? → when there is a back-transferred invariant!
  - Cases with: 2D NS, 2D MHD, 3DMHD c. helicity
- Other cases? → When another quantity is "almost invariant"
  - e.g., strong mean field 3D MHD with NO helicity; hydro with rotation, MHD with rotation…
- Role of nonlocality
- Influence on prediction
- can this be built into models?

Nonlinearity and waves in (incompressible) MHD

Velocity fluctuation v $\nabla \bullet v = 0$

Magnetic fluctuation b $\nabla \bullet B = 0$

Mean magnetic fld B_0 $B = B_0 + b$

Momentum

$$\frac{\partial \mathbf{v}}{\partial t} \sim - v \bullet \nabla v + J \times B + \nu \nabla^2 v$$

Induction

$$\frac{\partial \mathbf{b}}{\partial t} \sim - \mathbf{v} \bullet \nabla \mathbf{b} + \mathbf{b} \bullet \nabla \mathbf{v} + B_0 \bullet \nabla \mathbf{v} \;\; \mu \nabla^2 \mathbf{b}$$

nonlinear

$$\boldsymbol{b}(\boldsymbol{x}) = \int d^3k \, \boldsymbol{b}(\boldsymbol{k}) \, e^{\,ik \bullet x}$$

$$\partial \boldsymbol{b}(\mathbf{k})/\partial t \sim \sum_{\boldsymbol{k}=\boldsymbol{r}+\boldsymbol{p}} \boldsymbol{b}(\boldsymbol{r}) \boldsymbol{v}(\boldsymbol{p})$$

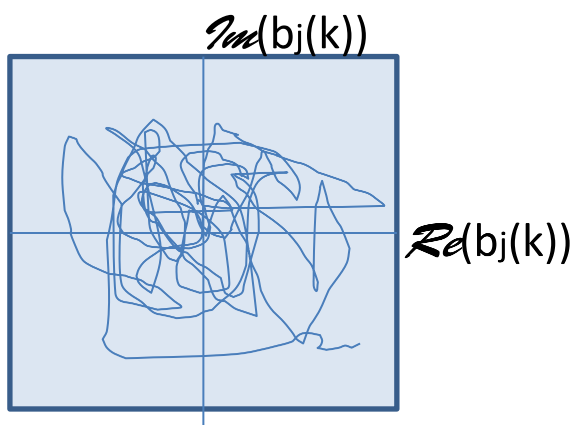
Cross scale couplings: chaos
Random walk in phase space
→ "turbulence"

linear

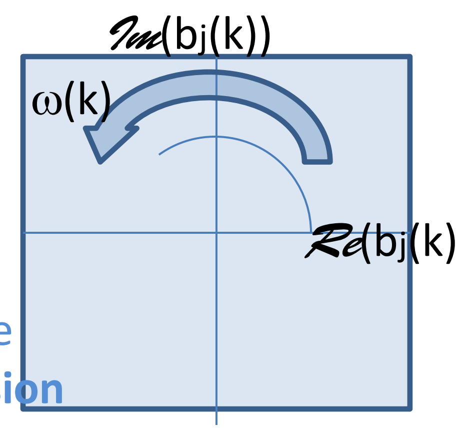
$$\mathbf{b}(\mathbf{k},t) \sim |\mathbf{b}(k,0)| \, e^{\,i\omega t}$$

eigenmodes (independent)
systematic rotation in
phase space → "waves"

Trajectory in time
of a Fourier mode
in strong
turbulence



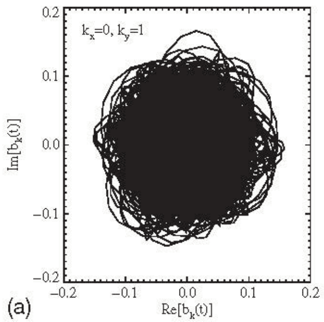
pure wave
Fourier mode
ω(k): dispersion relation



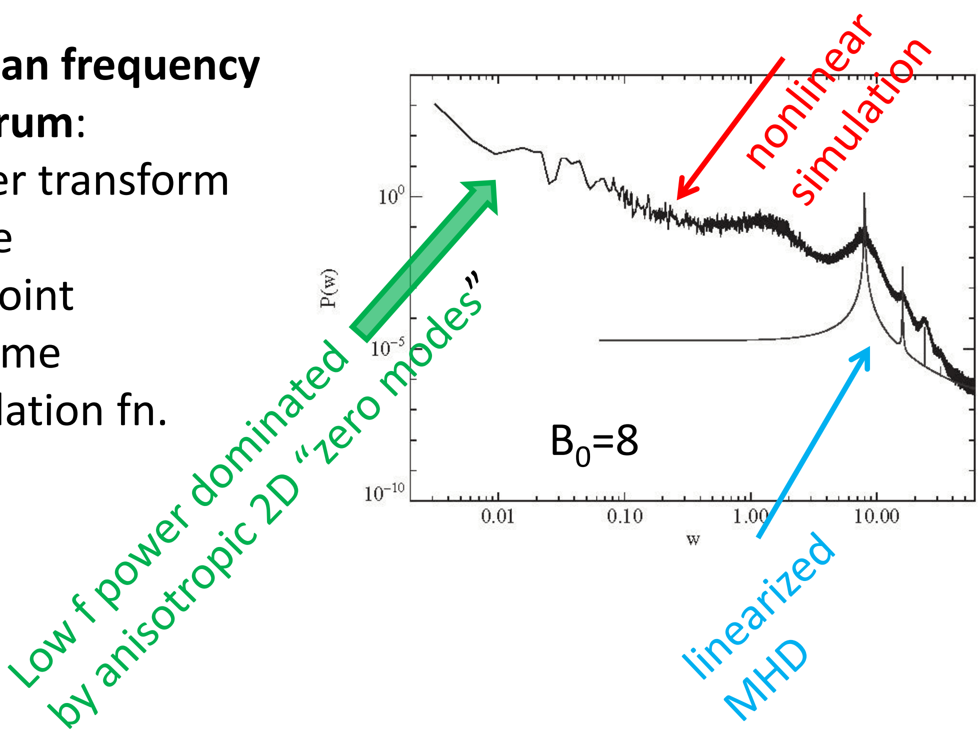
Numerical experiments on frequency spectrum of MHD Turbulence with mean field:

maximum of ~13% of energy in linear modes – that occurs when $dB/B_0 \sim 1/2$

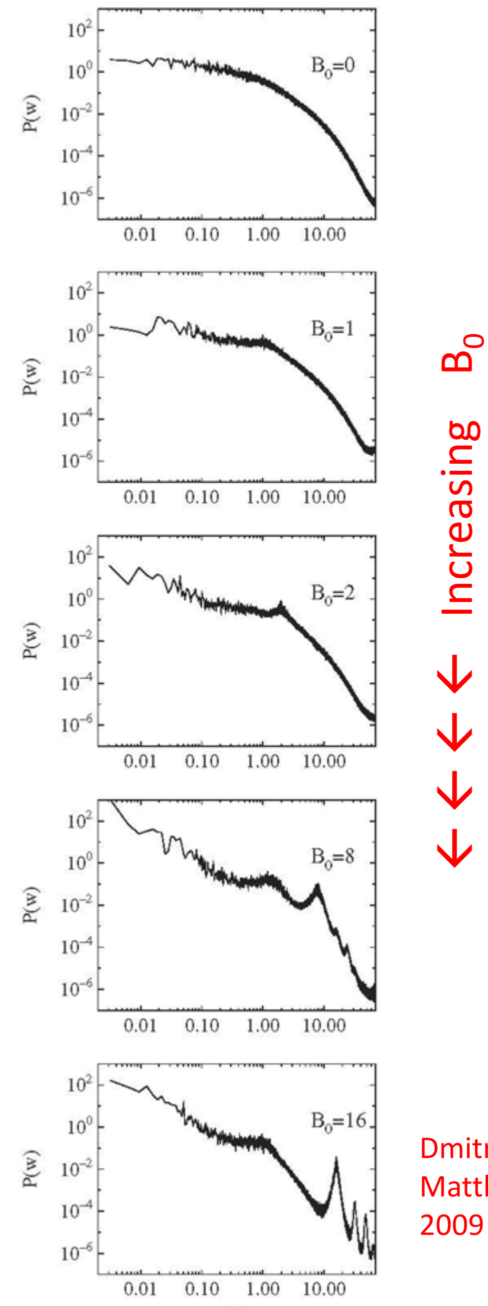
behavior of a Fourier mode In time, from simulation



Eulerian frequency spectrum: Fourier transform of the one point two time Correlation fn.



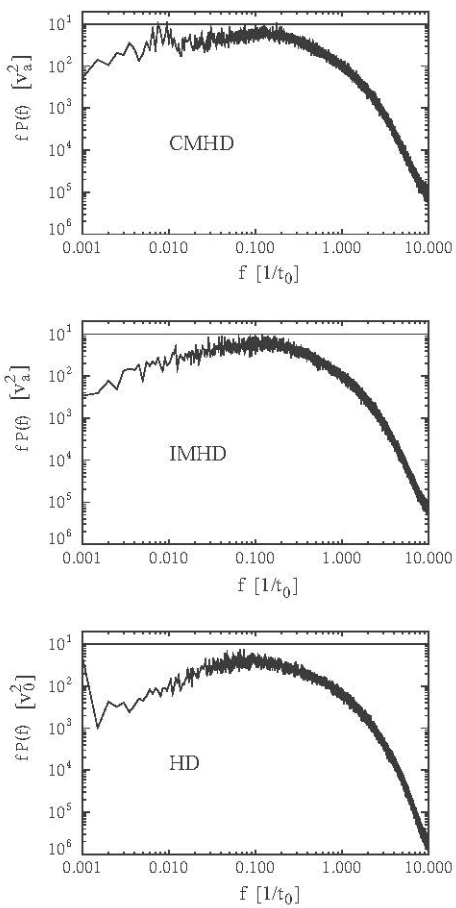
Eulerian frequency spectra



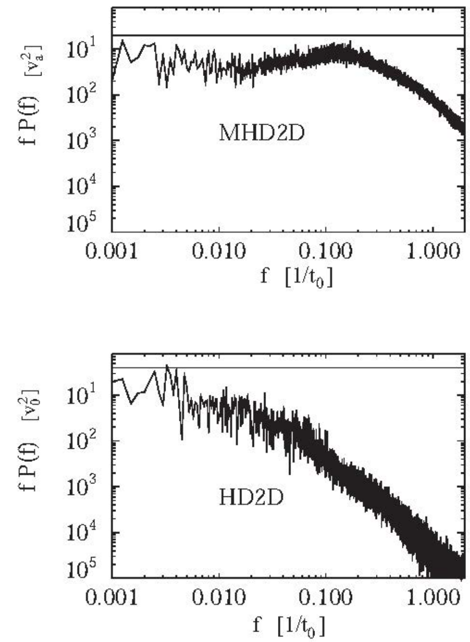
Dmitruk & Matthaeus, 2009

Eulerian frequency spectra: compensated fP(f)

3D cases:
direct cascade
only

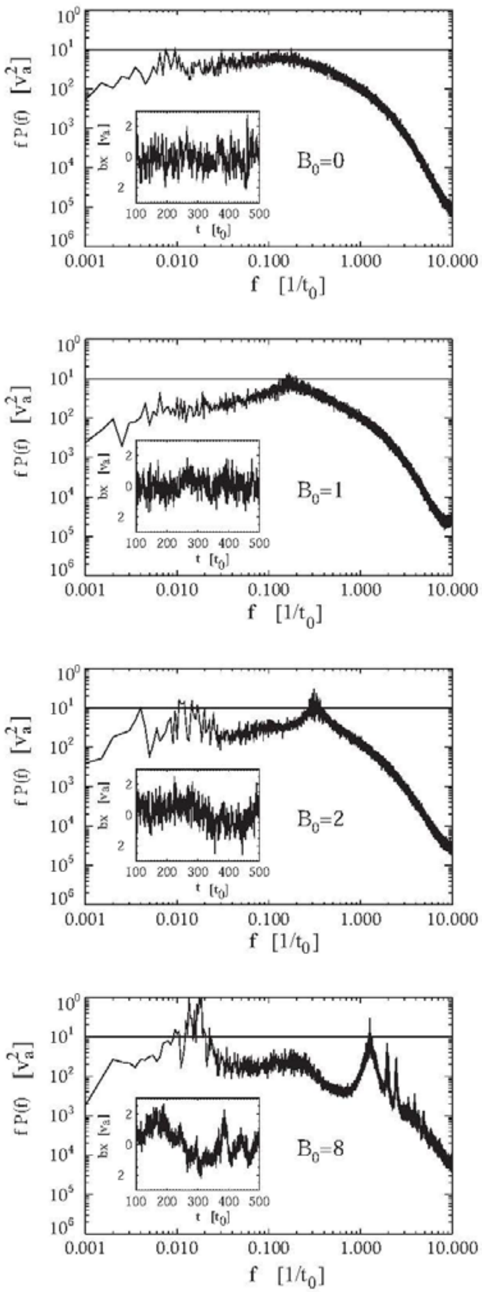


2D cases
(inverse cascade)\



3D MHD nonhelical driven runs , with increasing mean field B0: ***1/f "turns on"***

Compensated Eulerian frequency spectra fP(f)



Evidence of :

- very long time scales!
- Strong condensation
- Force-free in condensed scales

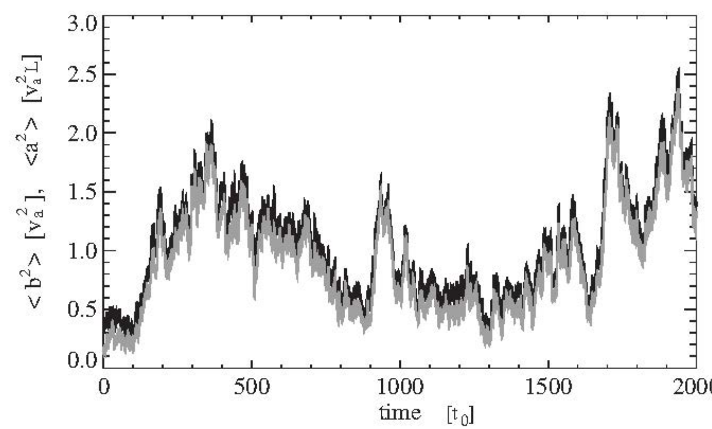


FIG. 7. Time history of fluctuation magnetic energy density $\langle b^2 \rangle$ (black) in units of V_a^2 and mean-square magnetic potential $\langle a^2 \rangle$ (gray) in units of $V_a^2 L$ (gray) for a driven 3D simulation with B_0 =8. Units of t_0 are used for time.

Relaxation times to thermal equilibrium

- Longer for “two dimensionalized” mode (rotation, or mean field)
  see Mininni et al, PRE 2011
- Increase in nonlocality of couplings
- Is it really an inverse cascade?

How can this effect be incorporated into turbulence modeling (LES/SGS) ??

- Must build in either second conservation law or control of nonlocal interactions
- Clues from shell models?
- Maybe someone knows how to do it already?