GTP Workshop on LES of MHD
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Self-consistent turbulence modeling on magnetic reconnection

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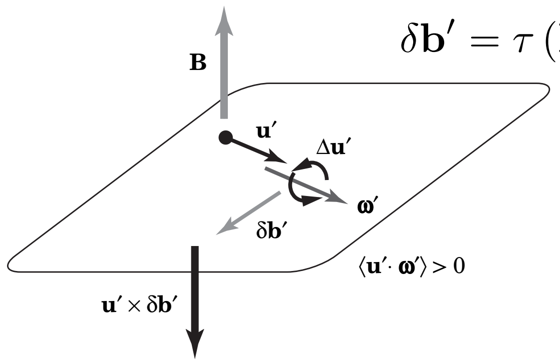
- Cross-helicity effects
- Transport enhancement and suppression
- A self-consistent model
- Numerical simulation

α and cross-helicity effects

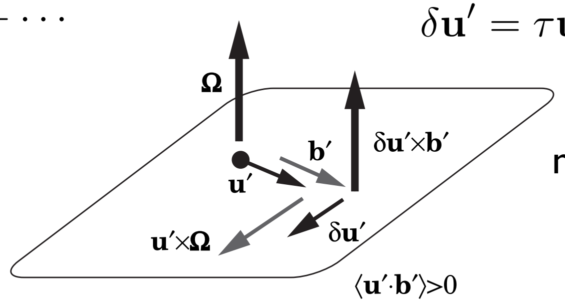
(Yokoi, GAFD **107**, 114, 2013)

$$\delta \mathbf{b}' = \tau (\mathbf{B} \cdot \nabla) \mathbf{u} + \cdots$$



kinetic helicity effect

$$\delta \mathbf{u}' = \tau \mathbf{u}' \times \boldsymbol{\Omega} + \cdots$$

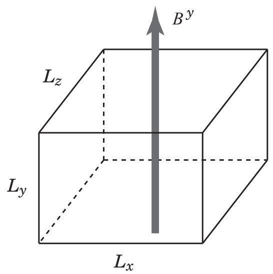


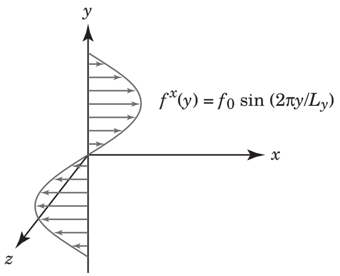
mean vorticity
$\boldsymbol{\Omega} = \nabla \times \mathbf{U}$

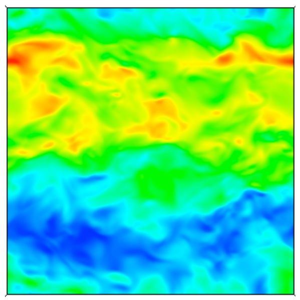
cross-helicity effect

$$\langle \mathbf{u}' \times \mathbf{b}' \rangle = \overbrace{\alpha \mathbf{B} - \beta \nabla \times \mathbf{B}}^{\alpha \text{ dynamo}} + \underbrace{\gamma \nabla \times \mathbf{U}}_{\text{cross-helicity dynamo}}$$

Test using DNS (Yokoi & Balarac, 2011)

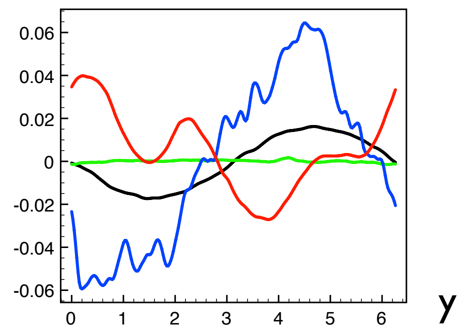






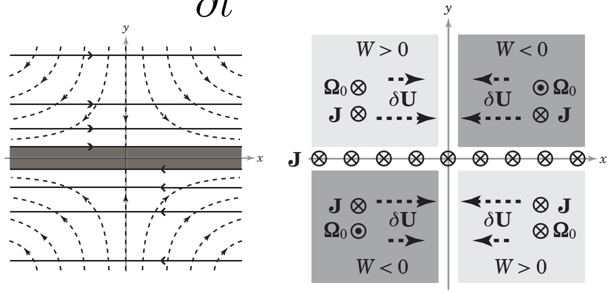
streamwise velocity

	$\langle \mathbf{u}' \times \mathbf{b}' \rangle$	black
helicity	$\alpha \mathbf{B}$	green
energy	$-\beta \mathbf{J}$	blue
cross helicity	$\gamma \boldsymbol{\Omega}$	red



Cross-helicity distribution

$$\frac{\partial W}{\partial t} = +\beta \mathbf{J} \cdot \mathbf{\Omega} + \cdots$$



(Yokoi & Hoshino, PoP, 2011)

Transport enhancement and suppression

turbulent electromotive force

$$\langle \mathbf{u}' \times \mathbf{b}' \rangle = -\beta \mathbf{J} + \gamma \mathbf{\Omega} + \alpha \mathbf{B}$$

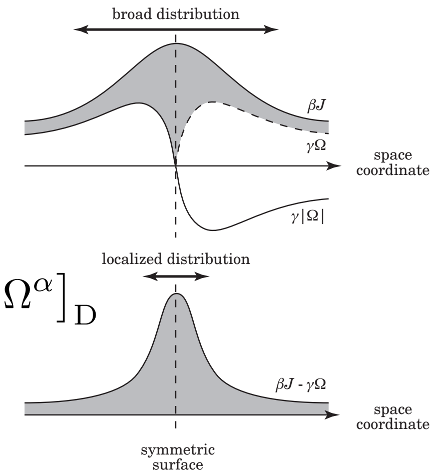
Reynolds (+ turb. Maxwell) stress

$$\left[\langle u'^\alpha u'^\beta - b'^\alpha b'^\beta \rangle \right]_{\mathrm{D}}$$

$$= -\nu_{\mathrm{K}} \mathcal{S}^{\alpha\beta} + \nu_{\mathrm{M}} \mathcal{M}^{\alpha\beta} + \left[\Gamma^\alpha \Omega^\beta + \Gamma^\beta \Omega^\alpha \right]_{\mathrm{D}}$$

$\mathcal{S}$: mean velocity strain

$\mathcal{M}$: mean magnetic strain



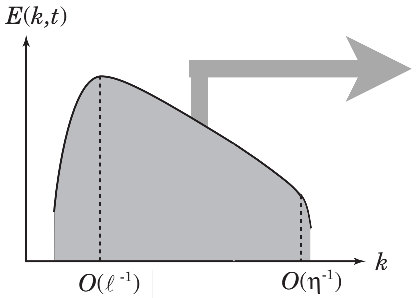
Self-consistent model

mean fields

$\overline{\rho}$

$\mathbf{U}$

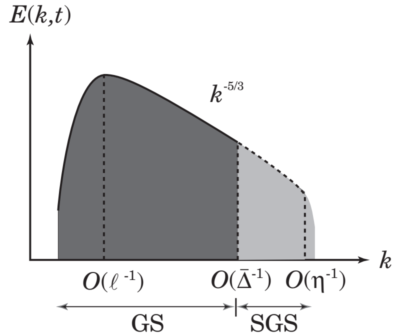
$\mathbf{B}$



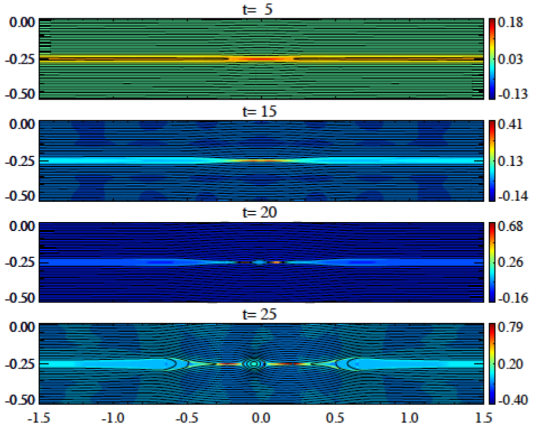
turbulent statistical quantities

$$\begin{cases} K \equiv \dfrac{1}{2} \langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle \\ \varepsilon \equiv \nu \left\langle \left(\dfrac{\partial u'^a}{\partial x^b} \right)^2 \right\rangle + \lambda \left\langle \left(\dfrac{\partial b'^a}{\partial x^b} \right)^2 \right\rangle \\ W \equiv \langle \mathbf{u}' \cdot \mathbf{b}' \rangle \\ \left[K_{\mathrm{R}} \equiv \dfrac{1}{2} \langle \mathbf{u}'^2 - \mathbf{b}'^2 \rangle \right] \\ \left[\varepsilon_W \equiv (\nu + \lambda) \left\langle \dfrac{\partial u'^a}{\partial x^b} \dfrac{\partial b'^a}{\partial x^b} \right\rangle \right] \\ \left[H \equiv \langle -\mathbf{u}' \cdot \boldsymbol{\omega}' + \mathbf{b}' \cdot \mathbf{j}' \rangle \right] \end{cases}$$

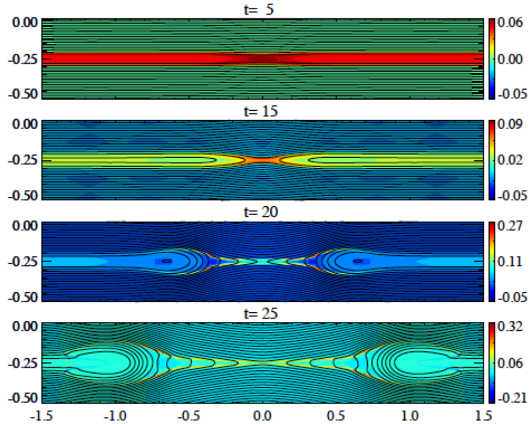
Extension to LES is straightforward



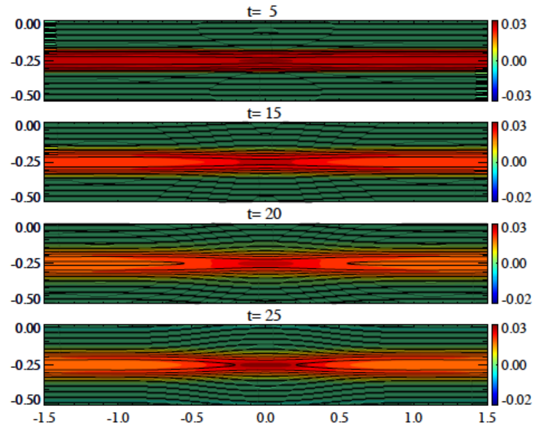
laminar reconnection

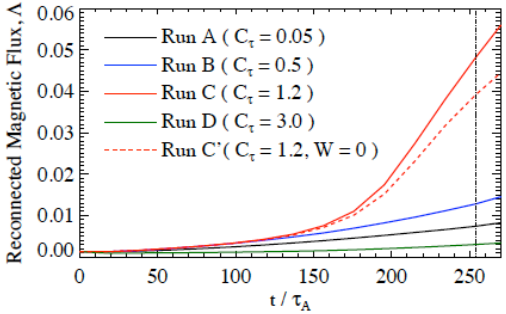


turbulent reconnection



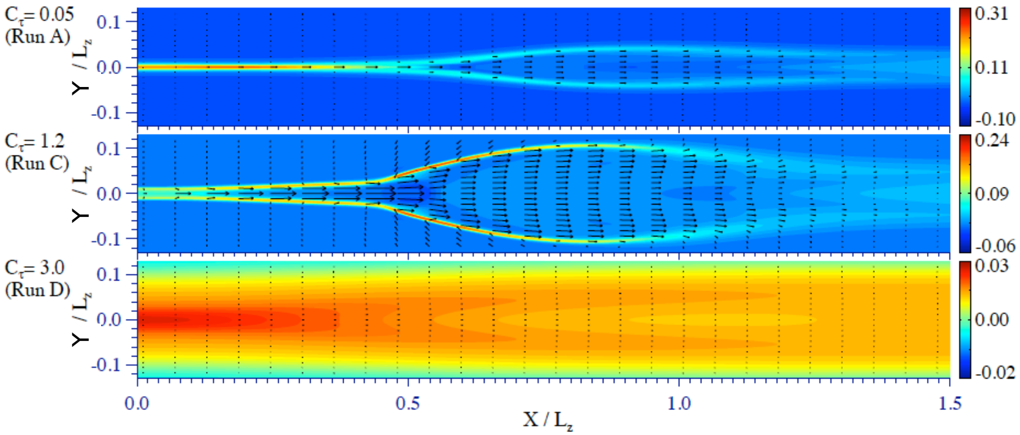
turbulent diffusion

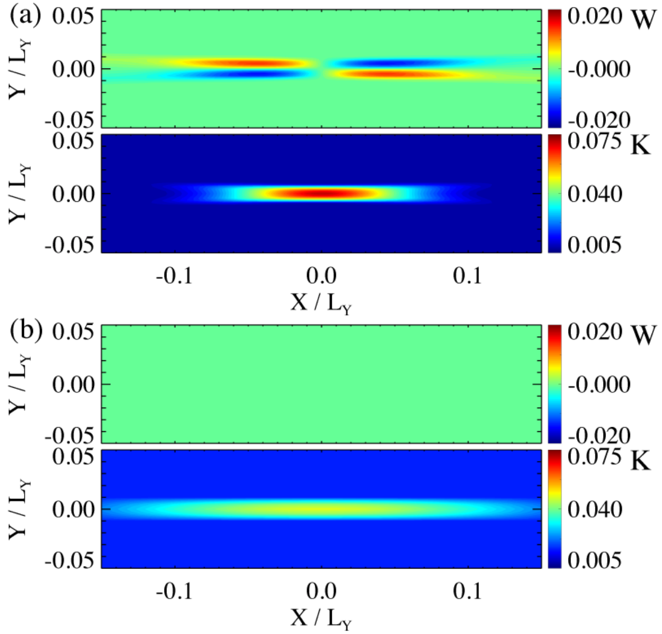




(Higashimori, Yokoi & Hoshino, 2013)

(Yokoi, Higashimori & Hoshino, 2013)





Basic equations to solve

Mean fields

4th-order Runge-Kutta scheme in time
4th-order centered difference in space

$$\frac{\partial \overline{\rho}}{\partial t} = -\nabla \cdot (\overline{\rho}\mathbf{U})$$

$$\frac{\partial}{\partial t}(\overline{\rho}\mathbf{U}) = -\nabla \cdot \left[\overline{\rho}\mathbf{U}\mathbf{U} - \mathbf{B}\mathbf{B} + \left(p + \frac{1}{2}\mathbf{B}^2\right)\boldsymbol{\mathcal{I}}\right]$$

$$\frac{\partial}{\partial t}\left(\frac{p}{\gamma - 1} + \frac{1}{2}\overline{\rho}\mathbf{U}^2 + \frac{1}{2}\mathbf{B}^2\right) = -\nabla \cdot \left[\left(\frac{\gamma}{\gamma - 1}p + \frac{1}{2}\overline{\rho}\mathbf{U}^2\right)\mathbf{U} + \mathbf{E} \times \mathbf{B}\right]$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} = -\mathbf{U} \times \mathbf{B} + \eta\mathbf{J} - \tau\left(W\boldsymbol{\Omega} - K\mathbf{J}\right)$$

Turbulent quantities

$$\beta = \tau K \qquad \frac{\partial K}{\partial t} = -\mathbf{U} \cdot \nabla K + \tau K \mathbf{J}^2 - \tau W \boldsymbol{\Omega} \cdot \mathbf{J} + \mathbf{B} \cdot \nabla W - \frac{K}{\tau}$$

$$\gamma = \tau W \qquad \frac{\partial W}{\partial t} = -\mathbf{U} \cdot \nabla W + \tau K \boldsymbol{\Omega} \cdot \mathbf{J} - \tau W \boldsymbol{\Omega}^2 + \mathbf{B} \cdot \nabla K - C_W \frac{W}{\tau}$$

with K. Higashimori & M. Hoshino, 2013

Initial Conditions

Adiabatic index: $\gamma = 5/3$

Initial plasma beta (inflow region) : $\beta_0 = 0.5$

Boundary conditions: Periodic in both X and Y

Number of grids: 2048 x 512

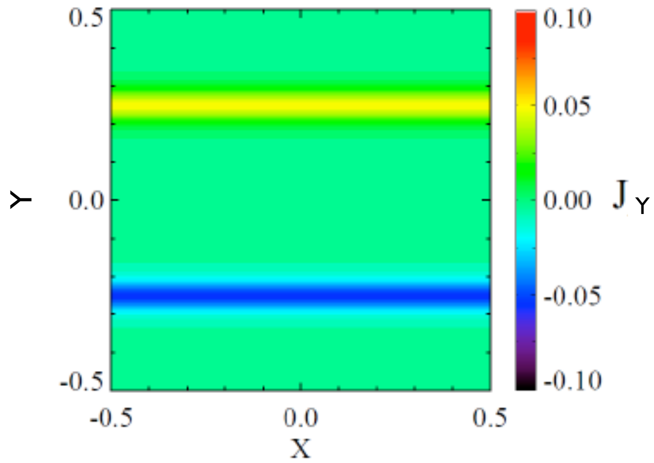
$$\mathbf{U} = 0$$

$$\mathbf{B} = \mathbf{e}_x B_{x0} \left[\tanh\left(\frac{y}{\delta}\right) - \tanh\left(\frac{y - 0.5L_y}{\delta}\right) - 1 \right] + \mathbf{e}_y B_{y0} \sum_{m=1}^{10} \sin\left(\frac{2\pi m x}{L_x}\right)$$

$$B_{y0}/B_{x0} = 1.0 \times 10^{-3} \qquad \delta = 0.02 L_y$$

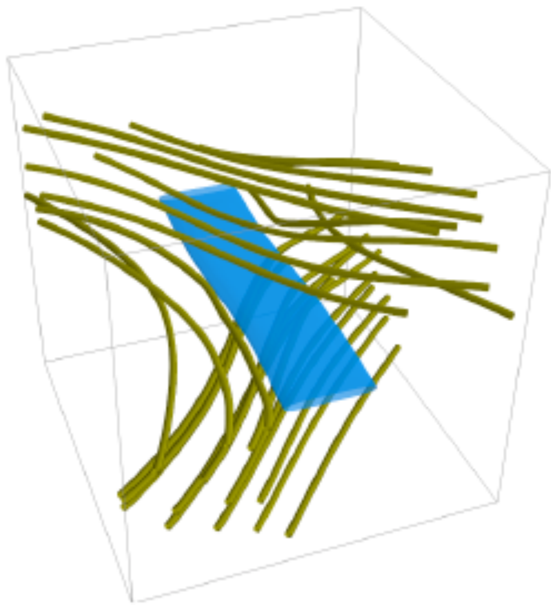
with small magnetic field perturbation in order to initiate reconnection

$$W = 0$$

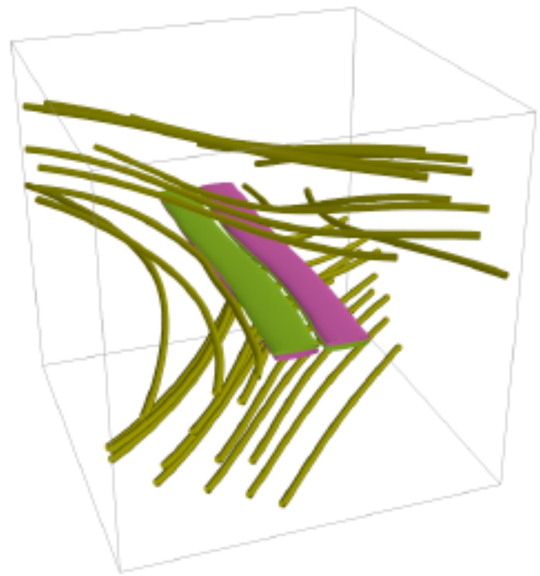


Only the portion of the simulation box Y<0, are shown in the results.

3D simulation



Turbulent energy



Turbulent cross helicity

Mean fields

$$\frac{\partial\overline{\rho}}{\partial t}+\nabla\cdot(\overline{\rho}\mathbf{U})=-\nabla\cdot\langle\rho'\mathbf{u}'\rangle$$

$$\frac{\partial}{\partial t}\overline{\rho}U^\alpha+\frac{\partial}{\partial x^a}\overline{\rho}U^aU^\alpha=-\frac{\partial P}{\partial x^\alpha}+\frac{\partial}{\partial x^\alpha}\mu\mathcal{S}^{a\alpha}+(\mathbf{J}\times\mathbf{B})^\alpha$$
$$-\frac{\partial}{\partial x^a}\left(\overline{\rho}\left\langle u'^a u'^\alpha\right\rangle-\frac{1}{\mu_0}\left\langle b'^a b'^\alpha\right\rangle+U^a\left\langle\rho' u'^\alpha\right\rangle+U^\alpha\left\langle\rho' u'^a\right\rangle\right)+R_U^\alpha,$$
$$\left[R_U^\alpha=-\frac{\partial}{\partial t}\langle\rho' u'^\alpha\rangle-\frac{\partial}{\partial x^a}\langle\rho' u'^a u'^\alpha\rangle-\frac{1}{2\mu_0}\frac{\partial}{\partial x^\alpha}\langle\mathbf{b}'^2\rangle\right]$$

$$\frac{\partial}{\partial t}\left(\frac{P}{\gamma_\mathrm{s}-1}+\frac{1}{2}\overline{\rho}\mathbf{U}^2+\frac{1}{2\mu_0}\mathbf{B}^2+\frac{1}{2}\overline{\rho}\left\langle\mathbf{u}'^2\right\rangle+\frac{1}{2\mu_0}\left\langle\mathbf{b}'^2\right\rangle+\langle\rho'\mathbf{u}'\rangle\cdot\mathbf{U}\right)$$
$$=-\nabla\cdot\left[\left(\frac{\gamma_\mathrm{s}}{\gamma_\mathrm{s}-1}P+\frac{1}{2}\overline{\rho}\mathbf{U}^2+\frac{1}{2}\overline{\rho}\langle\mathbf{u}'^2\rangle+\langle\rho'\mathbf{u}'\rangle\cdot\mathbf{U}\right)\mathbf{U}\right.$$
$$\left.+\left\langle\left(\frac{\gamma_\mathrm{s}}{\gamma_\mathrm{s}-1}p'+\overline{\rho}\mathbf{U}\cdot\mathbf{u}'+\frac{1}{2}\rho'\mathbf{U}^2\right)\mathbf{u}'\right\rangle+\frac{\mathbf{E}\times\mathbf{B}}{\mu_0}\right]+R_E,$$
$$\left[R_E=-\frac{\partial}{\partial t}\langle\rho'\mathbf{u}'^2\rangle-\nabla\cdot\left(\frac{1}{2}\langle\rho'\mathbf{u}'^2\rangle\mathbf{U}+\frac{1}{2}\overline{\rho}\langle\mathbf{u}'^2\mathbf{u}'\rangle+\langle\rho'\mathbf{u}'\cdot\mathbf{U}\mathbf{u}'\rangle+\frac{1}{2}\langle\rho'\mathbf{u}'^2\mathbf{u}'\rangle+\frac{\langle\mathbf{e}'\times\mathbf{b}'\rangle}{\mu_0}\right)\right]$$

$$\frac{\partial\mathbf{B}}{\partial t}=-\nabla\times\mathbf{E}$$

$$\mathbf{E}=-\mathbf{U}\times\mathbf{B}+\eta\mathbf{J}-\mathbf{E}_\mathrm{M}$$

$$\mathcal{S}^{\alpha\beta}=\frac{\partial U^\beta}{\partial x^\alpha}+\frac{\partial U^\alpha}{\partial x^\beta}-\frac{2}{3}\nabla\cdot\mathbf{U}\delta^{\alpha\beta}$$

Turbulent statistical quantities

$$\frac{D}{Dt}\left\langle \rho'^2 \right\rangle = -2\left\langle \rho' \mathbf{u}' \right\rangle \cdot \nabla\overline{\rho} - 2\left\langle \rho'^2 \right\rangle \nabla\cdot\mathbf{U}$$

$$\frac{D}{Dt}\frac{1}{2}\left\langle \mathbf{b}'^2 \right\rangle = -\frac{\mu_0}{2}\left\langle \mathbf{u}'\times\mathbf{b}' \right\rangle\cdot\mathbf{J} + \left\langle b'^a b'^b \right\rangle \frac{\partial U^a}{\partial x^b} - \frac{1}{2}\left\langle u'^a b'^b \right\rangle \left(\frac{\partial B^b}{\partial x^a} + \frac{\partial B^a}{\partial x^b}\right)$$
$$- \left\langle \mathbf{b}'^2 \right\rangle \nabla\cdot\mathbf{U} - \varepsilon_b + T_b$$

$$\frac{D}{Dt}\frac{1}{2}\left\langle \mathbf{u}'^2 \right\rangle = -\frac{1}{2\overline{\rho}}\left\langle \mathbf{u}'\times\mathbf{b}' \right\rangle\cdot\mathbf{J} - \left\langle u'^a u'^b \right\rangle \frac{\partial U^a}{\partial x^b} + \frac{1}{2\mu_0\overline{\rho}}\left\langle u'^a b'^b \right\rangle \left(\frac{\partial B^b}{\partial x^a} + \frac{\partial B^a}{\partial x^b}\right)$$
$$- (\gamma_\mathrm{s} - 1)\frac{1}{\overline{\rho}}\left(\left\langle \rho'\mathbf{u}' \right\rangle\cdot\nabla Q + \left\langle q'\mathbf{u}' \right\rangle\cdot\nabla\overline{\rho}\right) - \frac{1}{\overline{\rho}}\left\langle \rho'\mathbf{u}' \right\rangle\cdot\frac{D\mathbf{U}}{Dt} - \varepsilon_u + T_u$$

$$\frac{D}{Dt}\left\langle \mathbf{u}'\cdot\mathbf{b}' \right\rangle = -\left\langle \mathbf{u}'\times\mathbf{b}' \right\rangle\cdot\mathbf{\Omega} - \frac{1}{2}\left\langle u'^a u'^b - \frac{1}{\mu_0\overline{\rho}} b'^a b'^b \right\rangle \left(\frac{\partial B^b}{\partial x^a} + \frac{\partial B^a}{\partial x^b}\right) + \mathbf{B}\cdot\nabla\left\langle \frac{1}{2}\mathbf{u}'^2 \right\rangle$$
$$- (\gamma_\mathrm{s} - 1)\frac{1}{\overline{\rho}}\left(\left\langle \rho'\mathbf{b}' \right\rangle\cdot\nabla Q + \left\langle q'\mathbf{b}' \right\rangle\cdot\nabla\overline{\rho}\right)$$
$$- \left\langle \mathbf{u}'\cdot\mathbf{b}' \right\rangle \nabla\cdot\mathbf{U} - \frac{1}{\overline{\rho}}\left\langle \rho'\mathbf{b}' \right\rangle\cdot\frac{D\mathbf{U}}{Dt} - \varepsilon_W + T_W$$

$$\frac{D\varepsilon}{Dt} = C_{\varepsilon 1}\frac{\varepsilon}{K}P_K - C_{\varepsilon 2}\frac{\varepsilon}{K}\varepsilon + \nabla\cdot\left(\frac{\nu_\mathrm{K}}{\sigma_\varepsilon}\nabla\varepsilon\right)$$

Mean induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B} + \underline{\mathbf{E}_{\mathrm{M}}}) + \eta \nabla^2 \mathbf{B}$$

Turbulence

$$\mathbf{E}_{\mathrm{M}} = -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \mathbf{\Omega}$$

$$\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}, \quad \mathbf{J} = \mathbf{J}_0 + \delta \mathbf{J}$$

Reference $$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}_0) + \nabla \times (\alpha \mathbf{B}_0 - \beta \nabla \times \mathbf{B}_0)$$

Modulation $$\frac{\partial \delta \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \delta \mathbf{B}) - \nabla \times \left[\beta \nabla \times \left(\delta \mathbf{B} - \frac{\gamma}{\beta} \mathbf{U} \right) \right]$$

$$\longrightarrow \quad \delta \mathbf{B} = \frac{\gamma}{\beta} \mathbf{U} = C_{\mathrm{W}} \frac{W}{K} \mathbf{U}$$

$$\frac{|W|}{K} = \frac{|\langle \mathbf{u}' \cdot \mathbf{b}' \rangle|}{\langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle / 2} \leq 1$$

c.f. $$\nabla \times \left(\frac{\gamma}{\beta} \mathbf{U} \right) = \frac{\gamma}{\beta} \nabla \times \mathbf{U} + \nabla \left(\frac{\gamma}{\beta} \right) \times \mathbf{U}$$

Mean momentum equation

$$\frac{\partial \mathbf{U}}{\partial t} = \mathbf{U} \times \boldsymbol{\Omega} + \mathbf{J} \times \mathbf{B} \underline{- \nabla \cdot \boldsymbol{\mathcal{R}}} + \mathbf{F} - \nabla \left(P + \frac{1}{2}\mathbf{U}^2 + \left\langle \frac{1}{2}\mathbf{b}'^2 \right\rangle \right)$$

Turbulence

$$\mathbf{J} = \sigma \left(\mathbf{E} + \mathbf{U} \times \mathbf{B} + \underline{\mathbf{E}_\mathrm{M}} \right)$$

Turbulence

$$\begin{cases} \mathcal{R}^{\alpha\beta} = \frac{2}{3} K_\mathrm{R} \delta^{\alpha\beta} - \nu_\mathrm{K} \mathcal{S}^{\alpha\beta} + \nu_\mathrm{M} \mathcal{M}^{\alpha\beta} \\ \mathbf{E}_\mathrm{M} = -\beta \mathbf{J} + \alpha \mathbf{B} + \gamma \boldsymbol{\Omega} \end{cases}$$

Mean Lorentz force $\mathbf{J} \times \mathbf{B} = \frac{1}{\beta} (\mathbf{U} \times \mathbf{B}) \times \mathbf{B} + \frac{\gamma}{\beta} \boldsymbol{\Omega} \times \mathbf{B} - \frac{1}{\beta} \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla \varphi \right) \times \mathbf{B}$

$$\mathbf{U} = \mathbf{U}_0 + \delta \mathbf{U}, \quad \boldsymbol{\Omega} = \boldsymbol{\Omega}_0 + \delta \boldsymbol{\Omega}$$

Reference $\frac{\partial \boldsymbol{\Omega}_0}{\partial t} = \nabla \times \left[\mathbf{U}_0 \times \boldsymbol{\Omega}_0 + \nu_\mathrm{K} \nabla^2 \mathbf{U}_0 + \mathbf{F} - \frac{1}{\beta} \left(\frac{\partial \mathbf{A}}{\partial t} + \nabla \varphi \right) \times \mathbf{B} \right]$

Modulation $\frac{\partial \delta \boldsymbol{\Omega}}{\partial t} = \nabla \times \left[\left(\delta \mathbf{U} - \frac{\gamma}{\beta} \mathbf{B} \right) \times \boldsymbol{\Omega}_0 + \nu_\mathrm{K} \nabla^2 \left(\delta \mathbf{U} - \frac{\gamma}{\beta} \mathbf{B} \right) \right]$

$\longrightarrow$ $\delta \mathbf{U} = \frac{\gamma}{\beta} \mathbf{B} = C_\gamma \frac{W}{K} \mathbf{B}$ $\qquad \frac{|W|}{K} = \frac{|\langle \mathbf{u}' \cdot \mathbf{b}' \rangle|}{\langle \mathbf{u}'^2 + \mathbf{b}'^2 \rangle / 2} \leq 1$