LES of MHD Turbulence
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A mixed Eulerian-Lagrangian scalar transport model

Mark Rast
Laboratory for Atmospheric and Space Physics
Department of Astrophysical and Planetary Sciences
University of Colorado, Boulder, USA

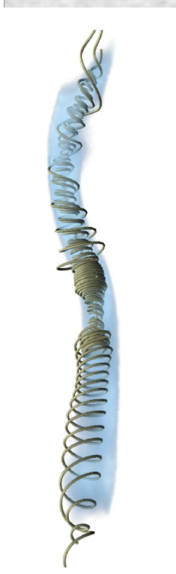
Jean-Francois Pinton
Laboratoire de Physique
Ecole Normale Supérieure de Lyon, CNRS
Université de Lyon, Lyon, France

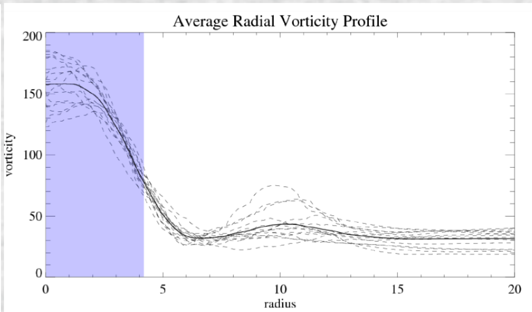
Is it possible to construct an exact scalar transport model for an idealized turbulence analog?

Assumption:

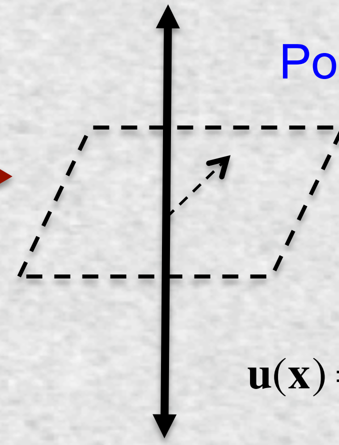
Turbulent transport is dominated by the vortical structures making up the flows

A Two-Dimensional Model of 3D Turbulence:





Gruchalla et al. (2009)

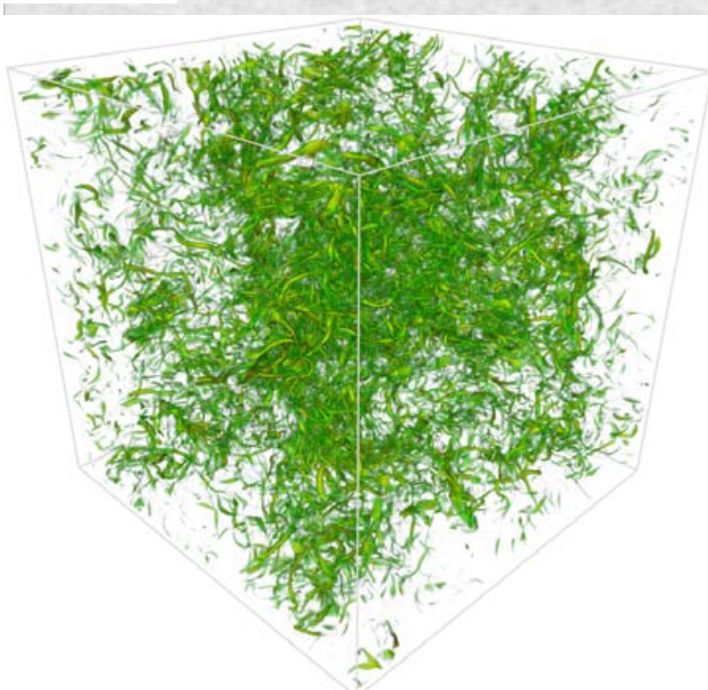


Point (line) vortex:

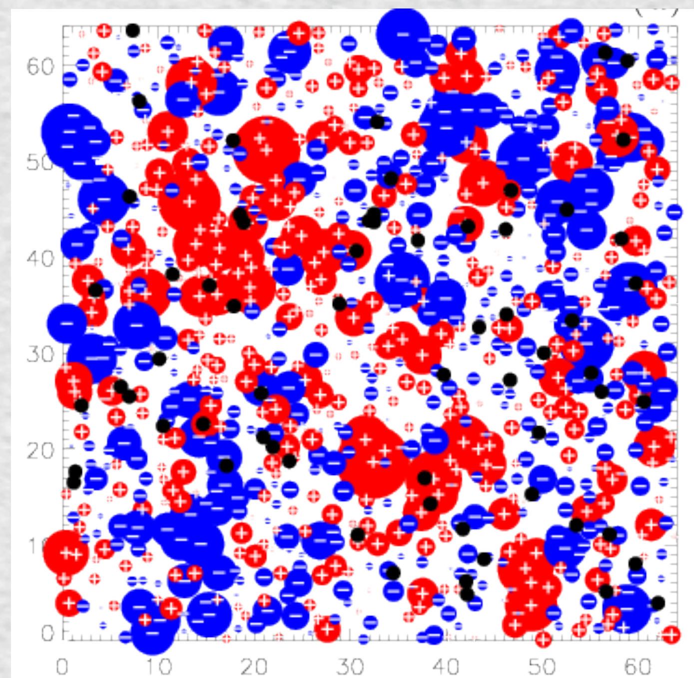
$$u_\theta \sim 1/r$$

$$u_r = u_z = 0$$

$$\mathbf{u}(\mathbf{x}) = \sum_{k=1}^{N} \frac{\Gamma_k}{2\pi |\mathbf{x}-\mathbf{x}_k|} \left(\hat{z} \times \left(\widehat{x - x_k} \right) \right)$$



Mininni et al. (2008+)



Rast & Pinton (2009)

Point vortex simulations:

$$\mathbf{u}(\mathbf{x}) = \sum_{k=1}^{N} \frac{\Gamma_k}{2\pi|\mathbf{x}-\mathbf{x}_k|}\left(\hat{z}\times\left(\widehat{x-x_k}\right)\right)$$

$$u_\theta \sim 1/r$$

$$u_r \sim 0$$

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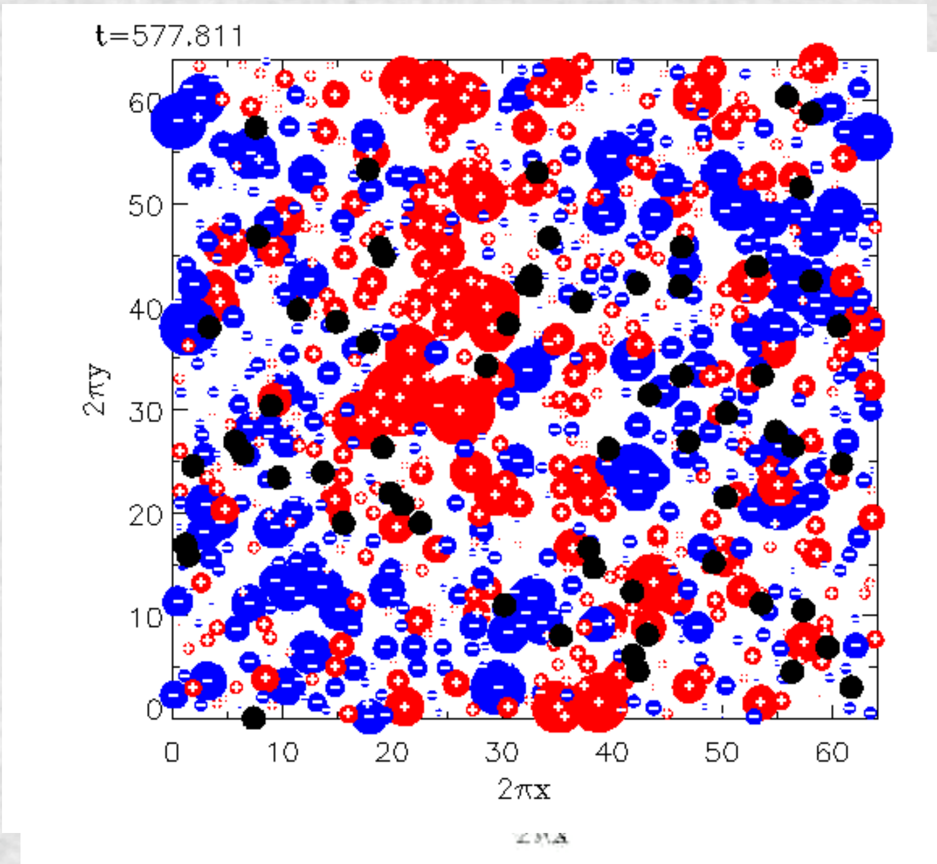
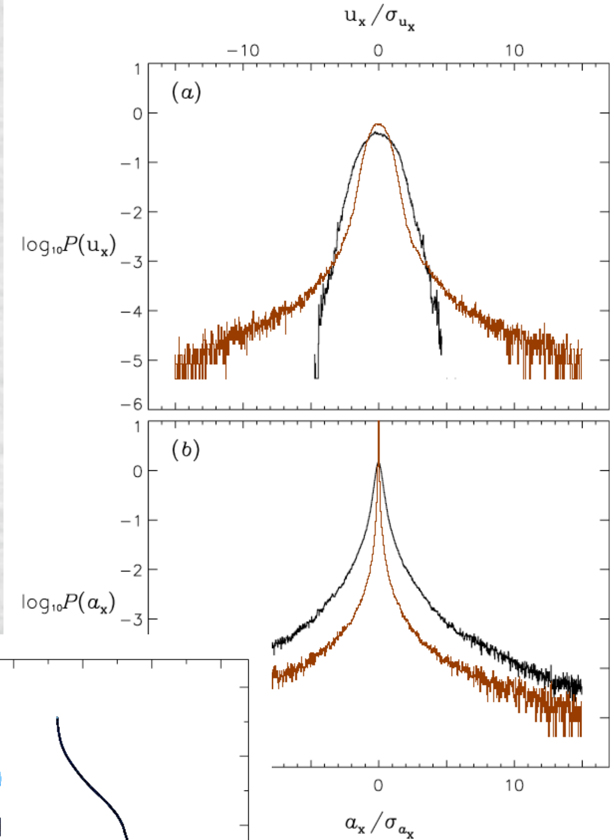
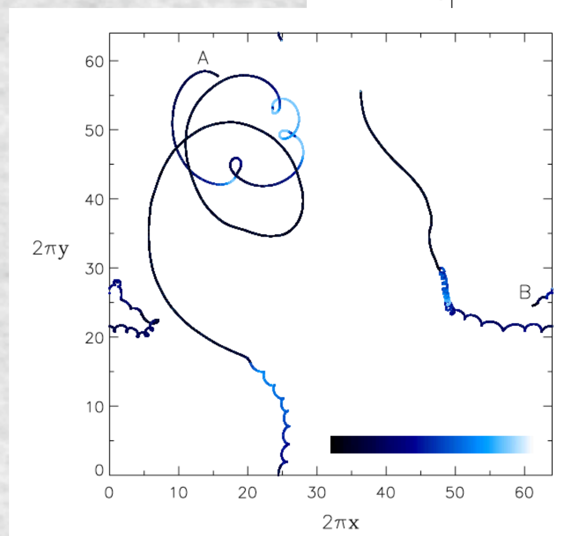
Merger of close vortices

Stirring by vortex creation

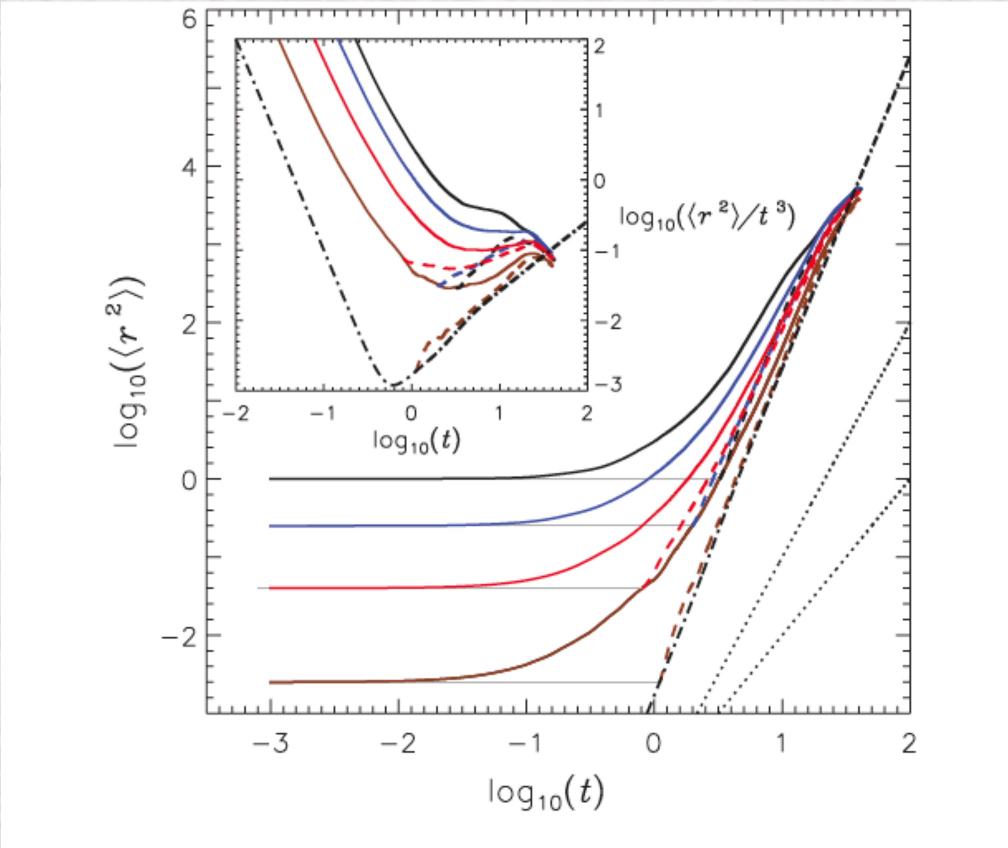






"Trapping" events at all scales dominate Lagrangian single point and pair dispersion statistics

Pair Dispersion (trapping dominates scaling):



$$r^2 = r_0^2 + \left(t - t_d\right)^\alpha$$

Delay time uniform distribution

then

$$\langle r^2 \rangle \propto \int \left(t - t_d\right)^\alpha dt \propto t^{\alpha+1}$$

With delay uniform distribution of delay times:

Underlying Batchelor scaling: $r^2 \sim t^2 \quad \Rightarrow \quad \langle r^2 \rangle \propto t^3$

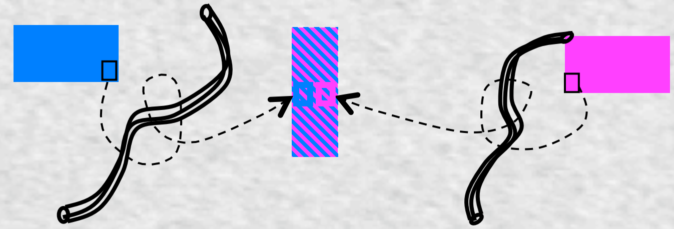
Underlying Richardson scaling: $r^2 \sim t^3 \quad \Rightarrow \quad \langle r^2 \rangle \propto t^4$ Observed

Constructing a transport model:

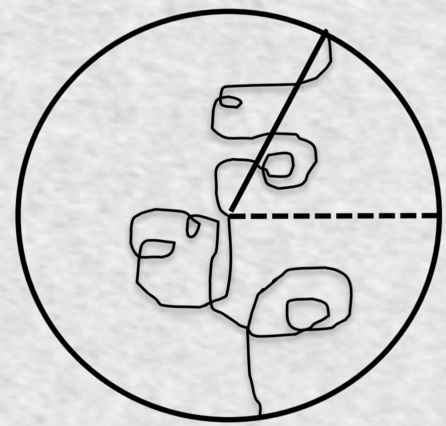
Consider transport of a scalar quantity, c.

$$\frac{\partial c}{\partial t} + \mathbf{u}\cdot\nabla c = S(\mathbf{x},t) \qquad c(\mathbf{x},t) = \int S(\mathbf{x}',t')\,\mathrm{G}(\mathbf{x},t\,|\,\mathbf{x}',t')\,d\mathbf{x}'dt'$$

$$\langle c(\mathbf{x},t)\rangle = \int S(\mathbf{x}',t')\,P(\mathbf{x},t\,|\,\mathbf{x}',t')\,d\mathbf{x}'dt'$$



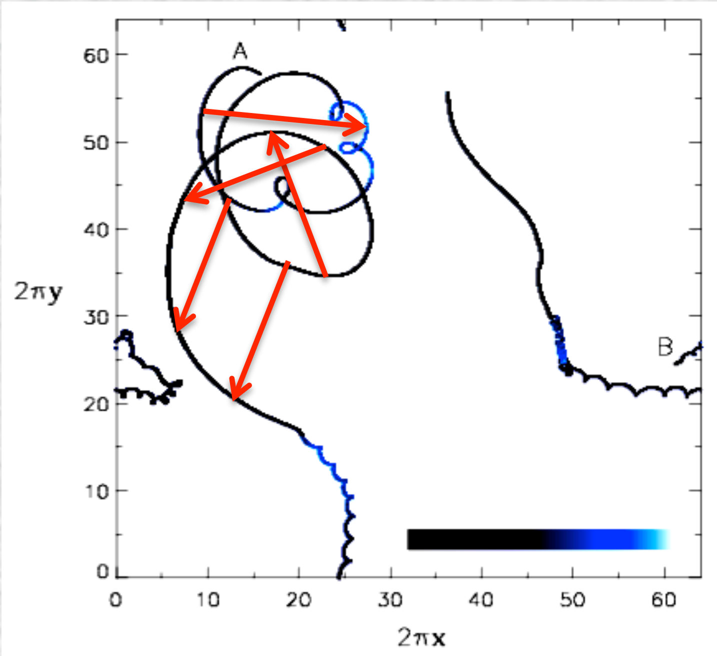
Measure $P(\mathbf{x},t\,|\,\mathbf{x}',t')$:



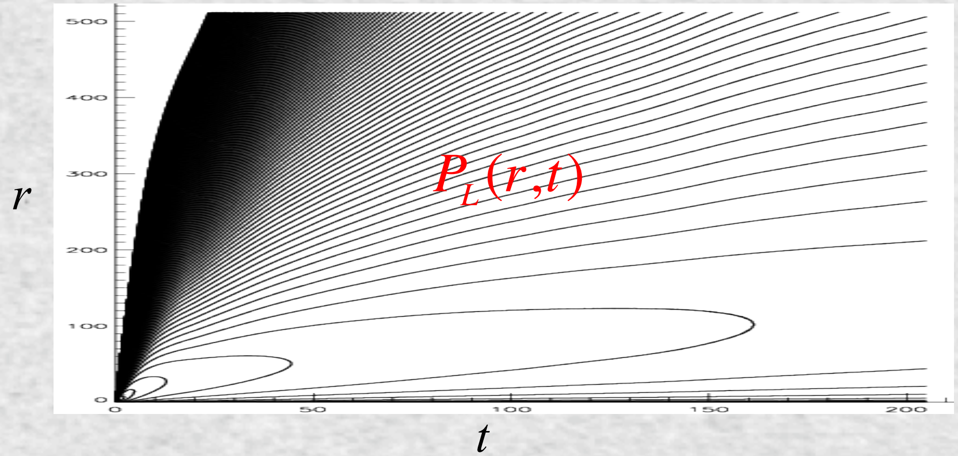
If one had this completely, the problem would be solved.

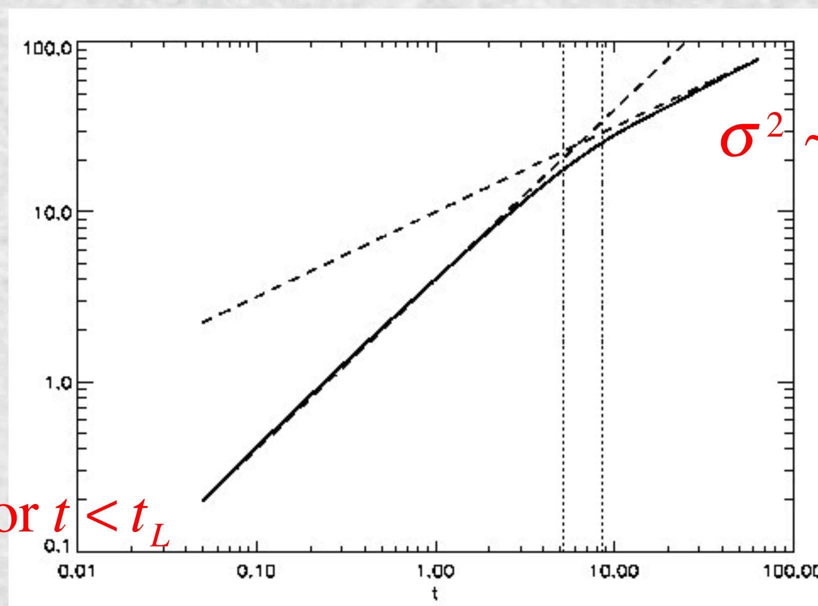
Instead take: $P(\mathbf{x},t\,|\,\mathbf{x}',t') = \dfrac{P(r,t)}{2\pi r}$ (isotropy)

From Lagrangian trajectories: $P(r,t)$, the probability of traveling an Eulerian distance r in time t along a Lagrangian path.



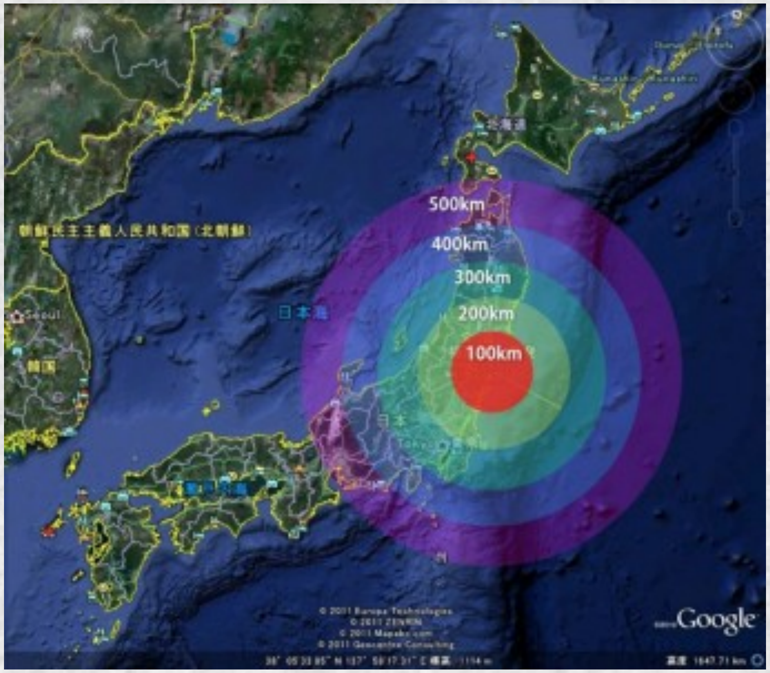
$$P(r,t) \sim r e^{-r^2/\sigma(t)^2}$$



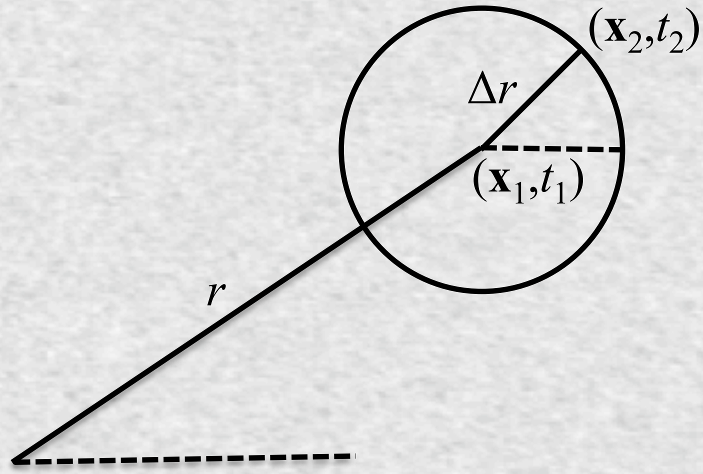


$\sigma^2 \sim t$ for $t > t_L$

$\sigma^2 \sim t^2$ for $t < t_L$



What about the variance of c ?



$$\langle c^2(\mathbf{x},t)\rangle = \int S(\mathbf{x}_1,\mathbf{x}_2,t_1,t_2)P(\mathbf{x},\mathbf{x},t,t \mid \mathbf{x}_1,\mathbf{x}_2,t_1,t_2)d\mathbf{x}_1 d\mathbf{x}_2 dt_1 dt_2$$

$$P(\mathbf{x},\mathbf{x},t,t \mid \mathbf{x}_1,\mathbf{x}_2,t_1,t_2) = P(\mathbf{x}_1,\mathbf{x}_2,t_1,t_2 \mid \mathbf{x},\mathbf{x},t,t)$$

Reversible, statistically steady flow

$$P(\mathbf{x},\mathbf{x},t,t \mid \mathbf{x}_1,\mathbf{x}_2,t_1,t_2) = \frac{1}{4\pi r\Delta r}P(r,t)P(\Delta r,\Delta t \mid r,t)$$

$\uparrow$

Pair dispersion problem generalized to include time.

Toward a turbulent transport model:

1. Model turbulent transport using the statistics of Lagrangian trajectories in a point vortex flow.

2. Identify coherent vortical structures in simulations of real three-dimensional turbulence.

3. Use the Lagrangian statistics of in presence of these coherent structures in place of those due to point vortices in a transport model.

For LES must also:

Relate the statistics of the coherent vortical structures to the large scale flow producing them.