Analysis of droplet motion

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In this talk we study the quality and features of a family of approximations to the solution of the unsteady Navier-Stokes equation for an unbounded domain with a finite number of spherical particles in the domain. This problem has been traditionally approximated by embedding Stokian particles in the domain [WAKG05]. Our initial motivation is to discuss how different approximations to the steady Navier-Stokes equation for a sphere in an unbounded domain can be used and how it affects the global approximation. More particularly we would like to determine if embedding uniformly valid approximations taking into account the spherical particles in the domain is mathematically consistent with using Stokian particles. However, we decide to provide a general framework for how different approximations of this form capture the solution of the original problem.

We define the framework as the family of approximations defined by the sum of a background velocity field and a linear combination of velocity fields. This is mathematically translated to the following: $\mathbf{v}(x; t) \approx \mathbf{U}(x; t) + \Sigma_i u^{(i)} (\mathbf{x} - \mathbf{x}_i(t))$, where $\mathbf{v}(\mathbf{x}, t)$ is the solution of the original problem and $\mathbf{U}(\mathbf{x}, t)$ and $\mathbf{u}^{(i)} (\mathbf{x} - \mathbf{x}_i(t))$ are the two pieces of our approximation. The velocity field used for the linear combination comes from an approximation to the steady Navier-Stokes equation for a sphere in an unbounded domain. We initially consider the case that the background velocity field is not affected by the presence of the particles, called one-way coupling [Sto96, Lot00, Mic03]. This assumption provides us great insight into how well we are solving the original problem and how the approximation can be improved. Based on this result, we theoretically analyze an extension of the initial model that lets the particles interact with the background fluid flow, called two-way coupling [Sto96, Lot00, Mic03]

This work has been inspired by [KD73] and [Max83]. They mathematically carefully develop how droplets behave in a background fluid flow. Since then, there have been great interest [Mil77, GS80, BW82, MW84, Lov93, Wan93, MKL94, Sto96, RM97, FM98, KES98, Lot00, Mic03, Pie04, HZ05, AR06, LD09, RMSRSGG11, HL11], to mention a few authors.

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